CHIN. PHYS. LETT. Vol. 34, No. 7 (2017) 074702

Natural Frequency of Oscillating Gaseous Bubbles in Ventilated Cavitation *

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(Received 18 December 2016)

An improved formula is proposed for the prediction of natural frequency of oscillating gaseous bubbles in the ventilated cavitation by considering the liquid compressibility and the thermal effects. The differences between the previous formula and ours are quantitatively discussed in terms of both dimensional parameters (e.g., frequency and bubble radius) and non-dimensional parameters (e.g., the Péclet number). Our analysis reveals that our formula is superior to the existing formula in the low-frequency excitation regions.

PACS: 47.55.dd, 43.35.Ei, 62.60.+v DOI: 10.1088/0256-307X/34/7/074702

Ventilated cavitation is being intensively investigated due to its great importance for the drag reduction of various kinds of marine vehicles[1–5] (including torpedo and surface ships). One of the current obstacles toward a highly efficient drag reduction method is the unstable nature of this method.[2] Under some working conditions, an unbalanced force will appear around the vehicles, leading to the severe fluctuations of the ships or the deviation of the target of the torpedoes. The underlying mechanisms behind this phenomenon involves the interaction of the gaseous bubbles with the fluid structure around the vehicles.

To solve the above problem, it is essential to understand the physical characteristics of gaseous bubbles. One of the important parameters of the gaseous bubbles is their natural frequency. If the external excitation is close to this natural frequency, oscillations and collapse of the bubbles will be greatly magnified, leading to prominent vibrations of the vehicles. Following a classical formula[1] based on the Rayleigh–Plesset equation,[6] the natural frequency of gaseous bubbles is

$$\omega_0 = \left[\frac{p_{\mathrm{in,eq}}}{\rho_{\mathrm{l}} R_0^2}\left(3\kappa - \frac{2\sigma}{p_{\mathrm{in,eq}} R_0}\right)\right]^{\frac{1}{2}}, \tag{1}$$

with

$$p_{\mathrm{in,eq}} = p_0 + \frac{2\sigma}{R_0}, \tag{2}$$

where ω_0 is the natural frequency of gaseous bubbles injected during ventilated cavitation, $p_{\mathrm{in,eq}}$ is the equilibrium pressure at the bubble interface in the gas side, ρ_{l} is the density of the liquid, R_0 is the equilibrium bubble radius, κ is the polytropic exponent during bubble oscillations (e.g., expansion and compression), σ is the surface tension coefficient, and p_0 is the ambient pressure (without disturbance). If the external excitation induced by the flow passing the vehicles is equal or close to the natural frequency of those ventilated bubbles, a significant rise of the bubble oscillations (in terms of amplitude) will be generated, possibly leading to the force unbalance of the vehicles.

If one closely examines Eq. (1) representing the natural frequency of gaseous bubbles, it could be revealed that effects of liquid compressibility are not included. Those effects are very well-known to produce some profound influences on the bubble oscillations in the case of high Mach numbers.[2,7] Hence, in this Letter, an improved formula for the predictions of the natural frequency of gaseous bubbles is derived with the corrections of liquid compressibility and other influencing parameters. Our results demonstrated that the corrections proposed here will be important for the regions with low frequency (or low Péclet number in terms of non-dimensional parameter).

Here formulas of natural frequency will be derived based on a bubble interface motion equation with liquid compressibility and thermal effects included. For that purpose, the equation given by Keller and Miksis[8] for oscillations of spherical gaseous bubbles is employed such as

$$\left(1 - \frac{\dot{R}}{c_{\mathrm{l}}}\right) R\ddot{R} + \frac{3}{2}\left(1 - \frac{\dot{R}}{3c_{\mathrm{l}}}\right)\dot{R}^2 = \left(1 + \frac{\dot{R}}{c_{\mathrm{l}}}\right)\frac{p_{\mathrm{ext}}(t) - p_{\mathrm{s}}(t)}{\rho_{\mathrm{l}}} + \frac{R}{\rho_{\mathrm{l}} c_{\mathrm{l}}}\frac{d[p_{\mathrm{ext}}(t) - p_{\mathrm{s}}(t)]}{dt}, \tag{3}$$

with

$$p_{\mathrm{ext}}(R,t) = p_{\mathrm{in}} - \frac{2\sigma}{R} - \frac{4\mu_{\mathrm{l}}}{R}\dot{R}, \tag{4}$$

$$p_{\mathrm{s}}(t) = p_0[1 + \varepsilon\cos(\omega t)], \tag{5}$$

where R is the instantaneous bubble radius of the ventilated gaseous bubble, overdot denotes the time

*Supported by the National Natural Science Foundation of China under Grant No 51506051, the Fundamental Research Funds for the Central Universities under Grant No JB2015RCY04, and the Incubation Project for Young Talents of Chinese Society for Electrical Engineering under Grant No JLB-2016-68.

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derivative, $c_{\rm l}$ is the speed of sound in the pure liquid (water here), t is the time, $p_{\rm in}$ is the instantaneous pressure at the gaseous side of the bubble interface, $\mu_{\rm l}$ is the viscosity of the liquid, ε is the non-dimensional amplitude of external excitation (i.e., characteristic waves induced by the flow passing the ventilated vehicles), and ω is the angular frequency of the above external excitation. The bubble–bubble interactions[9–11] are ignored in Eq. (3).

Apart from the Keller–Miksis equation, there are many other kinds of equation for bubble interface motion (e.g., the Gilmore and Herring equations). For the present purpose, the Gilmore equation will reduce to the Keller–Miksis equation as shown in the appendix D of Ref. [12]. For the Herring equation, it could be treated as a special case of the one-parameter family of the equations proposed by Prosperetti and Lezzi[7] up to the first order accuracy of the Mach number. In our previous work,[12,13] it has been proved that the resulting expressions will be identical as shown in the appendix B of Ref. [12].

In the following treatment, different from previous work, all the terms relating to the liquid compressibility (up to the first order of $c_{\rm l}^{-1}$) are kept during our derivations. For simplicity, we further assume that the oscillations of gaseous bubbles are with small amplitude,

$$R = R_0(1+x), \tag{6}$$

where x represents the non-dimensional oscillations of instantaneous radius of gaseous bubble. To close the model, the oscillations of gaseous bubble are assumed to be a polytropic process and $p_{\rm in}$ could be expressed as[14,15]

$$p_{\rm in} = \left(p_0 + \frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^{3\kappa} - \frac{4\mu_{\rm th}}{R}\dot{R}, \tag{7}$$

where κ is the polytropic exponent, $\mu_{\rm th}$ is the effective thermal-damping viscosity. Thermal effects shown here can refer to the heat transfer between the bubbles and the surrounding liquids during oscillations. For example, when the bubble collapses, the gases inside bubbles will be compressed, leading to the increase of temperature inside the bubbles. Hence, heat will be transferred from the inside of bubbles to the surrounding liquids. More details can be found in Ref. [14]. For convenience, in Eq. (7), the energy dissipation caused by the thermal effects is considered with an effective parameter ($\mu_{\rm th}$ term), which is an analogue to the effects of liquid viscosity. In our previous work,[14,15] the expressions of $\mu_{\rm th}$ were obtained. For details, researchers can refer to Eqs. (16)–(26) in Ref. [14]. In Eq. (5), the external excitation is assumed as one with monofrequency. For a complex wave with multiple frequencies, some new phenomena (e.g., combination resonances[16]) will appear in the dynamic oscillations of bubbles. In the ventilated cavitation, if cavitation vapor bubbles are involved, the phenomena will be much more complex.[17] For the present purpose, we further assume that the amplitude of external wave is limited (e.g., ε in Eq. (5) being far below 1). Hence, the solution is determinative rather than chaotic.[18]

Substituting Eqs. (6) and (7) into Eq. (3), one can obtain[12,13]

$$\ddot{x} + 2\beta\dot{x} + \tilde{\omega}_0^2 x = -\alpha\varepsilon\psi\cos(\omega t + \delta), \tag{8}$$

with

$$\begin{aligned}
\tilde{\omega}_0 &= \left(1 + \frac{R_0}{c_{\rm l}}\frac{4(\mu_{\rm l}+\mu_{\rm th})}{\rho_{\rm l}R_0^2}\right)^{-\frac{1}{2}} \\
&\quad \cdot \left[\frac{p_{\rm in,eq}}{\rho_{\rm l}R_0^2}\left(3\kappa - \frac{2\sigma}{p_{\rm in,eq}R_0}\right)\right]^{\frac{1}{2}}, \\
\alpha &= \frac{p_0}{\rho_{\rm l}R_0^2}, \\
\psi &= \left(1 + \frac{R_0}{c_{\rm l}}\frac{4(\mu_{\rm l}+\mu_{\rm th})}{\rho_{\rm l}R_0^2}\right)^{-1}[1+(\omega R_0/c_{\rm l})^2]^{1/2}, \\
\delta &= \arctan(\omega R_0/c_{\rm l}),
\end{aligned} \tag{9}$$

where β is the (total) damping constant (e.g., representing the total energy dissipations through various kinds of mechanisms), and $\tilde{\omega}_0$ is the natural frequency of gaseous ventilated bubbles with liquid compressibility and thermal effects included (referring to the second term in the first bracket of Eq. (9)). To distinguish from the notations in Eqs. (1) and (9), an overbar is employed for the notation of natural frequency in Eq. (9) derived by us. It should be further emphasized that energy dissipations (e.g., $\mu_{\rm l}$ and $\mu_{\rm th}$ terms) also contribute to the natural frequency. For the convenience of analysis and discussions, a non-dimensional parameter χ is further defined to reflect the differences between two kinds of formulas of the natural frequency of bubbles (Eqs. (1) and (9)),

$$\chi = \frac{\omega_0^2 - \tilde{\omega}_0^2}{\tilde{\omega}_0^2} = \frac{R_0}{c_{\rm l}}\frac{4(\mu_{\rm l}+\mu_{\rm th})}{\rho_{\rm l}R_0^2}. \tag{10}$$

For the predictions of $\mu_{\rm th}$, our previous formulas[14,15] are employed for the completeness. In Eq. (10), for low-frequency regions, the $\mu_{\rm l}$ term is negligible.[6,14] Based on the non-dimensional parameters, Eq. (10) becomes

$$\chi = Ma\delta_{\rm th}\frac{\tilde{\omega}_0^2}{\omega^2}, \tag{11}$$

with

$$Ma = \frac{\omega R_0}{c_{\rm l}}, \tag{12}$$

$$\delta_{\rm th} = \frac{4\mu_{\rm th}}{\rho_{\rm l}R_0^2}\frac{\omega}{\tilde{\omega}_0^2}, \tag{13}$$

where Ma is the bubble Mach number noticing that $\dot{R}$ is of the order of ωR_0, and $\delta_{\rm th}$ is the non-dimensional thermal damping constant.

In the following discussions, several demonstrating examples will be given to quantitatively show the differences between Eqs. (1) and (9) (or parameter χ). For the calculations, the air is chosen as the ventilated gas into the water through the surface of vehicles.

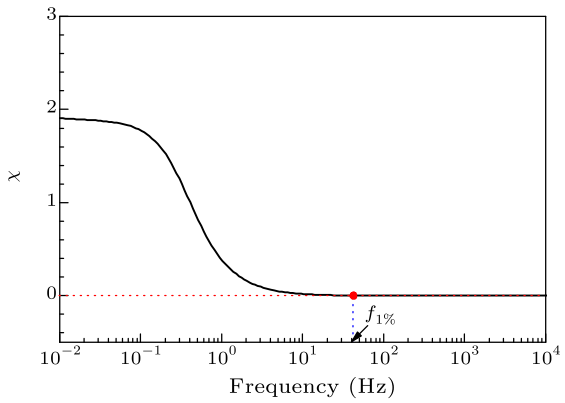

Fig. 1. Variations of non-dimensional error of natural frequency of oscillating gaseous bubbles (χ) versus frequencies ranging from 10^{-2} Hz to 10^{4} Hz. The solid red circle marks χ being 1%, the x-coordinate is defined as $f_{1\%}$. The dotted red line refers to $\chi = 0.0$, indicating that Eqs. (1) and (9) are the same. Here $R_0 = 10^{-2}$ m.

Figure 1 shows the variations of χ versus the frequency over a wide range (from 10^{-2} Hz to 10^{4} Hz) with bubble radius fixed. In Fig. 1, the solid red circle marks χ being 1%. For the convenience of further discussions, the x-coordinate (frequency) of χ being 1% is defined as $f_{1\%}$. The dotted red line refers to $\chi = 0.0$, representing no difference between Eqs. (1) and (9). With the decrease of the frequency, χ increases dramatically, reaching 2.0 for the lowest frequencies. Hence, for low-frequency regions, our formula plays an important role on the prediction of natural frequency. Based on the definition of χ (referring to Eq. (10)), the predictions by our newly proposed formula (Eq. (9)) is significantly lower than those in the literature (Eq. (1)).

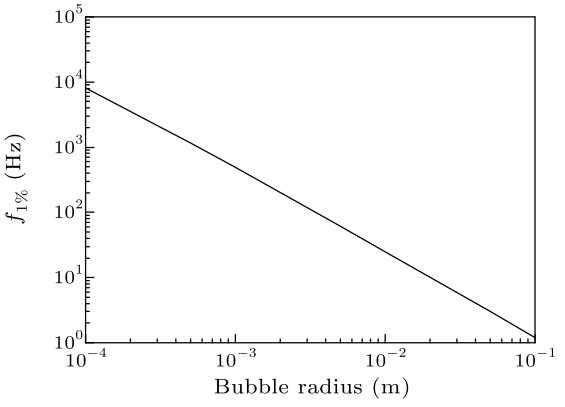

Fig. 2. Variations of $f_{1\%}$ versus bubble radius.

For the completeness, Fig. 2 shows a quantitative discussion of $f_{1\%}$ versus bubble radius ranging from 10^{-4} m to 10^{-1} m. With the increase of bubble radius, regions dominated by our formula shrink noticing that lower values of $f_{1\%}$ are observed for regions with larger bubbles. For example, for $R_0 = 10^{-4}$ m, $f_{1\%}$ is 8×10^{3} Hz and for regions with frequency smaller than this frequency, our corrections are significant. For $R_0 = 10^{-1}$ m, $f_{1\%}$ is about 1 Hz and the regions dominated by our formula shrink dramatically.

For a general discussion, a non-dimensional parameter (the Péclet number) will be introduced with the following definition

$$Pe = \frac{2\pi f R_0^2}{D}, \tag{14}$$

where Pe is the Péclet number, reflecting the square of the ratio between gaseous bubble radius and the penetration depth of thermal effects, f is the frequency of external excitation with $\omega = 2\pi f$, and D is the thermal diffusivity of gases inside bubbles. If we employ $f = f_{1\%}$ in Eq. (14), we define the corresponding values of Pe as $Pe_{1\%}$. Figure 3 shows the variations of $Pe_{1\%}$ versus bubble radius with the same range shown in Fig. 2. One can find that $Pe_{1\%}$ increases with the bubble radius.

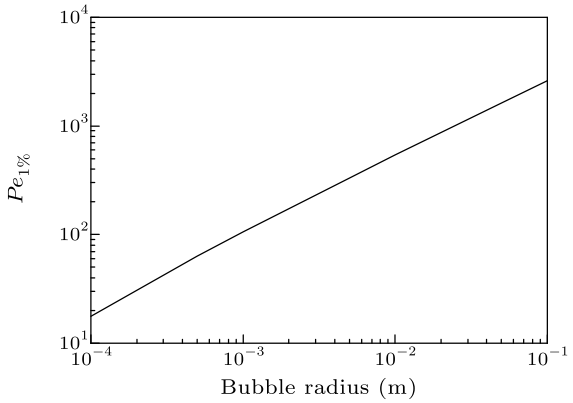

Fig. 3. Variations of $Pe_{1\%}$ versus bubble radius.

In conclusion, the natural frequency of oscillating gaseous bubbles in ventilated cavitation has been derived with the corrections of liquid compressibility and thermal effects. Our findings reveal that our corrections could significantly affect the predictions of oscillating gaseous bubbles in wide ranges of parameters (in terms of bubble radius and frequency of external excitation). The differences between formula in the literature and ours are revealed in terms of both dimensional parameters (e.g., frequency and bubble radius) and non-dimensional parameter (e.g., the Péclet number). In the present study, the Keller–Miksis equation is employed (referring to Eq. (3)). However, exactly identical formula could be obtained using any bubble interface motion equation up to the first order of c_1.[7,12] The bubble–bubble interactions are not considered due to the limited space. As shown in our

recent work,[9] both the sign and amplitude of the interaction forces could be significantly influenced by the natural frequency. Other possible applications of the present findings include the bubble-induced particle dynamics[19] and the hydroturbine instability.[20,21]

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