CHIN. PHYS. LETT. Vol. 34, No. 7 (2017) 070401

Accretion onto the Magnetically Charged Regular Black Hole

M. Azam**, A. Aslam

Division of Science and Technology, University of Education, Lahore 54590, Pakistan

(Received 6 April 2017)

We investigate the accretion process for static spherically symmetric geometry, i.e., magnetically charged regular black hole with isotropic fluid. We obtain generalized expressions for the velocity ($u(r)$), speed of sound (c_s^2), energy density ($\rho(r)$) and accretion rate ($\dot{M}$) at the critical point near the regular black hole during the accretion process. We also plot these physical parameters against fixed values of charge, mass and different values of equation of state parameter to study the process of accretion. We find that radial velocity and energy density of the fluid remain positive and negative as well as rate of change of mass is increased and decreased for dust, stiff, quintessence fluid and phantom-like fluid, respectively.

PACS: 04.70.Bw, 04.70.Dy, 95.35.+d DOI: 10.1088/0256-307X/34/7/070401

New evolution in cosmology reports that our universe is expanding, and the expansion rate is increasing step-by-step, i.e., the universe is sustaining an accelerating phase. The supernova type Ia[1] and large-scale structure[2,3] also confirmed the expansion of the universe. This behavior of the universe is provoked by some exotic matter that has positive energy density and negative pressure feature, noted as dark energy (DE). Dark energy has anti-gravitational properties. By observational estimations, 74%, 22% and 4% of our universe are engaged by DE, dark matter (DM) and ordinary matter, respectively. In the current era, DE is the most pressing issue in astrophysics. The cosmological constant (Λ) of DE candidates will satisfy the equation of state (EoS) $p = \omega\rho$ (ρ is density) when $\omega = -1$,[4–6] while for quintessence ($-1 < \omega < \frac{-1}{3}$) and $\omega < -1$ for phantom-like fluid.[7,8]

The presence of essential singularities is a dominant problem in the subject of general relativity (GR). To avert the singularities, regular black holes (RBHs) have been proposed. The metrics of these RBHs are regular and are the solution of Einstein's field equations with no singularities. Somewhere in space-time,[9,10] these RBHs violate the strong energy condition (SEC) and satisfy the weak energy condition (WEC).

There are many physical features of BHs, which have been discussed in different phenomena. One of them is accretion onto BHs, when extensive objects (stars, neutron stars/x-ray binaries etc.) try to catch a molecule of fluid from its neighbor, formerly the mass of that object has been changed and this process is noted as accretion. The issue of accretion onto compact objects was introduced by Bondi[11] adopting the Newtonian theory of gravity. Michel retrieved the accretion onto the Schwarzschild BH.[12] Babichev *et al.*[13,14] worked out the accretion onto the static Schwarzschild BH of phantom DE, which reveals that the mass of BH will decrease by cause of negative pressure of energy and at the end turns to zero near large slit singularity. DM and DE accretion onto some static BHs has been investigated by Kim *et al.*[15] Furthermore, Debnath[16] formulated a framework of accretion onto modified Hayward BH. Abbas discussed the accretion techniques for three-dimensional BHs, which draws on theory of Einstein–Power–Maxwell.[17] Martín-Moruno *et al.* extended the accretion formalism for the model given by Babichev–Dokuchaev–Eroshenko in the case of non-static metric and also analyzed for positive cosmological constant.[18,19] The change in mass of the Schwarzschild BH due to scalar field is reviewed by Rodrigues and Bernardiniz.[20] In this context, Bahamonde and Jamil[21] as well as Jawad and Umair[22] probed accretion onto different regular BHs. Many researchers have also presented the accretion formalism of DE onto different types of BHs.[23–27]

In this work, we have drawn out the formalism for magnetically charged BH. We investigate the consequences of regular BH mass by electing different values of parameter (ω). The formalism for accretion is studied. Moreover, we reveal velocity profile ($u(r)$), energy density ($\rho(r)$) and rate of change of RBH mass ($\dot{M}$) by plotting a graph. Finally, we discuss the results.

The magnetically charged regular black hole is given by[28]

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \tag{1}$$

where

$$f(r) = 1 + \frac{M}{\pi r}\Big[\ln\frac{32M^2r^2 + 8\pi Mq^2r + \pi^2q^4}{32M^2r^2 - 8\pi Mq^2r + \pi^2q^4} - 2\arctan\left(1 + \frac{8Mr}{\pi q^2}\right) + 2\arctan\left(1 - \frac{8Mr}{\pi q^2}\right)\Big],$$

where M and q are the mass and charge of BH, respectively. The energy-momentum tensor for the isotropic fluid is illustrated as follows:

$$T^{\alpha\beta} = (\rho + p)u^\alpha u^\beta + pg^{\alpha\beta}, \tag{2}$$

where p and ρ are the pressure and energy density, respectively. The 4-velocity u^α is given by $u^\alpha =$

**Corresponding author. Email: azam.math@ue.edu.pk

© 2017 Chinese Physical Society and IOP Publishing Ltd

$\frac{dx^\alpha}{ds} = (u^t, u^r, 0, 0)$, with u^t and u^r being non-zero components of velocity that satisfy the normalization condition $u_\alpha u^\alpha = -1$, and as a result we obtain $u^t = \sqrt{[(u^1)^2 + f(r)]/f^2(r)}$. We use the radial velocity component of fluid as $u^1 = u$ and $u_0 = g_{00}u^0 = -\sqrt{u^2 + f(r)}$, $\sqrt{-g} = r^2 \sin\theta$ to obtain $T^1_0 = (\rho + p)u_0 u$ from Eq. (2).

The conservation of Bernoulli energy equation ($T^{\alpha\beta}_{;\beta} = 0$) leads to

$$r^2 u(\rho + p)[u^2 + f(r)]^{1/2} = c_1, \tag{3}$$

where c_1 is a constant of integration. The energy flux equation can be determined by the extension of conservation law onto the fluid 4-velocity, i.e., $u_\alpha T^{\alpha\beta}_{;\beta} = 0$, which yields

$$ur^2 \exp\Big[\int_{\rho_\infty}^{\rho_h} \frac{d\rho}{\rho + p}\Big] = c. \tag{4}$$

From Eqs. (3) and (4), we have

$$(\rho + p)[u^2 + f(r)]^{1/2} \exp\Big[-\int_{\rho_\infty}^{\rho_h} \frac{d\rho}{\rho + p}\Big] = c_2, \tag{5}$$

where we have used $c_2 = c_1/c$. The equation of mass flux ($J^\alpha_{;\alpha} = 0$) yields

$$\rho u r^2 \sin\theta = A_1 \Longrightarrow \rho u r^2 = c_3, \tag{6}$$

where $c_3 = A_1/\sin\theta$. Substituting Eq. (6) into Eq. (3), we have

$$\frac{(\rho + p)}{\rho}[u^2 + f(r)]^{1/2} = c_4, \tag{7}$$

where $c_1/c_3 = c_4$. Differentiating Eqs. (6) and (7) and using $V^2 = \frac{d\ln(\rho+p)}{d\ln\rho} - 1$, we have

$$[V^2 - \frac{u^2}{u^2 + f(r)}]\frac{du}{u} + \Big[2V^2 - \frac{rf'(r)}{2(u^2 + f(r))}\Big]\frac{dr}{r} = 0. \tag{8}$$

The velocities of the fluid flow at the critical point can be obtained by putting the coefficient of du/u and dr/r equal to zero, which yields

$$V_c^2 = \frac{u_c^2}{u_c^2 + f(r_c)}, \tag{9}$$

$$\frac{4V_c^2}{r_c} = \frac{f'(r_c)}{u_c^2 + f(r_c)}, \tag{10}$$

where u_c is the critical speed of fluid. Equations (10) and (11) can be rewritten as

$$u_c^2 = \frac{1}{4} r_c f'(r_c), \tag{11}$$

$$V_c^2 = \frac{r_c f'(r_c)}{r_c f'(r_c) + 4f(r_c)}. \tag{12}$$

The speed of sound turns out to be

$$\frac{dp}{d\rho}\Big|_{r=r_c} = c_s^2 = c_4\sqrt{\frac{1}{u_c^2 + f(r_c)}} - 1. \tag{13}$$

Furthermore, the rate of change of mass of BH can be characterized as[16]

$$\dot{M}_{\rm acc} = 4\pi c_3 M^2(\rho + p), \tag{14}$$

where dot serves as the derivative of mass w.r.t. time. From the above equation, we observe that BH mass depends on the value of $(\rho + p)$. The mass of BH will increase (decrease) when $\rho + p > 0$ ($\rho + p < 0$). Generally, the BH mass will increase as a result of accretion and decrease in the case of the Hawking radiation.[21]

In the following we apply the above formalism to the RBHs defined in Eq. (1) with barotropic fluid EoS $p(r) = \omega\rho(r)$. From Eqs. (3) and (6), we have

$$u(r) = \Big[-f(r) + \frac{c_4^2}{(\omega + 1)^2}\Big]^{1/2}, \tag{15}$$

$$\rho(r) = \frac{c_3}{r^2[-f(r) + \frac{c_4^2}{(\omega+1)^2}]^{1/2}}, \tag{16}$$

which represent the velocity and energy density of fluid, respectively. The mass for magnetically charged RBH as a result of accretion using Eqs. (14) and (16) leads to

$$\dot{M} = \frac{4\pi c_4 c_3^2(\omega + 1)}{r^2[-f(r)(\omega + 1)^2 + c_4^2]^{1/2}}. \tag{17}$$

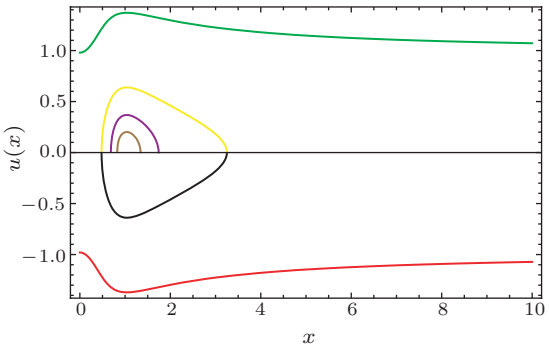

Fig. 1. Velocity profile of fluid against $x = r/M$ for $c_4 = 0.7$, $M = 1$, $q = 1.055$ and various values of ω i.e., -2 (black), -1.5 (red), -0.5 (green), 0 (yellow), 0.5 (purple) and 1 (brown) for the magnetically charged BH.

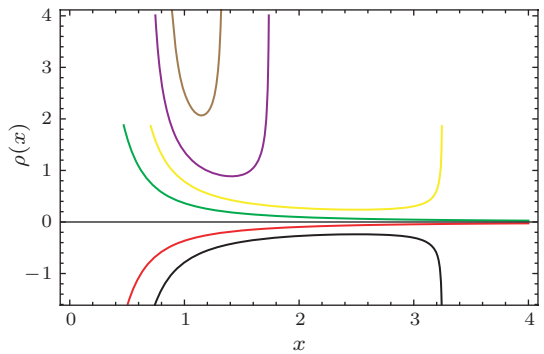

Fig. 2. Energy density of fluid against $x = r/M$ for $c_3 = 0.5$, $c_4 = 0.7$, $M = 1$, $q = 1.055$ and various values of ω i.e., -2 (black), -1.5 (red), -0.5 (green), 0 (yellow), 0.5 (purple) and 1 (brown) for the magnetically charged BH.

Now, we plot the graph for the above expressions to analyze the behavior of fluid near BH. Figure 1 represents the velocity profile for various values of ω. Here the value of ω represents different candidates of DE such that for cosmological constant ($\omega = -1$), dust ($\omega = 0$), stiff ($\omega = 1$), phantom energy ($\omega < -1$) and quintessence ($-1 < \omega < -1/3$). The behavior of velocity profile is compatible with the result discussed in Ref. [21]. It is noted that the velocity profile has negative behavior for phantom ($\omega < -1$) and positive for quintessence ($-1 < \omega < -1/3$) as well as dust and stiff fluid, respectively. Figure 2 represents the energy density graph of fluid flow near the RBH. It depicts that energy density decreases for phantom-like fluid when $\omega = -2$ and increases for $\omega = 0$, 0.5 and 1. For the value of $\omega = -1.5$ and 0.5, the energy density approaches to zero ($\rho \to 0$) at infinity.

Figure 3 shows the rate of change of mass of RBH for various values of ω. It is noted that the mass of RBH increases or decreases for different values of ω. The RBH mass will increase for $\omega = 0$, 0.5 and 1 at $x = 3.2$, 1.6 and 1.2, respectively, and decreases for $\omega = -2$ at $x = 3.2$, which shows that the mass of RBH as a result of accretion of dust, stiff and quintessence matter increases and the reverse case will occur for phantom energy.

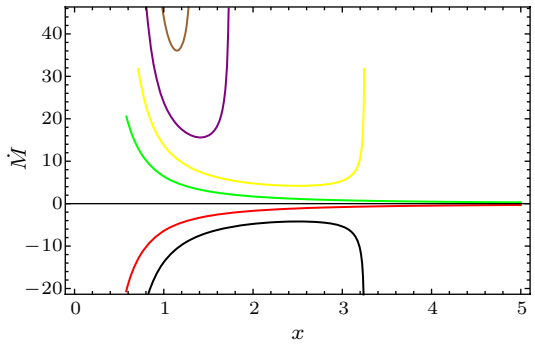

Fig. 3. The value of $\dot{M}$ against $x = r/M$ for $c_3 = 0.5$, $c_4 = 0.7$, $M = 1$, $q = 1.055$ and various values of ω, i.e., -2 (black), -1.5 (red), -0.5 (green), 0 (yellow), 0.5 (purple) and 1 (brown) for the magnetically charged BH.

In summary, we have investigated the accretion onto the magnetically charged RBH. In this scenario, we have adopted the technique of Bahamonde and Jamil [21] and obtain the critical velocities and nature of the speed of sound for RBH. Furthermore, we have evaluated the role of velocity, energy density and accretion rate $\dot{M}$ of RBH for different values of EoS parameters. We have evaluated the above quantities through conservation law and barotropic EoS. We have analyzed the behaviors of velocity profile, energy density and rate of change of mass corresponding to fixed values of $q = 1.055$, $M = 1$ and different values of equation of state parameter (EoS) ω, because for these values we have physical behavior of the given model which is similar to the results discussed in Refs. [21,22]. If we use different values of q and M then the above expressions for the given model do not show any graphical significance for some values of the EoS parameter. Hence it is worthwhile to mention that model parameters play a significant role on the accretion process. Also, the velocity profile of fluid is positive for $\omega > -1$ and negative for $\omega < -1$, which represent the dust, stiff, quintessence matter and phantom-like fluid, respectively. The graph of energy density represents negative and positive behaviors corresponding to the above candidates of DE like velocity profile. On the other hand, these candidates of DE become the cause of increase and decrease in the mass of RBH.

References

[1] Perlmutter S et al 1999 *Astrophys. J.* **517** 565
[2] Riess A G et al 1998 *Astron. J.* **116** 1009
[3] Eisenstein D J et al 2005 *Astrophys. J.* **633** 560
[4] Sahni V and Starobinsky A A 2000 *Int. J. Mod. Phys.* D **09** 373
[5] Padmanabhan T 2003 *Phys. Rep.* **380** 235
[6] Nojiri S and Odintsov S 2011 *Phys. Rep.* **505** 59
[7] Peebles P J E and Ratra B 1988 *Astrophys. J.* **325** L17
[8] Caldwell R R, Dave R and Steinhardt P J 1998 *Phys. Rev. Lett.* **80** 1582
[9] Elizalde E and Hildebrandt S R 2002 *Phys. Rev.* D **65** 124024
[10] Zaslavskii O B 2010 *Phys. Lett.* B **688** 278
[11] Bondi H 1952 *Mon. Not. R. Astron. Soc.* **112** 195
[12] Michel F C 1972 *Astrophys. Space Sci.* **15** 153
[13] Babichev E et al 2004 *Phys. Rev. Lett.* **93** 021102
[14] Babichev E, Dokuchaev V and Eroshenko Y 2005 *J. Exp. Theor. Phys.* **100** 528
[15] Kim S W and Kang Y 2012 *Int. J. Mod. Phys. Conf. Ser.* **12** 320
[16] Debnath U 2015 *Eur. Phys. J.* C **75** 129
[17] Abbas G 2013 *Chin. Phys. Lett.* **30** 100403
[18] Martín-Moruno P et al 2006 *Phys. Lett.* B **640** 117
[19] Martín-Moruno P et al 2009 *Gen. Relativ. Gravitation* **41** 2797
[20] Rodrigues M G and Bernardiniz A E 2012 *Int. J. Mod. Phys.* D **21** 1250075
[21] Bahamonda S and Jamil M 2015 *Eur. Phys. J.* C **75** 508
[22] Jawad A and Umair M 2016 *Eur. Phys. J.* C **76** 123
[23] Lima J A S 2010 *Phys. Lett.* B **693** 218
[24] Sharif M and Abbas G 2011 *Chin. Phys. Lett.* **28** 090402
[25] Sharif M and Abbas G 2012 *Chin. Phys. Lett.* **29** 010401
[26] Nayak B and Jamil M 2012 *Phys. Lett.* B **709** 118
[27] Dwivedee D et al 2014 *J. Astrophys. Astron.* **35** 97
[28] Ma M S 2015 *Ann. Phys.* **362** 529