CHIN. PHYS. LETT. Vol. 34, No. 7 (2017) 074209

Tight Focusing Properties of Azimuthally Polarized Pair of Vortex Beams through a Dielectric Interface

C. A. P. Janet[1], M. Lavanya[2], K. B. Rajesh[3]**, M. Udhayakumar[3], Z. Jaroszewicz[4], D. Velauthapillai[5]

[1]*Department of Physics, St.Xavier's Catholic College of Engineering, Nagercoil, Tamilnadu, India*
[2]*Department of Physics, PSGR Krishnammal College for Women, Coimbatore, Tamilnadu, India*
[3]*Department of Physics, Chikkanna Government Arts College, Tiruppur, Tamilnadu, India*
[4]*Institute of Applied Optics, Department of Physical Optics, Warsaw, Poland and National Institute of Telecommunications, Warsaw, Poland*
[5]*Faculty of Engineering and Business Administration, Bergen University College, Bergen, Norway*

(Received 24 April 2017)

Tight focusing properties of an azimuthally polarized Gaussian beam with a pair of vortices through a dielectric interface is theoretically investigated by vector diffraction theory. For the incident beam with a pair of vortices of opposite topological charges, the vortices move toward each other, annihilate and revive in the vicinity of focal plane, which results in the generation of many novel focal patterns. The usable focal structures generated through the tight focusing of the double-vortex beams may find applications in micro-particle trapping, manipulation, and material processing, etc.

PACS: 42.25.Bs, 42.25.Ja, 42.79.Ag DOI: 10.1088/0256-307X/34/7/074209

In the past decades, the problem of light focusing by an objective through a plane interface between two media with different refractive indices has been extensively studied. In many practical applications, an objective is used to focus an incident light beam through an interface between different media of different refractive indexes. For instance, in the case of laser trapping, a light beam is focused through an interface between glass and water and in the application of semiconductor inspection, light beams are focused from air onto silicon substrates. Torok *et al.* systematically developed the theoretical method for studying the focusing of an electromagnetic wave through dielectric interfaces.[1] Biss and Brown discussed the focusing of cylindrical vector beams (CVBs) through a dielectric interface.[2] Optical vortex beams possess wave-front dislocations, where the phase values are undefined (also referred to as phase singularities) and the amplitude vanishes to zero. Such beams can be generated using the spatial light modulators (SLMs) or the spatial phases plates (SPPs)[3] and possess a helical phase structure of $\exp(in\varphi)$, where φ is the polar coordinate in the plane perpendicular to the beam axis, and n is the so-called topological charge. It is further known that such an optical vortex beam carries orbital angular momentum (OAM) and each photon has a magnitude of nh OAM.[4,5] As vortex beams have unique advantages in optical manipulation, the focusing property of vortex beams has been the topic of much research.[6–8] Recently, Zhang *et al.* studied the tight focusing of linearly, radially and azimuthally polarized vortex beams through a dielectric interface.[9,10] In an electromagnetic field, a dipole refers to a pair of electric charges or magnetic poles, of equal magnitude but opposite signs or polarities, separated by a small distance. A similar concept of 'optical vortex dipole' is introduced in the study of an optical vortex. A pair of optical vortices, with topological charges of $m = +1$ and $m = -1$, respectively, form an optical vortex dipole.[11,12] An optical vortex dipole is a pair of vortices (POV) with opposite topological charges propagating in a Gaussian beam. It is observed that such a dipole may produce a variety of possible trajectories different from that of canonical vortex beams and annihilate and revive periodically when propagated in graded-index media.[12] Expressions for the trajectories of vortices launched as a canonical dipole from arbitrary locations in a Gaussian beam are derived.[13] Recently, Chen *et al.* investigated the properties of a pair of vortices embedded in a Gaussian beam focused by a high numerical aperture lens system on the basis of the vector Debye integral.[14] They observed that for the incident beam with a pair of vortices with opposite topological charges, the vortices move towards each other, annihilate and revive in the vicinity of the focal plane. Fang *et al.* studied the focusing properties of the double-vortex beams through a high numerical-aperture objective based on the vectorial Debye theory.[15] Recently, we numerically analyzed the tight focusing properties of a pair of vortices of the same topological charge nested in a radially polarized Gaussian beam. We observed that the evolution of focal field pattern from one annular pattern to a highly confined focal spot in the transverse direction can be obtained by changing the location the optical vortex in the input plane.[16] We also noted

**Corresponding author. Email: rajeskb@gmail.com
© 2017 Chinese Physical Society and IOP Publishing Ltd

that by properly manipulating the position of two vortices of opposite topological charge nested in an azimuthally polarized Gaussian beam, one can generate many novel focal patterns such as focal hole, transversely polarized focal spot and flat top focus of sub wavelength scale when tightly focused with high NA parabolic mirror.[17] It is observed that the strong longitudinal component generated by the tightly focused radially polarized beam suffers discontinuity at the interface of two neighbouring media and enlarges the focusing spot in the high-NA medium. This implies that the subwavelength focused spot with longitudinal polarization is limited in applications such as imaging of silicon integrated circuits.[18,19] To solve this problem, suppression of longitudinal components by using an azimuthally polarized beam has been suggested.[20] Recently, it is reported that the focal hole generated by an azimuthally polarized beam can be changed into a significantly sharper focal spot when a vortical phase is encoded on the azimuthally polarized beam, which shows the intriguing prospect in practical applications due to its subwavelength lateral spot size and the purely transverse electric field.[21] Recently, many methods to improve the axial extent and to reduce the transverse spot size of the transversely polarized focal field are suggested.[22–25] However, all these works investigated the focal field properties without dielectric interface. For applications such as optical trapping and semiconductor inspection, the presence of dielectric interface is unavoidable and hence the main aim of this study is to analyze the focal properties of POV of opposite topological charge nested in an azimuthally polarized Gaussian beam and tightly focused through a dielectric interface. It is interesting to investigate the focusing properties of the POV of opposite topological charges, since the vortices move towards each other, annihilate and revive in the vicinity of the focal plane, which may alter the intensity distribution of different field components in the vicinity of focus and may generate many novel focal patterns and expand its applications. Here the focusing properties of the azimuthally polarized optical vortex dipole through a dielectric interface are investigated theoretically by the vector diffraction theory.

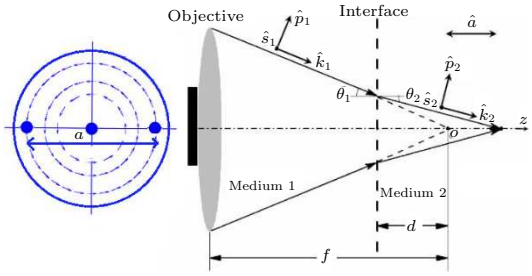

Fig. 1. Schematic diagram of the optical system.

The schematic diagram of the focusing system for a pair of vortices nested in an azimuthally polarized Gaussian beam is shown in Fig. 1.

For the numerical calculation, it is assumed that the interface is between two dielectric media of refractive indices $n_1 = 1$ and $n_2 = 3.55$, such as focusing in air onto a silicon substrate in applications of optical trapping and semiconductor inspection. The geometric focus of the objective without the interface is located at the origin O of the coordinate system. The distance between the interface and the geometric focus is d, which is referred to as the probe depth in Ref. [26]. For azimuthally polarized beams, the Cartesian components of the electric field vector in the focal region then could be written as[27]

$$E(r,\psi,z) = \begin{bmatrix} E_x(r,\psi,z) \\ E_y(r,\psi,z) \\ E_z(r,\psi,z) \end{bmatrix}$$
$$= \frac{-iE_0}{\pi} \int_{\delta\cdot\alpha}^{\alpha} \int_0^{2\pi} \exp[-ik_0 \Phi(\theta_1,\theta_2)]$$
$$\times \sin\theta_1 \sqrt{\cos\theta_1} A(\theta_1) \times t_s \exp[ik_2 z\cos\theta_2$$
$$+ ik_1 r \sin\theta_1 \cos(\psi-\varphi)] \times \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix} d\varphi d\theta_1, \quad (1)$$

where $k_i = n_i k_0$ is the wave number, $J_n(x)$ is the Bessel function of the first kind of order n, $\alpha = \arcsin(\mathrm{NA})$ is the maximal angle determined by the NA of the objective, δ is the ratio of the inner focusing minimum angle $\theta_{\min} to \alpha$, and t_s is the amplitude transmission coefficients for perpendicular polarization states, which is given by the Fresnel equation[13]

$$t_s = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1+\theta_2)}. \quad (2)$$

The function $\Phi(\theta_1,\theta_2)$ is given by

$$\Phi(\theta_1,\theta_2) = -d(n_1\cos\theta_1 - n_2\cos\theta_2), \quad (3)$$

representing the so-called aberration function caused by the mismatch of the refractive indices n_1 and n_2. Here θ_1 and θ_2 are related by the well-known Snell law. In turn, $A(\theta_1)$ describes the amplitude transmission function of a pair of vortices nested in a Gaussian beam. It is considered that electric field of two vortices with topological charge $m = \pm 1$ located at $x = \pm a$ and embedded in a Gaussian beam can be expressed as[14]

$$A(\theta_1) = \exp\left(-\frac{r^2}{w^2}\right)[r\exp(i\phi) - a][r\exp(-i\phi) + a], \quad (4)$$

where w is a constant denoting the beam size, and the distance between the two vortices is denoted by a. The two vortices will separate with the increment of a, and two dark cores emerge gradually in the beam. Here we consider that the topological charges of the

two vortices have the same magnitude. Since the objectives are often designed to obey the sine condition, we obtain $r = f \sin\theta$, where f is the focal length of the high NA objective, and θ is the numerical aperture.

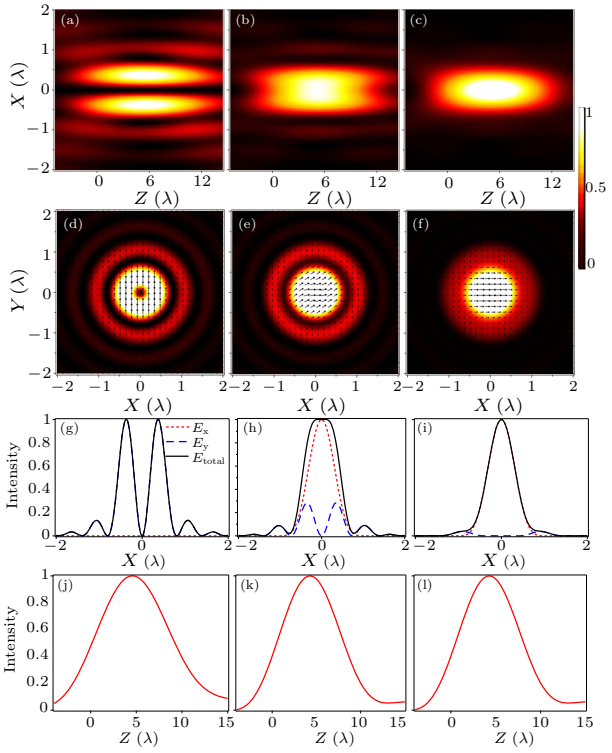

Fig. 2. Intensity distributions in the X–Z plane as a Gaussian beam with a pair of vortices of opposite charges focused by a high NA objective: (a) $a = 0$, (b) $a = 0.22w$, and (c) $a = 0.35w$. Here (d)–(f) show the same in the X–Y plane, (g)–(i) show the corresponding intensity distribution in the transverse direction at the point of maximum axial intensity, and (j)–(l) show the axial intensity distribution.

Without loss of generality and validity, it is proposed that the parameters are chosen as $\lambda = 632.8\,\text{nm}$, $w = 2\,\text{mm}$, $f = 2\,\text{mm}$, NA $= 0.9$, $m = 1$. The effect of location of the vortices on the beam pattern in the focal region is shown in Fig. 2. It is observed that when $a = 0$, the beam becomes a non-vortex azimuthally polarized beam and generates a focal hole as shown in Fig. 2(a). It is noted from Fig. 2(j) that the presence of dielectric interface shifts the position of an axial maximum intensity to 4.4λ and the depth of focus (DOF) of the generated focal hole segment is measured as 8λ. Here DOF is the full width at half-maximum (FWHM) of normalized axial intensity distribution. Figures 2(d) and 2(g) show that the FWHM of the generated focal hole is 1.18λ and the E_y component having central minimum is found to dominate the entire focal structure. It is observed that by setting $a = 0.22w$, the weights of E_x and E_y components are adjusted in such a way that the generated focal spot has a flat top profile having DOF as 8.47λ and the spot size as 0.942λ as shown in Figs. 2(b), 2(e), 2(h) and 2(k), respectively. We also observe that with further increasing a to $0.35w$, the E_y component with central minimum reduces and a highly confined focal spot is generated to have a dominating E_x component. The FWHM of the focal spot is measured as 0.68λ and the focal depths of the FWHM is 8.71λ, as shown in Figs. 2(c), 2(f) and 2(i), respectively. Such a transversely polarized focal spot having the minimum longitudinal component is highly useful in applications such as imaging the silicon integrated circuit. This is due to the fact that, though the radially polarized beam produced sharper focal spot, the strong longitudinal component of the radially polarized beam suffers discontinuity at the interface of two neighbouring media and enlarges the focal spot in the high NA medium. Hence the beam with strong longitudinal polarization is limited in applications such as silicon integrated circuits.[28] Thus by manipulating the location of the vortices one can tune the focal pattern from a sub wavelength focal hole to a sub wavelength transversely polarized focal spot. This is because initially only one dark hole is observed in the focal plane when two vortices with opposite topological charges are both located in the center of the incident beam. It then gradually disappears to form a sharp focal spot structure as the vortices move away from the center.

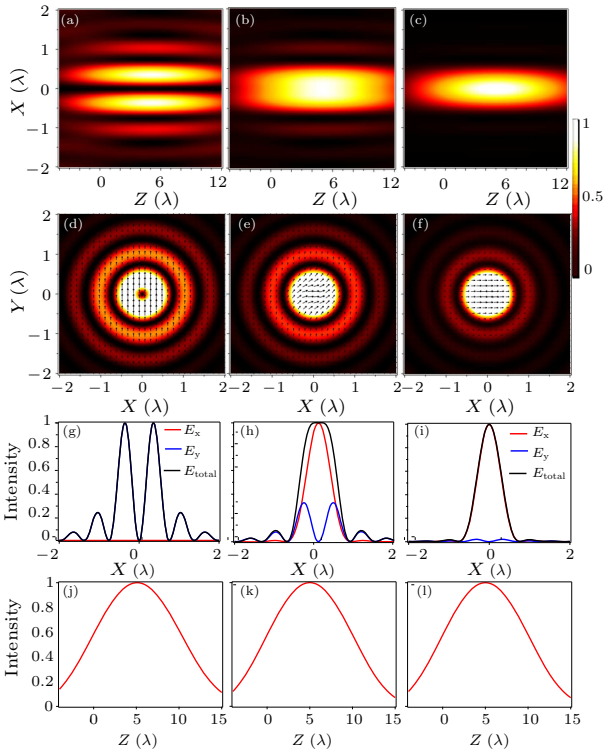

Fig. 3. Intensity distributions in the focal plane as a Gaussian beam with a pair of vortices of opposite charges focused by a high NA objective. Using annular obstruction $\delta = 0.5$ (a) $a = 0$, (b) $a = 0.25w$, and (c) $a = 0.35w$. Here (d)–(f) show the same in the X–Y plane, (g)–(i) show the corresponding intensity distribution in the transverse direction at the point of maximum axial intensity, and (j)–(l) show the axial intensity distribution.

Figure 3 shows the effect of annular obstruction

on the focal structure corresponding to the location of vortex in the Gaussian beam. It is observed for the annular obstruction with $\delta = 0.5$ and $a = 0$, the generated focal hole size slightly reduces to 1.15λ and focal depth is improved to 12.7λ as shown in Figs. 3(a), 3(d) and 3(g), respectively. It is noted from Fig. 3(j) that the position of maximum intensity is shifted to 5.3λ. However, by setting $a = 0.25w$, one can generate a flat top profile with focal depth of 11.9λ and spot size of 0.97λ as shown in Figs. 3(b), 3(e), 3(h) and 3(k), respectively.

We also observe that by setting $a = 0.35$, a highly confined focal spot with spot size of 0.68λ and focal depth of 12.4λ is observed. Thus using annular obstruction and properly tuning the distance between the vortices, one can generate a highly confined focal hole, focal spot and flat top profile with a large focal depth.

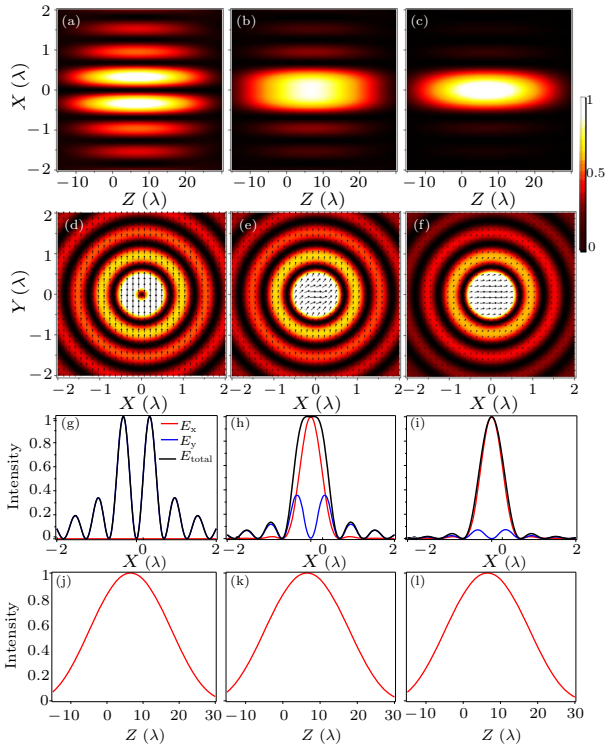

Fig. 4. Intensity distributions in the focal plane as a Gaussian beam with a pair of vortices of opposite charges focused by a high NA objective. Using annular obstruction $\delta = 0.75$: (a) $a = 0$, (b) $a = 0.27w$, and (c) $a = 0.35w$. Here (d)–(f) show the same in the X–Y plane, (g)–(i) show the corresponding intensity distribution in the transverse direction at the point of maximum axial intensity, and (j)–(l) show the axial intensity distribution.

Figure 4 shows the same as Fig. 3 but for the effect of annular obstruction with $\delta = 0.75$. It is observed that the further increase of the annular obstruction will increase the focal depth and reduce the spot size. It is noted that when $a = 0$, the generated focal hole has spot size of 1.15λ and the focal depth is much improved to 23.5λ, as shown in Figs. 4(a), 4(d) and 4(g). Such a dark channel may have many applications, ranging from atomic optics to single molecule detection, in which the dark regions of zero intensity are required.[28–31] Figures 4(b), 4(e), 4(h) and 4(k) show that by setting $a = 0.27$, a flat top focal structure with spot size of 0.91λ and focal depth of 24.5λ is observed. The flat top focus obtained above may also find other applications such as improving printing filling factor and improving uniformity and quality in materials processing and microlithography.

It is also observed when $a = 0.35$, Figs. 4(c), 4(f) and 4(g) show that the smallest transversely polarized focal spot having FWHM of 0.61λ and much improved focal depth of 24.5λ is observed. The DOF achieved here is much greater than the previously proposed method of generating transversely polarized focal field using the azimuthally polarized vortex beam.[21] Thus by properly tuning the distance between the vortex and by properly choosing the annular obstruction, one can manipulate focal structure for the incident pair of vortices. It observed that one can generate many novel focal structures such as transversely polarized focal hole and focal spot of sub wavelength size with super long focal depth and a highly confined flat top profile with the proposed system. Most existing optical tweezers use a focused Gaussian beam which has the highest intensity at the center. Thus they are only suitable for trapping and manipulating particles with a dielectric constant higher than the ambient. For particles with a dielectric constant lower than the ambient, a specifically designed laser mode such as a donut mode needs to be applied. However, using the proposed focus tailoring method, we can easily change the focal intensity distribution from a donut shape to a flat top profile and then to a peak-centered shape by properly tuning the distance between the vortex dipoles. This enables trapping and manipulating a wide variety of particles in the same optical system. Such a method may also find applications in material processing, semiconductor inspection, etc.

In conclusion, the focusing properties of the azimuthally polarized optical vortex dipole through a dielectric interface have been investigated theoretically by the vector diffraction theory. It is observed that by properly manipulating the distance between the vortex dipoles one can generate many novel focal patterns such as focal hole, flat top profile and highly confined focal spot suitable for micro-particle trapping, manipulation and material processing. It is also observed that by adding annular obstruction to the incident dipole vortex beam can improve the focal depth of the focal patterns generated.

References

[1] Torok P, Varga P, Laczik Z and Booker G J 1995 *J. Opt.*

Soc. Am. A **12** 325
[2] Biss D P and Brown T G 2001 *Opt. Express* **9** 490
[3] Palacios D M et al 2004 *Phys. Rev. Lett.* **92** 143905
[4] Tao S H, Yuan X C, Lin J and Burge R E 2006 *Opt. Express* **14** 535
[5] Zhan Q W 2006 *Opt. Lett.* **31** 867
[6] Singh R K, Senthilkumaran P and Singh K 2009 *J. Opt. Soc. Am.* A **26** 576
[7] Chen B S and Pu J X 2009 *Appl. Opt.* **48** 1288
[8] Molina-Terriza G, Torner L, Wright E M, García-Ripoll J J and Pérez-García V M 2001 *Opt. Lett.* **26** 1601
[9] Zhang Z, Pu J X and Wang X 2008 *Opt. Commun.* **281** 3421
[10] Zhang Z, Pu J X and Wang X 2008 *Chin. Phys. Lett.* **25** 1664
[11] Indebetouw G 1993 *J. Mod. Opt.* **40** 73
[12] Roux F S 2004 *J. Opt. Soc. Am.* B **21** 655
[13] Born M and Wolf E 1999 *Principles Opt.* 7th edn (Cambridge: Cambridge University Press)
[14] Chen Z, Pu J X and Zhao D M 2011 *Phys. Lett.* A **375** 2958
[15] Fang G J, Tian B and Pu J X 2012 *Opt. Laser Technol.* **44** 441
[16] Amala Prathiba Janet C, Udhayakumar M, Rajesh K B, Jaroszewicz Z and Pillai T V S 2016 *Opt. Quantum Electron.* **48** 521
[17] Amala Prathiba Janet C, Udhayakumar M, Rajesh K B, Jaroszewicz Z and Pillai T V S 2016 *Chin. Phys. Lett.* **33** 124206
[18] Huang K, Ye H, Liu H, Teng J, Yeo S P and Qiu C W 2014 arXiv:1406.3823 [physics.optics]
[19] Qin F, Huang K, Wu J, Jiao J, Luo X, Qiu C and Hong M 2015 *Sci. Rep.* **5** 1
[20] Machavariani G and Lumer Y 2007 *Opt. Lett.* **32** 1468
[21] Hao X, Kuang C F, Wang T and Liu X 2010 *Opt. Lett.* **35** 3928
[22] Lalithambigai K, Anbarasan P M and Rajesh K B 2015 *Opt. Quantum Electron.* **47** 1027
[23] Sundaram, C M, Prabakaran, K, Anbarasan, P M, Rajesh, K B and Musthafa A M 2016 *Chin. Phys. Lett.* **33** 064203
[24] Prabakaran C M, Rajesh K K B, Udhayakumar M, Anbarasan P M and Musthafa A M 2016 *Opt. Quantum Electron.* **48** 507
[25] Sundaram C M, Prabakaran K, Anbarasan P M, Rajesh K B, Musthafa A M and Aroulmoji V 2017 *Opt. Quantum Electron.* **49** 11
[26] Torok P, Varga P and Booker G R 1995 *J. Opt. Soc. Am.* A **12** 2136
[27] Young W K S and Brown T G 2000 *Opt. Express* **7** 77
[28] Qin F, Huang K, Wu J, Jiao J, Luo X, Qiu C and Hong M 2015 *Sci. Rep.* **5** 9977
[29] Helseth L E 2002 *Opt. Commun.* **212** 343
[30] Klar T A, Engel E and Hell S W 2001 *Phys. Rev.* E **64** 066613
[31] Engel E, Huse N, Klar T A and Hell S W 2003 *Appl. Phys.* B **77** 11