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Jeans Gravitational Instability with κ-Deformed Kaniadakis Distribution *

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The Jeans instabilities in an unmagnetized, collisionless, isotropic self-gravitating matter system are investigated in the context of κ-deformed Kaniadakis distribution based on kinetic theory. The result shows that both the growth rates and critical wave numbers of Jeans instability are lower in the κ-deformed Kaniadakis distributed self-gravitating matter systems than the Maxwellian case. The standard Jeans instability for a Maxwellian case is recovered under the limitation $\kappa = 0$.

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Jeans instability is a very hot topic in both gravitational[1,2] and plasma[3,4] as well as the complex fluid community,[5–7] since such an instability has been recognized as the key mechanism to explain the gravitational formation of structures and their evolution in the linear regime. In his classical work, Jeans[1] gave the first quantitative description of collapse of matter due to self-gravity based on an ideal hydrodynamics model, and showed that a self-gravitating infinite uniform gas at rest should be unstable against small perturbation in the forms of $\exp[i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$ with the dispersion equation $\omega^2 = k^2c_{\rm s}^2 - \Omega_{\rm G}^2$, where $\Omega_{\rm G} = \sqrt{4\pi G\rho}$ is the Jeans gravitational frequency, ρ is the matter density, G is the gravitational constant, and $c_{\rm s}$ is the adiabatic sound velocity. From the dispersion equation, one can find that when the perturbation wave number k is less then some certain critical values, i.e., $k < k_{\rm J} \equiv \sqrt{\Omega_{\rm G}/c_{\rm s}}$, the value of ω^2 becomes negative and the Jeans instability arises. Mace *et al.*[7] proposed that $c_{\rm s}$ can be seen as the typical velocity v of the system under consideration, then the critical wave number for the Jeans instability can be reset as $k_{\rm J} \equiv \sqrt{\Omega_{\rm G}/v}$.

Indeed, such typical velocity v varies under different systems, thus Jeans' criterion lengths and growth rates should be modified by the distribution of the self-gravitating matter system. Lima *et al.*[8] studied Jeans instability in a nonextensive distributed self-gravitating matter system within the kinetic regime, and showed that both the growth rate and critical wavenumber depend on the nonextensive parameter q. Based on the fluid mode, Du[9] restudied Jeans' criterion and gave a different critical wavenumber for the nonextensive distributed systems. Compared with the work of Lima *et al.*[8] Qian *et al.*[10] adopted the kappa distribution to model the dust particles to study the Jeans instability of the self-gravitating dusty plasmas systems. Recently, along the method of Du,[9] Abreu *et al.*[11] derived Jeans' criterion from the viewpoint of Kaniadakis statistics, and found that the criterion of Jeans instability depends on the parameter κ of Kaniadakis' statistics. However, Abreu *et al.*[11] have not given the growth rates of Jeans instabilities correspondingly. Thus in the present work, the characteristics of criterion and growth rate of Jeans instability will be presented based on the kinetic regime, as carried out by Lima *et al.*[8] for nonextensive distributed systems and Qian *et al.*[10] for Kappa distributed systems.

Kaniadakis' statistics, or so-called κ-deformed distribution, introduced by Kaniadakis[12] in 2001, can cover the conventional Maxwell–Boltzman distribution and nonextensive distribution. Later on, Beck and Cohen[13] proposed that this κ-deformed distribution can be viewed as the resultant of a more generalized statistics, which have been named superstatistics. Ourabah *et al.*[14] have also demonstrated that the nonthermal and suprathermal empirical distributions can be recovered from the Beck–Cohen superstatistics. Thus in this sense, the κ-deformed distribution can be viewed as a more general form of those distribution functions. Now, the Kaniadakis statistics or the κ-deformed distribution has been applied to black-body radiation,[15] quantum entanglement,[16] cosmic rays,[17] quark-gluon plasma formation,[18] kinetics of interaction atoms and photons,[19] nonlinear kinetics,[20,21] and even the financial systems,[22] etc. Very recently, Gougam and Tribeche[23] adopted the κ-deformed distribution function to study the electron-acoustic solitary waves, and showed that the parameter κ of the distribution function altered the characteristics of electron-acoustic solitary waves.

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In the present work, the Jeans instabilities in an unmagnetized, collisionless, isotropic self-gravitating matter system are investigated in the context of κ-deformed distribution. Similar to plasma systems, as a many-particle system, the kinetic dynamics of self-gravitating system can be expressed by[3,8,24]

$$\frac{\partial f(\boldsymbol{v},\boldsymbol{r},t)}{\partial t}+\boldsymbol{v}\cdot\frac{\partial f(\boldsymbol{v},\boldsymbol{r},t)}{\partial \boldsymbol{r}}+\nabla\varphi\cdot\frac{\partial f(\boldsymbol{v},\boldsymbol{r},t)}{\partial \boldsymbol{v}}=0, \quad (1)$$

where $f(\boldsymbol{v},\boldsymbol{r},t)$ is the matter distribution function, and φ is the gravitational potential, which satisfies the Poisson equation

$$\nabla^2\varphi=4\pi G\int f(\boldsymbol{v},\boldsymbol{r},t)d\boldsymbol{v}. \quad (2)$$

Along the standard procedure, the distribution function can be separated into equilibrium and perturbation, i.e., $f(\boldsymbol{v},\boldsymbol{r},t)=f_0(\boldsymbol{v})+f_1(\boldsymbol{v},\boldsymbol{r},t)$, then one can obtain the linearized Vlasov equation and the Poisson equation as follows:

$$\frac{\partial f_1(\boldsymbol{v},\boldsymbol{r},t)}{\partial t}+\boldsymbol{v}\cdot\frac{\partial f_1(\boldsymbol{v},\boldsymbol{r},t)}{\partial \boldsymbol{r}} +\nabla\varphi_1\cdot\frac{\partial f_0(\boldsymbol{v},\boldsymbol{r},t)}{\partial \boldsymbol{v}}=0, \quad (3)$$

$$\nabla^2\varphi_1=4\pi G\int f_1(\boldsymbol{v},\boldsymbol{r},t)d\boldsymbol{v}. \quad (4)$$

Setting $f_1(\boldsymbol{v},\boldsymbol{r},t)$ and $\varphi_1(\boldsymbol{r},t)$ proportional to $\exp[i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$, and using the Fourier transforms of Eqs. (3) and (4), we obtain the dielectric constant for the gravitational response as

$$\varepsilon^G(\omega,\boldsymbol{k})=-1+\frac{4\pi G}{k^2}\int\frac{\boldsymbol{k}\cdot\frac{\partial f_0(\boldsymbol{v})}{\partial \boldsymbol{v}}}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}}\cdot d\boldsymbol{v}=0. \quad (5)$$

Without loss of generality, letting v_x in the direction of $\boldsymbol{k}$, then Eq. (5) can be rewritten as

$$\varepsilon^G(\omega,k)=-1+\frac{4\pi G}{k^2}\int\frac{k\cdot\frac{\partial f_0(v_x)}{\partial v_x}}{\omega-kv_x}\cdot dv_x=0, \quad (6)$$

where $f_0(v_x)$ is the one-dimensional distribution function defined as $f_0(v_x)\equiv\int f_0(\boldsymbol{v})dv_ydv_z$.

The κ-deformed Kaniadakis distribution for free mass-particles is given by[11,12]

$$f_\alpha(\boldsymbol{v})=A_\kappa\Big(\sqrt{1+\kappa^2\frac{v^4}{4\sigma^4}}-\kappa\frac{v^2}{2\sigma^2}\Big)^{1/\kappa}, \quad (7)$$

where σ is the velocity dispersion that can be regarded as the thermal velocity of the matter system,[8] and A_κ is the normalized coefficient given by

$$A_\kappa=\rho_0\frac{|\frac{\kappa}{\pi}|^{\frac{3}{2}}}{\sigma^3}\Big(1+\frac{3}{2}|\kappa|\Big)\frac{\Gamma(\frac{1}{|2\kappa|}+\frac{3}{4})}{\Gamma(\frac{1}{|2\kappa|}-\frac{3}{4})}, \quad (8)$$

through the normalized condition $\int_{-\infty}^{\infty}f_0(\boldsymbol{v})d\boldsymbol{v}=\rho_0$, with ρ_0 being the equilibrium density. Before proceeding further, it is important to show the second velocity moment

$$\langle v^2\rangle\equiv 3\sigma_{\rm eff} =\frac{3}{2}\frac{1+\frac{3}{2}|\kappa|}{(1+\frac{5}{2}|\kappa|)\kappa}\frac{\Gamma(\frac{1}{|2\kappa|}-\frac{5}{4})}{\Gamma(\frac{1}{|2\kappa|}+\frac{5}{4})}\frac{\Gamma(\frac{1}{|2\kappa|}+\frac{3}{4})}{\Gamma(\frac{1}{|2\kappa|}-\frac{3}{4})}\sigma^2, \quad (9)$$

where $\sigma_{\rm eff}$ is the root-mean-square speed and can be seen as the effect velocity in the κ-deformed Kaniadakis distribution function. Figure 1 shows the ratio of the effect velocity to dispersive velocity. It can be seen that as $\kappa\to 0.4$, the ratio is diverged. Thus one has $|\kappa|<0.4$ to ensure the physical sense of the root-mean-square speed $\sigma_{\rm eff}$.

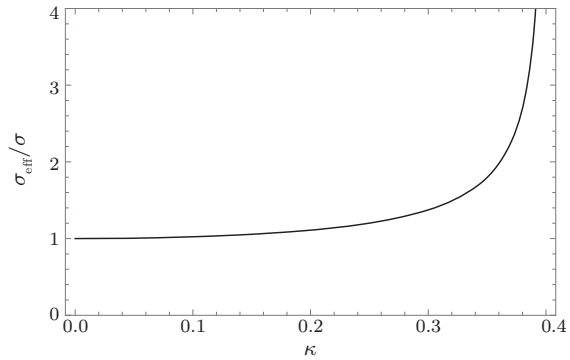

Fig. 1. The ratio of the effective velocity $\sigma_{\rm eff}$ to the dispersive velocity σ versus the distribution function parameter κ.

Substituting Eq. (7) into the definition of one-dimensional distribution function, we obtain the one-dimensional κ-deformed Kaniadakis distribution function as

$$f_0(v_x)=-\frac{2\sigma^2A_\kappa\pi}{\kappa^2-1}\Big[-\kappa\frac{v_x^2}{2\sigma^2}+\sqrt{1+\kappa^2\Big(\frac{v_x^2}{2\sigma^2}\Big)^2}\Big]^{1/\kappa} \cdot\Big[\kappa^2\frac{v_x^2}{2\sigma^2}+\sqrt{1+\kappa^2\Big(\frac{v_x^2}{2\sigma^2}\Big)^2}\Big]. \quad (10)$$

Putting $\partial f_0(v_x)/\partial v_x$ into Eq. (6), we obtain

$$\varepsilon^G(\omega,k)=-1+\frac{k_J^2}{C_\kappa k^2}[B_\kappa-Z(\xi)]=0, \quad (11)$$

where $k_J^2=4\pi G\rho_0/\sigma^2$ is the classical Jeans wavenumber, the coefficients B_κ and C_κ read

$$B_\kappa=\frac{|2\kappa|^{-\frac{1}{2}}}{1+\frac{1}{2}|\kappa|}\frac{\Gamma(\frac{1}{|2\kappa|}-\frac{1}{4})}{\Gamma(\frac{1}{|2\kappa|}+\frac{1}{4})}\sqrt{\pi},$$

$$C_\kappa=\frac{|\kappa|^{-\frac{3}{2}}}{1+\frac{3}{2}|\kappa|}\frac{\Gamma\Big(\frac{1}{|2\kappa|}-\frac{3}{4}\Big)}{\Gamma(\frac{1}{|2\kappa|}+\frac{3}{4})}\frac{\sqrt{\pi}}{2\sqrt{2}},$$

and $Z(\xi)$ is the modified dispersion function

$$Z(\xi)=\int_{-\infty}^{\infty}\frac{\xi}{\xi-x}\Big(-\kappa x^2+\sqrt{1+\kappa^2x^4}\Big)^{1/\kappa}\cdot dx, \quad (12)$$

with the argument $\xi = \omega/\sqrt{2}k\sigma$. In obtaining the coefficients A_κ, B_κ, and C_κ, we have used the integral relation[11,17,23]

$$\int_0^\infty x^{r-1}\left(-\kappa x^2+\sqrt{1+\kappa^2x^4}\right)^{1/\kappa}dx = \frac{|2\kappa|^{-r}}{1+r|\kappa|}\frac{\Gamma(\frac{1}{|2\kappa|}-\frac{r}{2})}{\Gamma(\frac{1}{|2\kappa|}+\frac{r}{2})}\Gamma(r). \quad (13)$$

In the study of the Jeans instability, the boundary between stable and unstable modes is obtained by setting $\omega = 0$, i.e., $\xi = 0$ in Eq. (11), then one has κ-parameterized critical wave-numbers k_κ given by

$$k_\kappa^2 = \frac{B_\kappa}{C_\kappa}k_J^2, \quad (14)$$

which is identical to the results of Abreu *et al.*[11] For more details about the effects of the parameter on the critical wave-numbers k_κ, one can refer to the works of Abreu *et al.*[11]

If assuming that the wavenumber k is real, and setting $\omega = i\gamma$ for the unstable modes, where γ is real positive, then from Eq. (11), the dispersion relation for Jeans' mode is obtained as

$$\frac{k^2}{k_\kappa^2} = 1 - \frac{1}{B_\kappa}Z(\beta). \quad (15)$$

The dispersion function $Z(\beta)$ is reformed as

$$Z(\beta) = \int_{-\infty}^{\infty}\frac{\beta^2}{\beta^2+x^2}\left(-\kappa x^2+\sqrt{1+\kappa^2x^4}\right)^{1/\kappa}\cdot dx \quad (16)$$

where $\beta = \gamma/\sqrt{2}k\sigma$. From Eq. (16) one can find that $Z(\beta)$ is always real positive with real positive γ and real wavenumber k. It should be pointed out that the image part of the dispersion function, i.e., $i\beta\int_{-\infty}^{\infty}\frac{x}{\beta^2+x^2}(-\kappa x^2+\sqrt{1+\kappa^2x^4})^{1/\kappa}\cdot dx$, goes to zero due to the odd integrand function, which also ensures that $Z(\beta)$ is always real positive.

It should be noticed that under the limit $\kappa \to 0$, the distribution function $(-\kappa x^2+\sqrt{1+\kappa^2x^4})^{1/\kappa} \to \exp(-x^2)$,[11,23] then the dispersion Eq. (15) reduces to

$$\frac{k^2}{k_J^2} = 1 - \sqrt{\pi}\beta e^{-\beta^2}[1-\mathrm{erf}(\beta)], \quad (17)$$

which is the standard dispersion relation for the stellar system with the Maxwellian distribution function.[2,8] For an arbitrary value of κ, the dispersion Eq. (15) can be numerically evaluated to give the growth rates of Jeans' modes. Figure 2 shows the normalized growth rate ($\Gamma \equiv \gamma/\Omega_\mathrm{G}$) versus the normalized wave-number ($K \equiv k/k_J$) with different κ. It can be found that as κ increases, the growth rate and the critical wavenumber for the instability decrease, i.e., compared with the Maxwellian case, the Jeans instability is suppressed in the κ-deformed Kaniadakis distributed self-gravitating matter systems. According to the work of Mace *et al.*,[7] the typical velocity $v \sim \sigma_\mathrm{eff}$ in our present model, and as shown in Fig. 1, σ_eff increases with κ, then the critical wave number $k_\mathrm{J} \equiv \sqrt{\Omega_\mathrm{G}/\sigma_\mathrm{eff}}$ decreases. Physically, the Jeans instability occurs when the thermal pressure is not strong enough to prevent gravitational collapse of the matter system. As the distribution parameter κ increases, then the effect velocity σ_eff and the related thermal pressure increase, thus the growth rate of the Jeans instability decreases. It should be pointed out that the growth rate with $\kappa = 0$ is evaluated under the guidance of the classical Maxwellian case,[2,8] and overlaps with the one of $\kappa = 0.01$, which indicates that the corresponding result of the Maxwellian case is well recovered in the present model, as is expected.

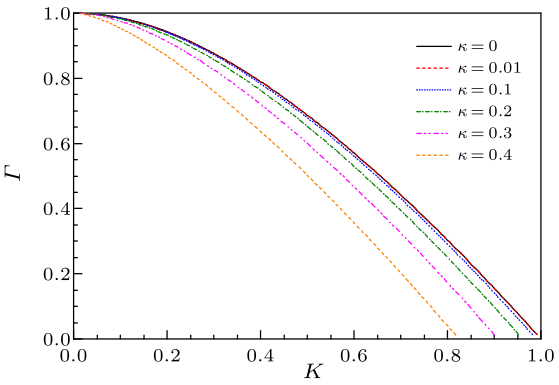

Fig. 2. The normalized growth rate ($\Gamma \equiv \gamma/\Omega_\mathrm{G}$) versus the normalized wave-number ($K \equiv k/k_J$) with different κ.

In summary, the Jeans instabilities in an unmagnetized, collisionless, isotropic self-gravitating matter system are investigated in the context of κ-deformed distribution based on kinetic theory. The result shows that the Jeans instability is suppressed in the κ-deformed Kaniadakis distributed self-gravitating matter systems, compared with the Maxwellian case. The standard Jeans instability for the Maxwellian case is recovered under the limitation $\kappa = 0$. Since the Jeans instability remains basically meaningful for the celestial system, plasma and complex fluid, moreover the κ-deformed Kaniadakis statistics have been viewed as more generalized statistics, the adoption of such a distribution and the related results of the present study may have interesting consequences for the relevant galaxy formation process,[2,8] dust agglomeration,[25] etc.

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