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Energy Conditions and Constraints on the Generalized Non-Local Gravity Model *

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We study and derive the energy conditions in generalized non-local gravity, which is the modified theory of general relativity obtained by adding a term $m^{2n-2}R\Box^{-n}R$ to the Einstein–Hilbert action. Moreover, to obtain some insight on the meaning of the energy conditions, we illustrate the evolutions of four energy conditions with the model parameter ε for different n. By analysis we give the constraints on the model parameters ε.

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According to recent observational data sets,[1–4] it has been believed that our universe is flat and undergoing a phase of accelerating expansion at the present time. Explaining this problem is a challenge of modern cosmology. As we know, the Einstein equation governs the interplay between the geometry of the spacetime and the matter distribution. Different ways of addressing the problem of accelerating expansion can be classified as the gravity side or the matter side of the Einstein equation. On the one hand, it is reasonable to believe that the acceleration of the Universe is probably driven by dark energy, which is an exotic component with negative pressure and provides the main contribution to the energy budget of the universe today. The simplest candidate for the dark energy is the cosmological constant Λ,[5,6] but the cosmological observations indicate that the dark energy may not be a constant. Several candidates for dark energy have been extensively investigated.[7–12] Unfortunately, a satisfactory answer to the questions concerning what dark energy is and where it comes from has not yet been obtained. On the other hand, as an alternative to dark energy, modified theories of gravity are extremely attractive (see Refs. [13–15] for reviews). The challenge is to theoretically construct a consistent theory more effectively explaining the acceleration data, and the significant deviations from general relativity (GR) are neglected by the data inside the solar system. There are numerous ways to modify Einstein's theory of GR. Among these theories, $f(R)$ gravity is one of the competitive candidates in modified theories of gravity,[16,17] in which $f(R)$ is an arbitrary function of the Ricci scalar R. Inspired by Ref. [18], another interesting alternative modified theory of gravity is the non-local gravity model proposed by Deser and Woodard.[19] The model is constructed by adding a non-local term of $f(\Box^{-1}R)$ to the Einstein–Hilbert action, where the $\Box^{-1}$ operator is a formal inverse of the d'Alembertian $\Box$ in the scalar representation, which can be expressed as the convolution with a retarded Green function.[20–24] The form of the non-local distortion function $f(\Box^{-1}R)$ can be chosen to reproduce the ΛCDM background cosmology exactly,[25,26] and the relief of the cosmological constant problem in nonlocal gravity theory has been studied in Refs. [27,28]. The Ricci scalar R vanishes during radiation dominance and $\Box^{-1}R$ cannot begin to grow until the onset of matter dominance while its growth becomes logarithmic. Moreover, the great advantage of this class of models is to trigger late time acceleration by the transition from radiation domination. This theory has attracted some attention theoretically and phenomenologically, as a possible alternative to dark energy that presents the universe as accelerated at late times. The model can be compared with observations, but fails miserably to account for structure formation data. Furthermore, another new non-local model has been proposed by Maggiore and Mancarella, which is obtained by adding a specific form $m^2\Box^{-2}R$ to the Einstein–Hillbert action.[29] A natural way to proceed is to introduce a mass scale m which is in the order of the Hubble parameter present value H_0. In contrast with the Deser–Woodard model, the non-local term is controlled by a mass parameter m. Currently this model is receiving more attention,[30–32] because it can be compared with a wide set of observations,[33–37] and it is so far the only model which is as good as ΛCDM, using the same number of free parameters. Recently, we extend the model to the generalized non-local gravity, which is defined by the action[38]

$$S_{\rm GNL}=\frac{1}{16\pi G}\int d^{d+1}x\sqrt{-g}R\Big(1-\lambda\frac{m^{2n-2}}{\Box^n}R\Big),\quad(1)$$

where n takes integer ($n\geq 2$), $\lambda=(d-1)/4d$ is the normalized coefficient, and d is the number of spatial dimensions. The convenient normalization of the mass parameter m which is in the order of the Hubble parameter present value H_0. A natural way to proceed is to set the mass scale $m=\varepsilon H_0$, where ε is a constant, which is called the model parameter. Thus when con-

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sidering four-dimensional spacetime (i.e., $d = 3$), the action (1) can be rewritten as

$$S_{\rm GNL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \cdot\Big(1-\frac{H_0^{2n-2}}{6}\varepsilon^{2n-2}\Box^{-n}R\Big). \quad (2)$$

On the other hand, since many models of modified theories of gravity have been proposed, it gives rise to the problem how to constrain them from theoretical aspects. One possibility is by imposing the so-called energy conditions,[39–44] namely, the strong energy condition (SEC), the null energy condition (NEC), the dominant energy condition (DEC) and the weak energy condition (WEC). In this study, we tackle the problem of the energy conditions in generalized non-local gravity. Considering these energy conditions, one is allowed not only to establish gravity which remains attractive, but also to keep the demands that the energy density is positive and cannot flow faster than light. This issue is extremely delicate since a standard approach is to consider the gravitational field equations as effective Einstein equations. To obtain the SEC and NEC, the Raychaudhuri equation which is their physical origin is used. It is worth stressing that the equivalent results can be obtained by taking the transformations $\rho \to \rho^{\rm eff}$ and $p \to p^{\rm eff}$ into $\rho + 3p \geqslant 0$ and $\rho + p \geqslant 0$, respectively. Thus by extending this approach to $\rho - p \geqslant 0$ and $\rho \geqslant 0$, the DEC and WEC are obtained. As is well known, at present the energy conditions have been used in various modified gravity theories widely.[45–47] Thus we wonder if the energy conditions for the generalized non-local gravity model as well as the constraints on the model could be obtained, which is our motivation in this work. The research results show that the energy conditions in generalized non-local gravity can be obtained, which can degenerate to the ones in GR as special cases. Moreover, the constraints on the model parameter ε can be given by means of the SEC, the NEC and DEC.

We consider the generalized non-local gravity model given by the action (2), where the covariant scalar d'Alembertian is

$$\Box \equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu). \quad (3)$$

In general, the EOM contains both retarded and advanced Green's function, because the variation of retarded operator $\Box_{\rm R}^{-1}$ always produces the advanced operator $\Box_A^{-1}$.[48] For the convenience of the discussions, following the process in Refs. [24,32] the nonlocal term was solved by the retarded Green function

$$(\Box^{-1}R)(x) = \int d^4x\sqrt{-g(x')}G(x,x')R(x'), \quad (4)$$

where $G(x,x')$ is the Green function of the $\Box^{-1}$ operator acting on scalar. We consider that the non-local effect only starts at cosmic time t_0, thus we must impose the boundary conditions. A coordinate system can be chosen in which the initial conditions take the simplest form

$$G(x,x')|_{x^0=t_0} = 0,\ \partial_0 G(x,x')|_{x^0=t_0} = 0. \quad (5)$$

For such a $G(x,x')$ we choose an initial time to lie in the deep radiation dominated period. Indeed, during radiation dominated period $R = 0$, which accords with the condition in general relativity. Taking the variation of the action (2) with respect to the metric $g^{\mu\nu}$, we obtain the modified Einstein equations

$$G_{\mu\nu} - \frac{H_0^{2n-2}}{6}\varepsilon^{2n-2}K_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (6)$$

with

$$\begin{aligned} K_{\mu\nu} = {} & \Big[2(G_{\mu\nu} - \nabla_\mu\nabla_\nu + g_{\mu\nu}\Box) + \frac{1}{2}g_{\mu\nu}R\Big](\Box^{-n}R) \\ & + \sum_{l=0}^{n-1}\Big\{\nabla_\nu(\Box^{l-n}R)\nabla_\mu(\Box^{-l-1}R) \\ & - \frac{1}{2}g_{\mu\nu}[\nabla_\sigma(\Box^{l-n}R)\nabla^\sigma(\Box^{-l-1}R) \\ & + (\Box^{l-n}R)(\Box^{-l}R)]\Big\}, \end{aligned} \quad (7)$$

where $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$, $K_{\mu\nu}$ comes from varying the non-local term in the above action, and the energy-momentum tensor of matter $T_{\mu\nu}$ is defined by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\rm M})}{\delta g^{\mu\nu}}. \quad (8)$$

We need to rewrite the original generalized non-local gravity model as a local form. Following the same definitions and localization procedure in Ref. [29], we introduce the following auxiliary scalar fields $U_1, \ldots, U_n$,

$$U_1 = -\Box^{-1}R, \quad (9)$$

$$U_i = -\Box^{-1}U_{i-1} = (-1)^i\Box^{-i}R,\ (i = 2, \ldots, n), \quad (10)$$

whose initial conditions all are zero. For convenience, we set a set of variables $V_i = H_0^2U_i$. One reads a set of coupled differential equations

$$U_1'' + U_1'(3+\zeta) = 6(2+\zeta), \quad (11)$$

$$V_i'' + V_i'(3+\zeta) = h^{-2}U_{i-1}, \quad (12)$$

where $h = H(t)/H_0$ with $H(t) = \dot{a}/a$ and H_0 being the present value of the Hubble parameter, $\zeta = h'/h$, and a prime denotes the derivative with respect to the time coordinate $x = \ln a$.

We consider a flat FRW background with metric

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2. \quad (13)$$

From the (00) component of the modified Einstein equation (6), we obtain

$$K_{00} = (-1)^n\Big[6h^2V_n' + 6h^2V_n - \frac{1}{2}h^2\Theta_1 + \frac{1}{2}\Theta_2\Big], \quad (14)$$

where

$$\Theta_1 = \sum_{l=0}^{n-1} V'_{n-l} U'_{l+1}, \tag{15}$$

$$\Theta_2 = \sum_{l=1}^{n-1} U_{n-l} U_l, \tag{16}$$

as well as

$$\Theta'_1 = \sum_{l=0}^{n-1} [-2(3+\zeta) V'_{n-l} U'_{l+1} - h^{-2}(U_{n-l-1} U'_{l+1} + U'_{n-l} U_l)], \tag{17}$$

$$\Theta'_2 = \sum_{l=1}^{n-1} (U'_{n-l} U_l + U_{n-l} U'_l). \tag{18}$$

Considering these definitions, one reads an effective dark energy density $\rho_{\rm DE} = \rho_0 \beta Y$, where $\rho_0 = 3H_0^2/(8\pi G)$, $\beta = \frac{H_0^{2n-4}}{9}\varepsilon^{2n-2}$ and

$$Y = (-1)^n \Big(3h^2 V'_n + 3h^2 V_n - \frac{1}{4} h^2 \Theta_1 + \frac{1}{4}\Theta_2\Big). \tag{19}$$

Thus we obtain

$$h^2 = \frac{\Omega_{\rm M} e^{-3x} + \Omega_{\rm R} e^{-4x} + \frac{1}{4}\beta(-1)^n \Theta_2}{1 - \beta(-1)^n (3V'_n + 3V_n - \frac{1}{4}\Theta_1)}, \tag{20}$$

and

$$\zeta = \Big[h^{-2}(-3\Omega_{\rm M} e^{-3x} - 4\Omega_{\rm R} e^{-4x}) - 3\beta(-1)^n \Big(- h^{-2} U_{n-1} + 4V'_n - \frac{1}{2}\Theta_1\Big)\Big] / \{2[1 - 3\beta(-1)^n V_n]\}, \tag{21}$$

where $\Omega_{\rm M}$ and $\Omega_{\rm R}$ are the fractional energy densities of matter and radiation, respectively. Also, with ζ and h^2, Eqs. (11) and (12) provide a closed set of second-order differential equations for $V_i = H_0^2 U_i$, $(i = 2, \ldots, n)$, whose numerical integration is straightforward.

We consider the energy conditions in the generalized non-local gravity. When n is taken as an arbitrary interger $(n \geq 2)$, Eq. (6) may be rewritten as the following effective gravitational field equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T^{\rm eff}_{\mu\nu}, \tag{22}$$

where

$$T^{\rm eff}_{\mu\nu} = T_{\mu\nu} + (-1)^n \frac{H_0^{2n-2}}{48\pi G}\varepsilon^{2n-2}\Big\{\Big[2(G_{\mu\nu} - \nabla_\mu\nabla_\nu + g_{\mu\nu}\Box) + \frac{1}{2} g_{\mu\nu} R\Big] U_n - \sum_{l=0}^{n-1}\Big[\nabla_\nu U_{n-l} \nabla_\mu U_{l+1} - \frac{1}{2} g_{\mu\nu}(\nabla_\sigma U_{n-l} \nabla^\sigma U_{l+1} - U_{n-l} U_l)\Big]\Big\}. \tag{23}$$

Moreover,

$$\rho^{\rm eff} = -T^{\rm eff}_{00} g^{00} = \rho + (-1)^n \frac{H_0^{2n-4}\rho_0}{18}\varepsilon^{2n-2}\Big(6h^2 V_n + 6h^2 V'_n - \frac{1}{2} h^2 \Theta_1 + \frac{1}{2}\Theta_2\Big), \tag{24}$$

$$p^{\rm eff} = \frac{1}{3} T^{\rm eff}_{ii} g^{ii} = p + (-1)^n \frac{H_0^{2n-4}\rho_0}{18}\varepsilon^{2n-2}\Big(- 4h^2\zeta V_n - 6h^2 V_n + 2h^2 V'_n - 2U_{n-1} - \frac{1}{2} h^2 \Theta_1 - \frac{1}{2}\Theta_2\Big), \tag{25}$$

where the energy density $\rho = \rho_{\rm M} + \rho_{\rm R}$, $p = p_{\rm R}$ and $\rho_0 = 3H_0^2/(8\pi G)$.

Note that the SEC and NEC are directly derived from the Raychaudhuri equation and the equivalent expressions can be obtained by taking the transformations $\rho \to \rho^{\rm eff}$ and $p \to p^{\rm eff}$ into $\rho + 3p \geqslant 0$ and $\rho + p \geqslant 0$, respectively.[40–47] By extending this approach to $\rho - p \geqslant 0$ and $\rho \geqslant 0$, we can give the corresponding DEC and WEC in the generalized non-local gravity. Hence, the four energy conditions, i.e., SEC, NEC, DEC, WEC, in generalized non-local gravity can be, respectively, given as

$$\begin{aligned}
{\rm SEC}: \ & \rho^{\rm eff} + 3p^{\rm eff} \geq 0 \Rightarrow \Omega_{\rm M} + \Omega_{\rm R}(1 + 3w_{\rm R}) + (-1)^n \frac{H_0^{2n-4}}{18}\varepsilon^{2n-2}(-12h^2\zeta V_n - 12h^2 V_n + 12h^2 V'_n - 6U_{n-1} - 2h^2\Theta_1 - \Theta_2) \geq 0, && (26)\\
{\rm NEC}: \ & \rho^{\rm eff} + p^{\rm eff} \geq 0 \Rightarrow \Omega_{\rm M} + \Omega_{\rm R}(1 + w_{\rm R}) + (-1)^n \frac{H_0^{2n-4}}{18}\varepsilon^{2n-2}(-4h^2\zeta V_n + 8h^2 V'_n - 2U_{n-1} - h^2\Theta_1) \geq 0, && (27)\\
{\rm DEC}: \ & \rho^{\rm eff} - p^{\rm eff} \geq 0 \Rightarrow \Omega_{\rm M} + \Omega_{\rm R}(1 - w_{\rm R}) + (-1)^n \frac{H_0^{2n-4}}{18}\varepsilon^{2n-2}(4h^2\zeta V_n + 12h^2 V_n + 4h^2 V'_n + 2U_{n-1} + \Theta_2) \geq 0, && (28)\\
{\rm WEC}: \ & \rho^{\rm eff} \geq 0 \Rightarrow \Omega_{\rm M} + \Omega_{\rm R} + (-1)^n \frac{H_0^{2n-4}}{18}\varepsilon^{2n-2}\Big(6h^2 V_n + 6h^2 V'_n - \frac{1}{2} h^2\Theta_1 + \frac{1}{2}\Theta_2\Big) \geq 0. && (29)
\end{aligned}$$

From the inequalities (26)–(29), we can easily find that the four energy conditions are functions of the time coordinate $x = \ln a$, the model parameter ε as well as n. We consider the value range of x, $x \in [-2, 1]$, i.e., the redshift $z \in [-0.632, 6.389]$, and for the fixed values of x and n, there are different evolutional curves of energy conditions with the model parameter ε. In Fig. 1, for simplicity, we take $n = 2, \ldots, 8$, $\Omega_{\rm M} = 0.308$, $\Omega_{\rm R} = 0.001$ and

$w_{\rm R} = 1/3$.[4] We only plot the evolutional curves corresponding to $x = 1$, i.e., $z = -0.632$, which have the largest deviations from the peak values of the curves, and can give the tightest constraints on the model parameter ε. From Fig. 1 it is not difficult to see that for the different even numbers n, the SEC and the NEC can give the constraints on the model parameters ε, respectively. However, for the different odd numbers n the DEC gives the constraints on the model parameters ε. In addition, Table 1 lists the constraints on the model parameters ε for different numbers of n. Here the symbol $-$ stands for no limits to the model parameter ε. From Table 1 we can easily see that when taking the even numbers of n, the larger the n is, the wider the range of ε is, which is determined by the SEC and the NEC, while the DEC and WEC cannot give any constraints on the model parameter ε. In other words, the DEC and WEC are always satisfied in any value ranges of the model parameter ε. However, when taking the odd numbers of n, the DEC constrains almost the same value ranges on ε in this model, while three other energy conditions (i.e., SEC, NEC and WEC) cannot give any constraints on the model parameter ε.

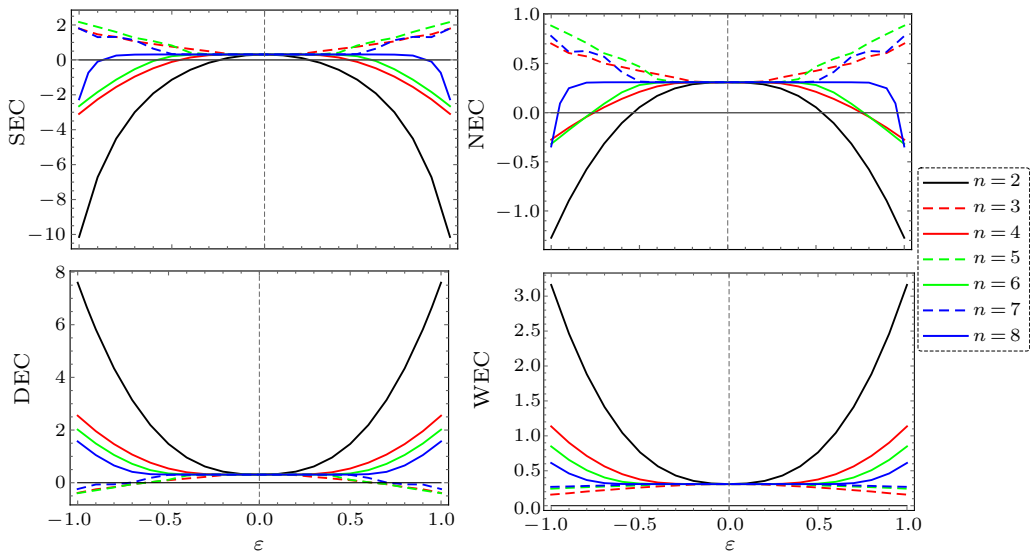

Fig. 1. The evolutions of SEC, NEC, DEC, WEC with the model parameter ε for different n ($n = 2, \ldots, 8$).

Table 1. The constraints on the model parameter ε by the four energy conditions for different n ($n = 2, \ldots, 8$).

	SEC	NEC	DEC	WEC	SEC&NEC&DEC&WEC
$n = 2$	$\lvert\varepsilon\rvert \leq 0.26$	$\lvert\varepsilon\rvert \leq 0.53$	$-$	$-$	$\lvert\varepsilon\rvert \leq 0.26$
$n = 3$	$-$	$-$	$\lvert\varepsilon\rvert \leq 0.63$	$-$	$\lvert\varepsilon\rvert \leq 0.63$
$n = 4$	$\lvert\varepsilon\rvert \leq 0.46$	$\lvert\varepsilon\rvert \leq 0.75$	$-$	$-$	$\lvert\varepsilon\rvert \leq 0.46$
$n = 5$	$-$	$-$	$\lvert\varepsilon\rvert \leq 0.62$	$-$	$\lvert\varepsilon\rvert \leq 0.62$
$n = 6$	$\lvert\varepsilon\rvert \leq 0.58$	$\lvert\varepsilon\rvert \leq 0.77$	$-$	$-$	$\lvert\varepsilon\rvert \leq 0.58$
$n = 7$	$-$	$-$	$\lvert\varepsilon\rvert \leq 0.72$	$-$	$\lvert\varepsilon\rvert \leq 0.72$
$n = 8$	$\lvert\varepsilon\rvert \leq 0.88$	$\lvert\varepsilon\rvert \leq 0.96$	$-$	$-$	$\lvert\varepsilon\rvert \leq 0.88$

In summary, we have derived the well known strong energy condition, the null energy condition, the dominant energy condition and the weak energy condition for the generalized non-local gravity model, which are obtained by adding a term $m^{2n-2}R\Box^{-n}R$ to the Einstein–Hilbert action. Moreover, to obtain some insights on the meaning of the energy conditions, we have illustrated the evolutions of the four energy conditions in terms of the model parameter ε for the fixed value of $x = 1$ (i.e., $z = -0.632$) and different n in Fig. 1, and as listed in Table 1 we have given the tightest constraints on the model parameters ε for different n in the generalized non-local gravity model satisfying the SEC, NEC, DEC and WEC, respectively. From Fig. 1 it is not difficult to see that for different even numbers n, the SEC and the NEC can give the constraints on the model parameters ε, respectively. However, for different odd numbers n, the DEC gives the constraints on the model parameters ε. From Table 1 we can easily see that when taking the even numbers of n, the larger the n is, the wider the range of ε is, which is determined by the SEC and the NEC, while the DEC and WEC cannot give any constraints on the model parameter ε. However, when taking the odd numbers of n, the DEC constrains almost the same value ranges on ε in this model, while SEC, NEC and WEC cannot give any constraints on the model parameter ε. It is worth stressing that as discussed in Ref. [49], in classical general relativity the SEC must be violated sometime between the epoch of galaxy formation and the present, because all normal matter satisfies the SEC. In this study the generalized non-local term acts the role of an effective dark energy. Thus the SEC in the generalized non-local

is not in confliction with the one in classical general relativity.

References

[1] Perlmutter S et al 1999 *Astrophys. J.* **517** 565
[2] Riess A G et al 1998 *Astron. J.* **116** 1009
[3] Hinshaw G et al 2013 *Astrophys. J. Suppl. Ser.* **208** 19
[4] Ade P A R et al 2016 *Astron. Astrophys.* **594** A13
[5] Sahni V and Starobinsky A A 2000 *Int. J. Mod. Phys.* D **9** 373
[6] Carroll S M 2001 *Living Rev. Relativ.* **4** 1
[7] Peebles P J E and Ratra B 2003 *Rev. Mod. Phys.* **75** 559
[8] Copeland E J, Sami M and Tsujikawa S 2006 *Int. J. Mod. Phys.* D **15** 1753
[9] Li M, Li X D, Wang S and Wang Y 2011 *Commun. Theor. Phys.* **56** 525
[10] Zeng X X, Hu X Y and Li L F 2017 *Chin. Phys. Lett.* **34** 010401
[11] Wu Y B, Tong H D, Yang H, Lu J B, Zhao Y Y, Lu J W and Zhang X 2014 *Chin. Phys. Lett.* **31** 029801
[12] Lu J B, Wu Y B, Xu L X and Wang Y T 2011 *Chin. Phys.* B **20** 079801
[13] Nojiri S and Odintsov S D 2011 *Phys. Rep.* **505** 59
[14] Clifton T, Ferreira P G, Padilla A and Skordis C 2012 *Phys. Rep.* **513** 1
[15] Capozziello S and Laurentis M D 2011 *Phys. Rep.* **509** 167
[16] Sotiriou T P and Faraoni V 2010 *Rev. Mod. Phys.* **82** 451
[17] Felice A D and Tsujikawa S 2010 *Living Rev. Relativ.* **13** 3
[18] Arkani-Hamed N, Dimopoulos S, Dvali G and Gabadadze G 2002 arXiv:hep-th/0209227
[19] Deser S and Woodard R P 2007 *Phys. Rev. Lett.* **99** 111301
[20] Koshelev N A 2009 *Grav. Cosmol.* **15** 220
[21] Koivisto T S 2010 *AIP Conf. Proc.* **1206** 79
[22] Barvinsky A O 2012 *Phys. Lett.* B **710** 12
[23] Barvinsky A O 2012 *Phys. Rev.* D **85** 104018
[24] Maggiore M 2014 *Phys. Rev.* D **89** 043008
[25] Koivisto T 2008 *Phys. Rev.* D **77** 123513
[26] Deffayet C and Woodard R P 2009 *J. Cosmol. Astropart. Phys.* **0908** 023
[27] Zhang Y L and Sasaki M 2012 *Int. J. Mod. Phys.* D **21** 1250006
[28] Nojiri S, Odintsov S D, Sasaki M and Zhang Y L 2011 *Phys. Lett.* B **696** 278
[29] Maggiore M and Mancarella M 2014 *Phys. Rev.* D **90** 023005
[30] Dirian Y, Foffa S, Khosravi N, Kunz M and Maggiore M 2014 *J. Cosmol. Astropart. Phys.* **06** 033
[31] Barreira A, Li B, Hellwing W A, Baugh C M and Pascoli S 2014 *J. Cosmol. Astropart. Phys.* **09** 031
[32] Dirian Y and Mitsou E 2014 *J. Cosmol. Astropart. Phys.* **10** 065
[33] Koivisto T S 2008 *Phys. Rev.* D **78** 123505
[34] Park S and Dodelson S 2013 *Phys. Rev.* D **87** 024003
[35] Jaccard M, Maggiore M and Mitsou E 2013 *Phys. Rev.* D **88** 044033
[36] Foffa S, Maggiore M and Mitsou E 2014 *Int. J. Mod. Phys.* A **29** 1450116
[37] Nesseris S and Tsujikawa S 2014 *Phys. Rev.* D **90** 024070
[38] Zhang X, Wu Y B, Li S, Liu Y C, Chen B H, Chai Y T and Shu S 2016 *J. Cosmol. Astropart. Phys.* **1607** 003
[39] Hawking S W and Ellis G F R 1973 *The Large Scale Structure of Space-Time* (Cambridge: Cambridge University Press)
[40] Santos J, Alcaniz J S, Reboucas M J and Carvalho F C 2007 *Phys. Rev.* D **76** 083513
[41] Garcia N M, Harko T, Lobo F S N and Mimoso J P 2011 *Phys. Rev.* D **83** 104032
[42] Liu D and Reboucas M J 2012 *Phys. Rev.* D **86** 083515
[43] Atazadeh K and Darabi F 2014 *Gen. Relativ. Gravitation* **46** 1664
[44] Sharif M and Zubair M 2013 *J. High Energy Phys.* **1312** 079
[45] Bertolami O and Sequeira M C 2009 *Phys. Rev.* D **79** 104010
[46] Wang J, Wu Y B, Guo Y X, Qi F, Zhao Y Y and Sun X Y 2010 *Eur. Phys. J.* C **69** 3
[47] Zhao Y Y, Wu Y B, Lu J B, Zhang Z, Han W L and Lin L L 2012 *Eur. Phys. J.* C **72** 1924
[48] Zhang Y L, Koyama K, Sasaki M and Zhao G B 2016 *J. High Energy Phys.* **1603** 039
[49] Visser M 1997 *Science* **276** 88