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Modelling Magnetoresistance Effect in Limited Anisotropic Semiconductors

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A macroscopic model of the magnetoresistance effect in limited anisotropic semiconductors is built. This model allows us to solve the problem of measurement of physical magnetoresistance components of crystals and films. Based on a unified mathematical model the method is worked out enabling us to measure tensor components of the specific electrical resistance and the relative magnetoresistance of anisotropic semiconductors simultaneously.

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Galvanomagnetic effects in semiconductors have attracted a great deal of interest since they were discovered. The following works contain the researches carried out in this field.[1–5] These works pay special attention to the Hall effect and the effect of magnetoresistance (Gauss effect), considering them to be the most informative phenomena. The peculiar feature of Hall and Gauss effects is their universalism, enabling us to study the properties of metals, semiconductors, composites and nanostructures.[1–6]

A number of additional side-effects appear while carrying out experimental research of Hall and Gauss effects. These side-effects must be analyzed and taken into account in the case of practical measurements. Quasi-Hall difference of potential,[7,8] concentration of current density in different fragments of the sample[8,9] and additional thermoelectromotive force caused by the tensor character of electroconductivity can be referred to such effects.[7,10,11]

To create the model of magnetoresistance in anisotropic semiconductor samples, we solve the problem of electric field distribution in the sample placed in a transverse electric field. Let us consider the flat square cut sample made in such a way that the main axes of the elecroconductivity tensor are directed along the x and y axes of the rectangular coordinate system (Fig. 1).

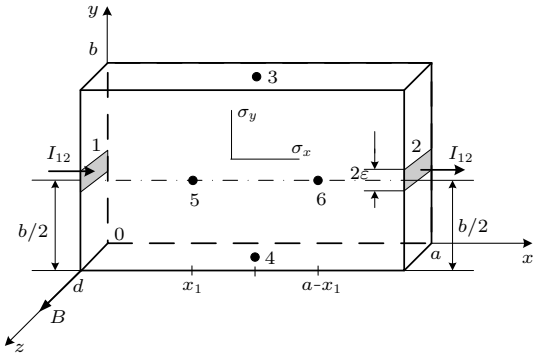

Fig. 1. The scheme of the location of current (1, 2), Hall (3, 4) contacts and the contacts for measurement of magnetoresistance components (5, 6) of the anisotropic sample placed into the magnetic field by dielectric displacement $\boldsymbol{B}$.

Direct current I_{12} is conveyed to the sample with the help of low-resistance electrodes 1, 2 with width 2ε,[8] the magnetic field dielectric displacement $\boldsymbol{B}$ is directed along the z axis (Fig. 1).

Under these conditions the electro-conductivity tensor of the material is[7,8,12]

$$\sigma_{\rm H} = \begin{pmatrix} \sigma_x & \sigma_x\sigma_y RB \\ -\sigma_x\sigma_y RB & \sigma_y \end{pmatrix}. \tag{1}$$

The condition of stationarity is[7,8,12]

$$(\nabla \cdot \boldsymbol{j}) = -(\nabla \cdot \sigma_{\rm H}[\nabla\varphi]) = 0. \tag{2}$$

The equations for the electric field potential in the semiconductor area are

$$\frac{\partial^2\varphi}{\partial x^2} + \gamma^2\frac{\partial^2\varphi}{\partial y^2} = 0,\ \gamma^2 = \frac{\sigma_y}{\sigma_x}. \tag{3}$$

The expressions of the constituents of the current density on the surface of the sample are

$$j_x = \sigma_x E_x + R_z\sigma_x\sigma_y B)E_y, \tag{4}$$
$$j_y = \sigma_y E_y - R_z\sigma_x\sigma_y B)E_x. \tag{5}$$

The Neumann boundary conditions for the potential in accordance with the scheme of the current electrode location (Fig. 1) are

$$\left[\frac{\partial\varphi}{\partial x} + \mu_y\frac{\partial\varphi}{\partial y}\right]|_{x=0,a} = \begin{cases} 0, & y \in [0, b/2-\varepsilon] \cup [b/2+\varepsilon, b], \\ -\frac{I_{12}\bar{\rho}_x}{2\varepsilon d}, & y \in [b/2-\varepsilon, b/2+\varepsilon], \end{cases} \tag{6}$$

$$\left[\frac{\partial\varphi}{\partial y} - \mu_x\frac{\partial\varphi}{\partial x}\right]|_{y=0,b} = 0, \tag{7}$$

where a, b and d are length, width and thickness of the sample, respectively, 2ε is the size of the current contacts, $\mu_x = \sigma_x R_z B$ and $\mu_y = \sigma_y R_z B$ are the tensor components of the Hall mobility of the charge carrier, R_z is the tensor component of the Hall quotient, $\bar{\rho}_x = \rho_{0x} + i\cdot\Delta\rho_x$ is the main component of the complex resistivity tensor, the imagery part of $\bar{\rho}_x$ is the physical magnetoresistance, ρ_{0x} is the main

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component of the resistivity tensor without the presence of the external magnetic field, $\sigma_x = \bar{\rho}_x/\rho_x^2$, and $\rho_x = \sqrt{\rho_{0x}^2 + \Delta\rho_{0x}^2}$. With formulation of the conditions (6) and (7), the requirement that the normal constituent of current density equals zero is met, except the points under the lateral electrodes where the current density is constant.

The general expression for the complex potential can be presented as a series[14]

$$\varphi(x,y) = \sum_{n=1}^{\infty} X_n(x) \cdot e^{i\alpha_n y}, \tag{8}$$

where $\alpha_n = \pi n/b$. The function X_n is determined by

$$\frac{\partial^2 X_n}{\partial x^2} + \beta_n^2 X_n = 0, \ \beta_n = \alpha_n \gamma, \tag{9}$$

and boundary conditions (6) and (7).

Let us put down the expression for electrical potential on the location line of the current contacts (Fig. 1), without mentioning lengthy mathematical calculations

$$\begin{aligned}\varphi(x)|_{y=b/2} &= -\frac{I_{12}\bar{\rho}_x}{db}x + \frac{2I_{12}\bar{\rho}_x}{db} \\ &\cdot \sum_{n=2,4,\ldots} \frac{\sin(\alpha_n\varepsilon)}{\gamma\alpha_n^2\varepsilon} \cdot \frac{\mathrm{ch}(\beta_n(a-x)) - \mathrm{ch}(\beta_n x)}{\mathrm{sh}(\beta_n a)} \\ &+ \frac{2I_{12}\bar{\rho}_x\gamma_{xy}^2}{db} \cdot \sum_{n=1,3,\ldots} \frac{\sin(\alpha_n\varepsilon)}{\gamma\alpha_n^2\varepsilon} \\ &\cdot \frac{\mathrm{ch}(\beta_n(a-x)) - \mathrm{ch}(\beta_n x)}{\mathrm{sh}(\beta_n a)}, \end{aligned} \tag{10}$$

$$\gamma_{xy} = \sigma_0 R_z B, \quad \sigma_0 = (\sigma_x\sigma_y)^{1/2}. \tag{11}$$

To identify the magnetoresistance value, we obtain the potential difference between points 5 and 6 on the location line of current contacts

$$U_{56} = \frac{I_{12}\bar{\rho}_x}{db}(a - 2x_1)1 + Z + \gamma_{xy}^2 \cdot G), \tag{12}$$

$$\begin{aligned} Z = \frac{4}{a-2x_1} &\sum_{n=2,4,\ldots} \frac{\sin(\alpha_n\varepsilon)}{\gamma\alpha_n^2\varepsilon} \\ &\cdot \frac{\mathrm{ch}(\beta_n(a-x_1)) - \mathrm{ch}(\beta_n x_1)}{\mathrm{sh}(\beta_n a)}, \end{aligned} \tag{13}$$

$$\begin{aligned} G = \frac{4}{a-2x_1} &\sum_{n=1,3,\ldots} \frac{\sin(\alpha_n\varepsilon)}{\gamma\alpha_n^2\varepsilon} \\ &\cdot \frac{\mathrm{ch}(\beta_n(a-x_1)) - \mathrm{ch}(\beta_n x_1)}{\mathrm{sh}(\beta_n a)}. \end{aligned} \tag{14}$$

Function Z takes into account the inhomogeneity of the current distribution in the sample when $2\varepsilon < b$, and function $\gamma_{xy}^2 G$ determines the influence of external field B on the distribution character of lines of the current density vector, which is the effect of geometrical magnetoresistance.[12,15]

The results of the boundary problem solution enable us to analyze the dependence of non-dimensional values Z and G on x_1/b for different parameters σ_y/σ_x of anisotropic samples, which has a practical meaning for the measurement of physical magnetoresistance.[15,17]

According to Fig. 2 the increase of ratio σ_y/σ_x leads to the decrease of size x_1/b of the area where the current is spreading close to the contacts. Figure 3 shows that the value of G diminishes with the increase of σ_y/σ_x. It shows the fact that the effect of geometrical magnetoresistance is weakening as a result of weakening of the short-circulating effect of current electrodes on the Hall electromotive force value.

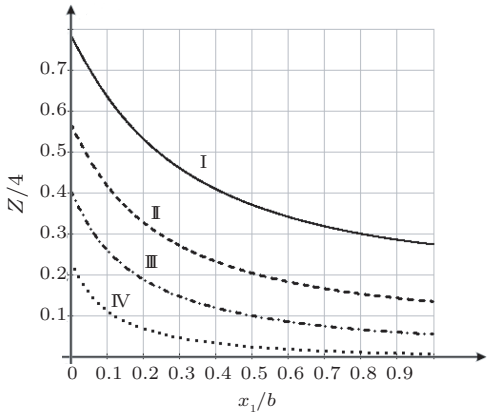

Fig. 2. Calculation of the dependence of multiplier Z on ratio x_1/b, where $a = 3b$, $\varepsilon = 0.1b$, $\sigma_y/\sigma_x = 0.25$ (I), 0.5 (II), 1 (III) and 3 (IV).

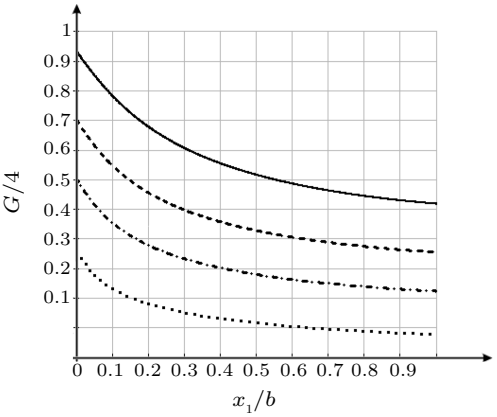

Fig. 3. Calculation of dependence of multiplier G on ratio x_1/b, where $a = 3b$, $\varepsilon = 0.1b$, $\sigma_y/\sigma_x = 0.25$ (I), 0.5 (II), 1 (III) and 3 (IV).

The methods of measurement of magnetoresistance $\Delta\rho_x/\rho_x$ of anisotropic samples imply that it is better to measure the tensor magnetoresistance component simultaneously with corresponding conductivity components.

After measuring potential difference U_{56}^0 without an external magnetic field and then potential difference U_{56} between the same contacts placed in the magnetic field with constant current I_{12}, we calculate the tensor component of relative physical mangetoresistance $\Delta\rho_x/\rho_x$ using

$$\frac{\Delta\rho_x}{\rho_{0x}} = \frac{\Delta U_{56}}{U_{56}^0} = \frac{U_{56} - U_{56}^0}{U_{56}^0}. \tag{15}$$

The main component of specific electrical resistance ρ_{0x} of the sample material is calculated using Eq. (12) when $\gamma_{xy}^2 G = 0$,

$$\rho_{0x} = U_{56}^0 \cdot bd/[I_{12}(a - 2x_1)(1 + Z)]. \qquad (16)$$

To identify the other main components of the magnetoresistance tensor using this method, it is necessary to cut the samples under study along the following directions.

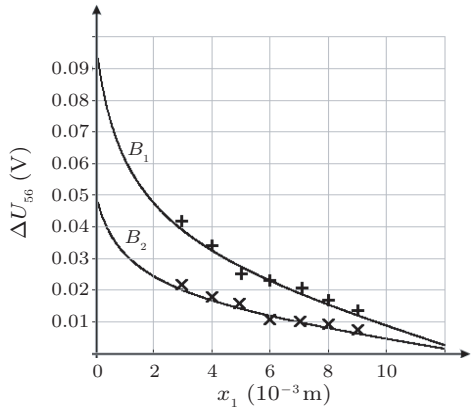

Fig. 4. Experimental testing of isotropic n-Si ($n = 7.8 \times 10^{20}\,\mathrm{m}^{-3}$), for $a = 25.5\,\mathrm{mm}$, $b = 23.8\,\mathrm{mm}$, $2\varepsilon = 1\,\mathrm{mm}$, $d = 3.7\,\mathrm{mm}$, $\sigma_x = \sigma_y = 0.189\,(\Omega{\cdot}\mathrm{m})^{-1}$, electric current $I_{12} = 1\,\mathrm{mA}$, magnetic fields $B_1 = 1\,\mathrm{T}$, $B_2 = 1.4\,\mathrm{T}$. Solid line: theory; +, ×: experiment.

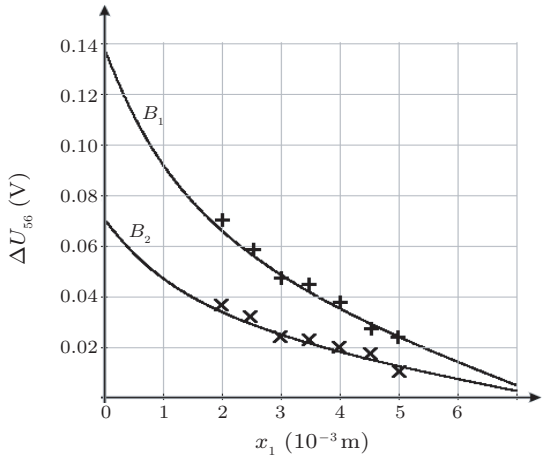

Fig. 5. Experimental testing of anisotropic p-$CdAs_2$ ($p = 4.1 \times 10^{21}\,\mathrm{m}^{-3}$), for $a = 15\,\mathrm{mm}$, $b = 10\,\mathrm{mm}$, $2\varepsilon = 1\,\mathrm{mm}$, $d = 1\,\mathrm{mm}$, $\sigma_x = 42.1\,(\Omega{\cdot}\mathrm{m})^{-1}$, $\sigma_y = 8.91\,(\Omega{\cdot}\mathrm{m})^{-1}$, electric current $I_{12} = 10\,\mathrm{mA}$, magnetic fields $B_1 = 1\,\mathrm{T}$, $B_2 = 1.4\,\mathrm{T}$. Solid line: theory; +, ×: experiment.

When measuring the Hall electromotive force value of lengthy samples, it is important to know how close one can place Hall measurement contacts 3 and 4 to shortening electrodes. This information is especially significant if one measures the tensor component of physical magnetoresistance, Hall ratio and electroconductivity along one plane of the sample.

The voltage Eq. (12) in the magnetic field was experimentally verified for isotropic of n-Si and tetragonal single crystals of $CdAs_2$ (Figs. 4 and 5). A dc electric current of 1 and 10 mA from a stabilized BSP-45 source was transmitted by the anisotropic samples placed in a transverse magnetic field with the induction $B = 1$–$1.4\,\mathrm{T}$. Then, using a movable probe and a V2-34 highresistance voltmeter, we measured this voltage. The voltage measurement error was no more than 6%. The experimental potential values are shown in Figs. 4 and 5 by points. Some deviations of the experimental points from the theoretical curves are explained by the error of determining the coordinate of the probe position and inhomogeneities at the boundaries of the sample.

To eliminate the influence of the magnetoresistance effect on the Hall electromotive force value one should try to place Hall contacts symmetrically about each other. The requirements concerning contact ohmic and the value of transition contact resistance are not rigid if measurements of the potential difference and their increase are carried out by the compensation metering method.[16]

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