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Implementing Classical Hadamard Transform Algorithm by Continuous Variable Cluster State *

Yu Wang(王宇)**, Qi Su(苏琦)

State Key Laboratory of Cryptology, Beijing 100878

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Measurement-based one-way quantum computation, which uses cluster states as resources, provides an efficient model to perform computation. However, few of the continuous variable (CV) quantum algorithms and classical algorithms based on one-way quantum computation were proposed. In this work, we propose a method to implement the classical Hadamard transform algorithm utilizing the CV cluster state. Compared with classical computation, only half operations are required when it is operated in the one-way CV quantum computer. As an example, we present a concrete scheme of four-mode classical Hadamard transform algorithm with a four-partite CV cluster state. This method connects the quantum computer and the classical algorithms, which shows the feasibility of running classical algorithms in a quantum computer efficiently.

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Quantum computation (QC) is expected to be more efficient than classical computation. A quantum computer is built from a circuit containing wires and elementary quantum gates to carry around and manipulate the quantum information.[1] In recent years, esoteric and attractive ideas about quantum computers have been incrementally converted into the visible realization, along with the experimental demonstrations of various quantum logic operations in both discrete variable (DV) and continuous variable (CV) domains.[2,3] Furthermore, some famous quantum algorithms have been realized theoretically and experimentally, such as Shor's algorithm,[4] Grover algorithm,[5] and solving linear equations algorithm.[6,7]

Measurement-based one-way QC performs computation by measurement and classical feedforward on a multipartite cluster entangled state.[8,9] Following the first theoretical proposal on universal quantum computation with CV cluster states,[9] four-partite and eight-partite CV cluster states have been successfully prepared.[10–13] Recently, up to 10000–qumode[14] optical CV cluster states have been prepared for QC. Based on the prepared CV cluster state, several quantum logic gates have been demonstrated, such as squeezing and Fourier transformation operation,[15,16] controlled-x operation,[17] controlled-phase[18] and a gate sequence.[19]

In the development of QC, some fundamental questions still need to be answered, such as which classical math algorithm can be speeded up in a quantum way, and which math problems are always run faster with the classical computer than a quantum one? The classical computer may be incapable of computing quantum algorithms, so can quantum computers carry out some classical algorithms more efficiently? If we find some classical algorithms which can be performed in quantum computers, the scope and application of the quantum computation will be expanded.

The classical Hadamard transform (HT), which can be regarded as being built out of size-2 discrete Fourier transform (DFT), is a special case of a classical example of fast Fourier transform (FFT).[20] The basic element of HT, whose transformation matrix is $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, is also called the Hadamard gate in quantum mechanics. Many research groups have carried out research on how to achieve the Hadamard gate. A probabilistic Hadamard gate has been demonstrated by coherent state qubits,[21] and squeezed superposition of coherent states.[22] However, higher order research on HT is very little, except in 2012 quantum interference observed as a function of Berry's phase in a Hadamard optical network.[23]

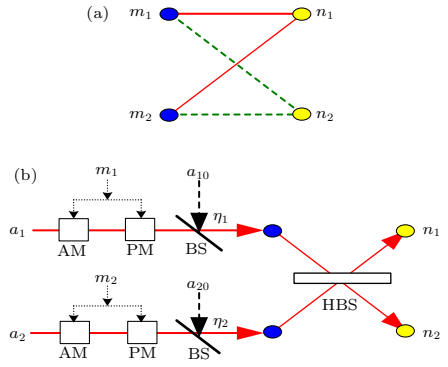

Fig. 1. (Color online) Basic element for achieving HT with the cluster state. (a) Two-mode classical HT algorithm, red lines (solid): addition operation; green lines (dashed): subtraction operation. (b) Theoretical scheme of the basic element HT algorithm with the EPR state, a_1 and a_2: EPR states; AM: amplitude modulator; PM: phase modulator; HBS: half beam splitter; BS: beam splitter; a_{10} and a_{20}: vacuum states; η_1 and η_2: transmittances of BS.

Although Gaussian operations can be simulated efficiently on a classical computer, it is not well understood whether Gaussian operations and preparations are sufficient to reproduce classical algorithms in the general case. In this work, we propose a method to

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**Corresponding author. Email: wangy@sklc.org

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achieve the HT algorithm with CV cluster states and Gaussian operations, which would expand the application scenarios of one-way QC. Actually our scheme is a realization of optical information processing, and it can be applied directly to the one-way QC model by using cluster states. We find that the use of one-way QC with the cluster state to achieve the HT algorithm requires fewer operations than classical computation, i.e., the one-way CV cluster QC needs only half the operations of the normal classical computation. As an example, we show the implementation of the four-mode HT algorithm with four-partite CV cluster states. The signal-to-noise ratio (SNR) is used to quantify the performance of the HT algorithm. It should be emphasized that we do not propose a new quantum algorithm for one-way QC, but propose a new method to realize a special classical algorithm (HT algorithm) by using the CV one-way QC.

The HT, which is also named as the Walsh–Hadamard transform, Hadamard–Rademacher–Walsh transform, Walsh transform, and Walsh–Fourier transform, is an example of a generalized class of Fourier transforms. It performs an orthogonal, symmetric, linear operation on 2^k real numbers.[20] HT is widely used in computer science and many other different applications, such as power spectrum analysis, digital signal processing, filtering, processing speech, medical signal fields, calculating large integer multiplication, multiplexing and coding in communications, characterizing non-linear signals, solving non-linear differential equations and calculating the linear approximation of any Boolean functions to the linear functions.

Given a $2^k \times 2^k$ matrix H_{2^k}, and a 2^k real-number input unit vector $\boldsymbol{M}$, we would like to calculate the 2^k real-number output vector $\boldsymbol{N}$, where the relationship between input vector and output vector satisfies $\boldsymbol{N} = H_{2^k} \times \boldsymbol{M}$. For $k \geq 1$, the matrix H_{2^k} is

$$H_{2^k} = H_2 \otimes H_{2^{k-1}}, \tag{1}$$

where we have $H_1 = 1$, $H_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, and the normalization coefficient is omitted.

The cluster state is a type of multipartite quantum entangled graph state corresponding to some mathematical graphs.[9,24–26] The CV cluster quadrature correlations (so-called nullifiers) can be expressed by[24–26]

$$(\hat{p}_a - \sum_{b \in N_a} \hat{x}_b) \to 0, \ \forall a \in G, \tag{2}$$

where $\hat{x}_a = (\hat{a} + \hat{a}^\dagger)/2$ and $\hat{p}_a = (\hat{a} - \hat{a}^\dagger)/2i$ represent the quadrature-amplitude and quadrature-phase operators of an optical mode $\hat{a}$, respectively. The subscript $a(b)$ expresses the designated mode $\hat{a}$ ($\hat{b}$). The modes of $a \in G$ denote the vertices of the graph G, while the modes of $b \in N_a$ are the nearest neighbors of mode $\hat{a}$. For an ideal cluster state, the left-hand side of Eq. (2) tends to zero, which represents a simultaneous zero eigenstate of the quadrature combination.[26] The CV cluster quantum entanglements generated by experiments are deterministic, but imperfect, the entanglement features of which have to be verified and quantified by the sufficient conditions for the full inseparability of multipartite CV entanglement.[10,11]

The classical HT algorithm can be decomposed into several basic elements (the transform matrix H_2) which is shown in Fig. 1(a). The basic element of classical HT algorithm consists of two input modes m_1 and m_2, two output modes n_1 and n_2, one addition operation and one subtraction operation. After utilizing addition and subtraction operations to the input vector $\boldsymbol{M} = \binom{m_1}{m_2}$, we obtain the output vector $\boldsymbol{N} = \binom{n_1}{n_2}$. If the input modes and output modes satisfy $n_1 = m_1 + m_2$ and $n_2 = m_1 - m_2$, the simplest and most basic HT algorithm is achieved.

Now we explain how to achieve the basic element of the HT algorithm with the basic optical element: a half beam splitter (HBS) and a two-partite CV cluster state. The quadrature components of the CV cluster state are used to compute and transmit information. The nullifiers of the two-partite CV cluster state can be obtained from those of the Einstein–Podolsky–Rosen (EPR) entangled state with a local phase shift, thus the two states are equivalent up to a phase shift in the infinite squeezing limit. For the EPR entangled states, the quadratures satisfy $\langle \Delta^2(\hat{x}_{a_1} + \hat{x}_{a_2}) \rangle = 2e^{-2r}$ and $\langle \Delta^2(\hat{p}_{a_1} - \hat{p}_{a_2}) \rangle = 2e^{-2r}$, where $0 < r < \infty$ is the correlation parameter of the EPR entangled states, $r = 0$ without any quantum correlation and $r \to \infty$ corresponding to the ideal correlation,[27] the entanglement degree of the EPR state is $-10 \log e^{-2r}$. Firstly we encode the information (m_1 and m_2) of the input vector, which can be generated from the signal generator, onto the quadrature components of the EPR state $\hat{a}_1$ and $\hat{a}_2$ by amplitude and phase modulators, respectively (Fig. 1(b)). The frequency of modulated modes is usually about several MHz, which is at the side bands of the optical frequency. The measurement frequency of the homodyne detection is chosen to be the same as the frequency of the modulated modes. After interference on the HBS, the amplitude quadratures and the phase quadratures of $\hat{n}_1$ and $\hat{n}_2$ can be expressed as

$$\begin{aligned}
\hat{x}_{n_1} &= \frac{1}{\sqrt{2}}(\hat{x}_{a_1} + \hat{x}_{a_2} + \hat{x}_{m_1} + \hat{x}_{m_2}), \\
\hat{p}_{n_1} &= \frac{1}{\sqrt{2}}(\hat{p}_{a_1} + \hat{p}_{a_2} + \hat{p}_{m_1} + \hat{p}_{m_2}), \\
\hat{x}_{n_2} &= \frac{1}{\sqrt{2}}(\hat{x}_{a_1} - \hat{x}_{a_2} + \hat{x}_{m_1} - \hat{x}_{m_2}), \\
\hat{p}_{n_2} &= \frac{1}{\sqrt{2}}(\hat{p}_{a_1} - \hat{p}_{a_2} + \hat{p}_{m_1} - \hat{p}_{m_2}).
\end{aligned} \tag{3}$$

Among the amplitude and phase quadratures of modes $\hat{n}_1$ and $\hat{n}_2$, the amplitude quadrature of $\hat{n}_1$ and the phase quadrature of $\hat{n}_2$ are used to realize the two-mode HT algorithm. For the ideal EPR entangled states, we can write variances of the amplitude quadrature of $\hat{n}_1$ and the phase quadrature of $\hat{n}_2$ in

the following form[28]

$$\langle\delta^2(\hat{x}_{n_1})\rangle = \frac{1}{2}\langle\delta^2(\hat{x}_{m_1}+\hat{x}_{m_2})\rangle,$$
$$\langle\delta^2(\hat{p}_{n_2})\rangle = \frac{1}{2}\langle\delta^2(\hat{p}_{m_1}-\hat{p}_{m_2})\rangle. \quad (4)$$

When the resource state is an ideal EPR entangled state, we can finally use the homodyne technology to obtain the element of the output vector: m_1+m_2 and m_1-m_2 simply by measuring the amplitude quadrature of $\hat{n}_1$ and the phase quadrature of $\hat{n}_2$. Here we have assumed that the variance of modulated signal $\hat{x}_{n_i}$ and $\hat{p}_{n_i}$ $(i=(1,2))$ are equal. The outputs $\hat{x}_{n_1}$, $\hat{p}_{n_2}$ represent the outputs (n_1 and n_2) of the HT algorithm, respectively. With the help of the optical interference on the HBS, we can achieve one addition and one subtraction operation at the same time, while the classical computer needs to carry out the addition and subtraction operation separately. Therefore, The number of operations needed by the HT algorithm using the CV one-way quantum computer with the cluster state is just half of the classical computer.

The calculation we discuss here is the classical data by using quantum computer, and just classical calculation result would be needed, which is quite different from quantum computation. In quantum computation theory, quantum states are the basic computational resources and carriers of quantum information. Through the evolution and operation of the quantum state one can obtain the required calculation results, and all the parameters of the quantum state in the evolution should be changed at the same time. For example, in CV quantum theory the amplitude quadrature $\hat{x}$ and the phase quadrature $\hat{p}$ could correspond to quadrature amplitudes of a mode in the electromagnetic field. In the Heisenberg picture, applying a Hamiltonian $\hat{H}$ gives a time evolution for operators $\dot{A}=i[\hat{H},\hat{A}]$. In our theory, we only need to use partial parameters of the quantum state to obtain the final calculation result. The remaining parameters of the quantum state would be used for identity authentication, eavesdropping detection, error correction and so on. Such as in the scheme of basic element for achieving HT with the cluster state (Fig. 1), we just use the amplitude quadrature of $\hat{n}_1$ $(\hat{x}_{n_1})$ and the phase quadrature of $\hat{n}_2$ $(\hat{p}_{n_2})$, but not the phase quadrature of $\hat{n}_1$ $(\hat{p}_{n_1})$ and the amplitude quadrature of $\hat{n}_2$ $(\hat{x}_{n_2})$.

Although our scheme can be considered as a realization of optical information processing, it should be applied directly to the one-way QC model by using cluster states, because the output of our scheme can be fed back directly to other cluster states which can perform the subsequent job. In our scheme, both input and output are classical data, thus we analyze the classical SNR[29] to compare the level of a desired signal to the level of background noise. SNR can be given by the following equation with the variance of the signal $\langle\delta^2(\text{sig})\rangle$ and noise $\langle\delta^2(\text{noise})\rangle$,[29]

$$\text{SNR} = 10\log_{10}\left(\frac{\langle\delta^2(\text{sig})\rangle}{\langle\delta^2(\text{noise})\rangle}\right). \quad (5)$$

The real cluster state is affected by ambient noise during transmission, which can be described by the beam splitter model (Fig. 1(b)). The transmittance of BSs are η_1 and η_2, we assume $\eta_1=\eta_2=\eta$. After interference at HBS, the amplitude quadrature of $\hat{n}_1$ and the phase quadrature of $\hat{n}_2$ can be calculated by

$$\hat{x}_{n_1} = \frac{1}{\sqrt{2}}(\sqrt{\eta}\hat{x}_{a_1}+\sqrt{\eta}\hat{x}_{a_2}+\sqrt{\eta}\hat{x}_{m_1}+\sqrt{\eta}\hat{x}_{m_2} + \sqrt{1-\eta}\hat{x}_{a_{10}}+\sqrt{1-\eta}\hat{x}_{a_{20}}),$$
$$\hat{p}_{n_2} = \frac{1}{\sqrt{2}}(\sqrt{\eta}\hat{p}_{a_1}-\sqrt{\eta}\hat{p}_{a_2}+\sqrt{\eta}\hat{p}_{m_1}-\sqrt{\eta}\hat{p}_{m_2} + \sqrt{1-\eta}\hat{p}_{a_{10}}-\sqrt{1-\eta}\hat{p}_{a_{20}}), \quad (6)$$

where $\hat{x}_{a_{10}}$ $(\hat{x}_{a_{20}})$ and $\hat{p}_{a_{10}}$ $(\hat{p}_{a_{20}})$ are the amplitude and phase quadratures of the vacuum state a_{10} (a_{20}), respectively. For the non-ideal EPR entangled states, the variances of amplitude quadrature of $\hat{n}_1$ and the phase quadrature of $\hat{n}_2$ can be written as

$$\langle\delta^2(\hat{x}_{n_1})\rangle = \frac{\eta}{2}\langle\delta^2(\hat{x}_{m_1}+\hat{x}_{m_2})\rangle + \eta e^{-2r}+1-\eta,$$
$$\langle\delta^2(\hat{p}_{n_2})\rangle = \frac{\eta}{2}\langle\delta^2(\hat{p}_{m_1}-\hat{p}_{m_2})\rangle + \eta e^{-2r}+1-\eta. \quad (7)$$

The first item of each equation in Eq. (7) ($\langle\delta^2(\hat{x}_{m_1}+\hat{x}_{m_2})\rangle$ and $\langle\delta^2(\hat{p}_{m_1}-\hat{p}_{m_2})\rangle$) can be regarded as the signal $\langle\delta^2(\text{sig})\rangle$ and the rest as the noise $\langle\delta^2(\text{noise})\rangle$.

Based on Eq. (5), Fig. 2(a) shows the dependence of the SNR on entanglement degree of the CV cluster state, where $\langle\delta^2(\text{sig})\rangle$ is chosen to be 10. When no entanglement exists, SNR will correspond to the classical limit. We compare the dependence of the SNR on entanglement degree of the CV cluster state when the transmittance equals 0.2, 0.4, 0.6 and 0.8, respectively.

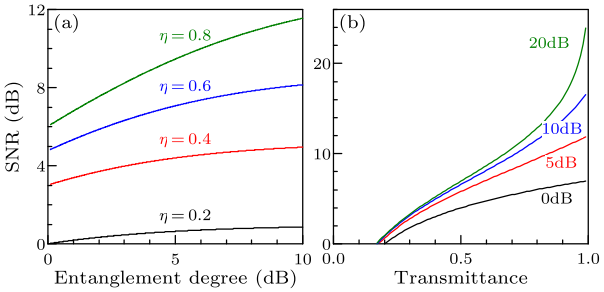

Fig. 2. (Color online) (a) The dependence of SNR on the entanglement degree of the cluster state. (b) The dependence of SNR on the transmittance η.

We compare the dependence of SNR on the transmittance in Fig. 2(b) when the entanglement degree equals 0 dB, 5 dB, 10 dB and 20 dB, respectively. In the high loss cases ($\eta<0.18$), we cannot distinguish signal from noise no matter what entanglement state is used. The simulation results in both Figs. 2(a) and 2(b) show that the use of the cluster state can distinguish the signal better.

Our scheme can be considered as a realization of optical information processing, where BSs are used to achieve addition and subtraction operations, and the cluster state is used to suppress noise. A BS can be

simply described by a transformation from two input mode operators, $\hat{a}_{1_{\rm in}}$ and $\hat{a}_{2_{\rm in}}$, into two output mode operators, $\hat{b}_{1_{\rm out}}$ and $\hat{b}_{2_{\rm out}}$. The relationship between input and output modes can be expressed as[30]

$$\begin{pmatrix}\hat{b}_{1_{\rm out}}\\ \hat{b}_{2_{\rm out}}\end{pmatrix} = M\begin{pmatrix}\hat{a}_{1_{\rm in}}\\ \hat{a}_{2_{\rm in}}\end{pmatrix} = \begin{pmatrix}\sqrt{\eta} & \sqrt{1-\eta}\\ \sqrt{1-\eta} & -\sqrt{\eta}\end{pmatrix}\begin{pmatrix}\hat{a}_{1_{\rm in}}\\ \hat{a}_{2_{\rm in}}\end{pmatrix}, \tag{8}$$

where we describe the properties of the BS by the intensity reflection coefficient η. When $\eta = 1/2$, the matrix M in Eq. (8) equals H_2 in Eq. (1).

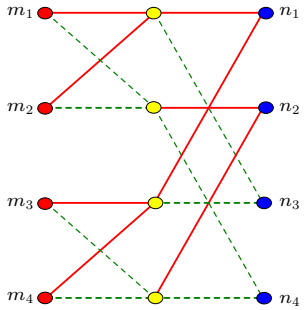

Fig. 3. (Color online) The input–output relation of the four-mode HT algorithm.

We have achieved the basic element of HT algorithm (the transform matrix H_2) utilizing the EPR state. HT algorithm can be decomposed into a combination of two-mode transformation (H_2). Take the four-mode HT algorithm as an example (Fig. 3). First, two-mode HT transformations (H_2) are performed on m_1, m_2 modes and m_3, m_4 modes, respectively. Secondly, two-mode HT transformations (H_2) are performed on the addition mode (the subtraction mode) of m_1, m_2 and the addition mode (the subtraction mode) of m_3, m_4, respectively. Then the output result $n_1 - n_4$ is obtained. Therefore for high-order cases, with suitable and large enough cluster states, arbitrary HT algorithms can be achieved in principle.

Here we present an example of achieving a four-mode HT algorithm by suitable cluster state. The four-mode HT algorithm consists of four input modes (m_1, m_2, m_3 and m_4), four output modes (n_1, n_2, n_3 and n_4), four addition operations and four subtraction operations. The relationship between input modes and output modes can be shown in Fig. 3.

When $k = 2$ the transform matrix of the four-mode HT algorithm is

$$H_4 = \begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix}. \tag{9}$$

After the HT algorithm transform, the input and output modes have the following relationship

$$\begin{aligned} n_1 &= m_1 + m_2 + m_3 + m_4,\\ n_2 &= m_1 - m_2 + m_3 - m_4,\\ n_3 &= m_1 + m_2 - m_3 - m_4,\\ n_4 &= m_1 - m_2 - m_3 + m_4. \end{aligned} \tag{10}$$

When using four p-squeezed states as input states, three kinds of four-partite CV cluster states can be achieved by linear optics.[11] They are linear cluster state, square cluster state and T-shape cluster state. We choose a four-partite T-shape cluster state as the resource state to implement the four-mode HT algorithm. The quantum correlations among the four-partite T-shape cluster state are given by

$$\begin{aligned} \hat{p}_1 - \hat{x}_2 - \hat{x}_3 - \hat{x}_4 &= 2e^{-r}\hat{p}_2^{(0)},\\ \hat{p}_2 - \hat{x}_1 &= \sqrt{2}e^{-r}\hat{p}_1^{(0)},\\ \hat{p}_3 - \hat{x}_1 &= \frac{1}{\sqrt{2}}e^{-r}\hat{p}_2^{(0)} + e^{-r}\hat{p}_3^{(0)} + \frac{1}{\sqrt{2}}e^{-r}\hat{p}_4^{(0)},\\ \hat{p}_4 - \hat{x}_1 &= \frac{1}{\sqrt{2}}e^{-r}\hat{p}_2^{(0)} + e^{-r}\hat{p}_3^{(0)} - \frac{1}{\sqrt{2}}e^{-r}\hat{p}_4^{(0)}, \end{aligned} \tag{11}$$

where four p-squeezed states, with mode operators $\hat{a}_i = e^{r_i}\hat{x}_i^{(0)} + ie^{-r_i}\hat{p}_i^{(0)}$ ($i = 1$–4) are used to create the four-partite T-shape cluster states. The superscript (0) denotes the initial vacuum modes, which has a noise variance $\langle\Delta^2\hat{x}_i^{(0)}\rangle = \langle\Delta^2\hat{p}_i^{(0)}\rangle = 1/4$, and $r_i \in [0, \infty)$ is the squeezing parameter of the ith mode.

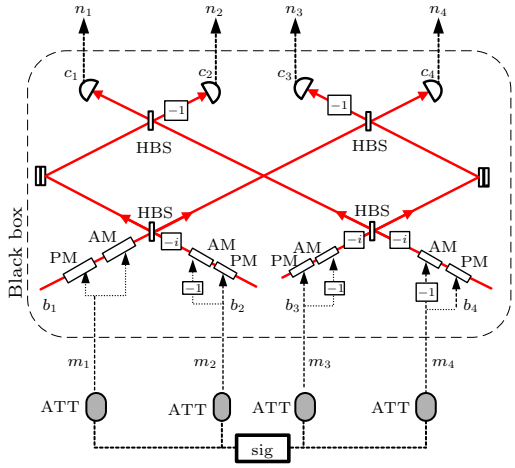

Fig. 4. (Color online) The scheme of realizing the four-mode HT algorithm with a four-partite T-shape cluster state. Here m_1–m_4: input data of HT algorithm, n_1–n_4: output data of HT algorithm, b_1–b_4: four-partite T-shape cluster state, c_1–c_4: homodyne detectors, -1 is a $-180°$ rotation, $-i$ is a $-90°$ rotation.

We design the network for achieving four-mode HT algorithm with four-partite T-shape cluster states just utilizing four HBSs. The schematic diagram of optical setups is shown in Fig. 4. The input information of m_1, m_2, m_3 and m_4 is encoded to the four submodes $\hat{b}_1$–$\hat{b}_4$ of the T-shape cluster state by using the amplitude (AM) and phase (PM) modulators. Every attenuator (ATT) in different lines controls the intensity of the four signals, which are generated from the same signal generator (sig), respectively. They satisfy the relationship $sig_1 : sig_2 : sig_3 : sig_4 = m_1 : m_2 : m_3 : m_4$.

For achieving the HT algorithm, the amplitude and phase quadratures of m_1 should be set to the same phase, while the amplitude and phase quadratures of m_2–m_4 should be set to the opposite phase. The amplitude and phase quadratures relationship between the optical modulated modes $\hat{d}_1$–$\hat{d}_4$ and the T-shape

cluster state $\hat{b}_1$–$\hat{b}_4$ are given by

$$\begin{aligned}
\hat{x}_{d_1} &= \hat{x}_{b_1} + \hat{x}_{m_1}, \quad \hat{p}_{d_1} = \hat{p}_{b_1} + \hat{p}_{m_1}, \\
\hat{x}_{d_2} &= \hat{x}_{b_2} - \hat{x}_{m_2}, \quad \hat{p}_{d_2} = \hat{p}_{b_2} + \hat{p}_{m_2}, \\
\hat{x}_{d_3} &= \hat{x}_{b_3} - \hat{x}_{m_3}, \quad \hat{p}_{d_3} = \hat{p}_{b_3} + \hat{p}_{m_3}, \\
\hat{x}_{d_4} &= \hat{x}_{b_4} - \hat{x}_{m_4}, \quad \hat{p}_{d_4} = \hat{p}_{b_4} + \hat{p}_{m_4},
\end{aligned} \tag{12}$$

where we have assumed that the variances of modulated input signals $\hat{x}_{m_i}$ and $\hat{p}_{m_i}$ $(i = 1$–$4)$ are equal. After combining the submodes $\hat{d}_1$–$\hat{d}_4$ on four half beam-splitters respectively, as shown in Fig. 4, we obtain the output modes $\hat{c}_1$–$\hat{c}_4$. From Fig. 4, we can easily obtain the relations between input and output modes,

$$\begin{aligned}
\hat{c}_1 &= \frac{1}{2}(\hat{d}_1 - i\hat{d}_2 - i\hat{d}_3 - i\hat{d}_4), \\
\hat{c}_2 &= \frac{1}{2}(\hat{d}_1 + i\hat{d}_2 - i\hat{d}_3 + i\hat{d}_4), \\
\hat{c}_3 &= \frac{1}{2}(\hat{d}_1 - i\hat{d}_2 + i\hat{d}_3 + i\hat{d}_4), \\
\hat{c}_4 &= \frac{1}{2}(\hat{d}_1 + i\hat{d}_2 + i\hat{d}_3 - i\hat{d}_4).
\end{aligned} \tag{13}$$

We only use half quadratures of $\hat{c}_1$–$\hat{c}_4$ to achieve the four-mode HT algorithm. They are phase quadrature of $\hat{c}_1$ and amplitude quadratures of $\hat{c}_2$–$\hat{c}_4$. Based on Eq. (11), the amplitude and phase quadratures of these modes can be expressed as

$$\begin{aligned}
\langle\delta^2(\hat{p}_{c_1})\rangle &= \frac{1}{4}\langle\delta^2(\hat{p}_{m_1} + \hat{x}_{m_2} + \hat{x}_{m_3} + \hat{x}_{m_4})\rangle \\
&\quad + e^{-2r}\langle\delta^2(\hat{p}_2^{(0)})\rangle, \\
\langle\delta^2(\hat{x}_{c_2})\rangle &= \frac{1}{4}\langle\delta^2(\hat{x}_{m_1} - \hat{p}_{m_2} + \hat{p}_{m_3} - \hat{p}_{m_4})\rangle \\
&\quad + \frac{1}{2}e^{-2r}\langle\delta^2(\hat{p}_1^{(0)})\rangle + \frac{1}{2}e^{-2r}\langle\delta^2(\hat{p}_4^{(0)})\rangle, \\
\langle\delta^2(\hat{x}_{c_3})\rangle &= \frac{1}{4}\langle\delta^2(\hat{x}_{m_1} + \hat{p}_{m_2} - \hat{p}_{m_3} - \hat{p}_{m_4})\rangle \\
&\quad + \frac{1}{2}e^{-2r}\langle\delta^2(\hat{p}_1^{(0)})\rangle + \frac{1}{2}e^{-2r}\langle\delta^2(\hat{p}_2^{(0)})\rangle \\
&\quad + e^{-2r}\langle\delta^2(\hat{p}_3^{(0)})\rangle, \\
\langle\delta^2(\hat{x}_{c_4})\rangle &= \frac{1}{4}\langle\delta^2(\hat{x}_{m_1} - \hat{p}_{m_2} - \hat{p}_{m_3} + \hat{p}_{m_4})\rangle \\
&\quad + \frac{1}{2}e^{-2r}\langle\delta^2(\hat{p}_1^{(0)})\rangle + \frac{1}{2}e^{-2r}\langle\delta^2(\hat{p}_4^{(0)})\rangle.
\end{aligned} \tag{14}$$

The output data n_1 is given by the phase quadrature of $\hat{c}_1$, and the output data n_2–n_4 are given by the amplitude quadratures of $\hat{c}_2$–$\hat{c}_4$, respectively. It is obvious that in the limit of infinite squeezing $(r \to \infty)$, the phase quadrature $\hat{p}_{c_1}$ and the amplitude quadrature $\hat{x}_{c_2}$–$\hat{x}_{c_4}$ are determined by the amplitude and phase quadratures of the input field, i.e., the input–output relationship of Eq. (10) is achieved. The four-mode HT transform is obtained only when the quantum correlations of the four-partite T-shape CV cluster state is utilized. If the experimental setup is regarded as a black box, the output data n_1–n_4 can be obtained by simply inputting the data m_1–m_4, which can complete the four-mode HT transformation.

In summary, we have presented the scheme to implement the classical HT algorithm with the CV cluster state as a resource state. As an example, the concrete scheme to implement the four-mode classical HT algorithm with a four-partite CV cluster state is analyzed, which only needs four HBSs operated on the input modes, while at least eight operations (four addition operations and four subtraction operations) are needed in the classical computation model. The HT algorithm is one of the widely used algorithms in normal classical computation, and the CV one-way cluster quantum computation would be the possible model of quantum computer. Our work shows that some special classical algorithms can be run in classical computer as well as quantum one. Moreover, by using the quantum interference, the operations we used for HT algorithm by the CV cluster quantum computer is just half of the classical computation. Once quantum computers can be applied in practice, both quantum algorithms and some special classical algorithms can be run in it. A complex computational problem may contain several algorithms. If we can summarize which classical algorithms can be run in quantum computers, the computational problem involving only quantum algorithms and those classical algorithms that can be run in quantum computers would be designed. This will improve the operating efficiency of quantum computers by avoiding the transmission of data between quantum computer and classical computer.

References

[1] Nielsen M A and Chuang I 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[2] Monroe C et al 1995 *Phys. Rev. Lett.* **75** 4714
[3] Miwa Y et al 2009 *Phys. Rev.* A **80** 050303
[4] Shor P W 1997 *SIAM J. Computing* **26** 1484
[5] Grover L K 1996 *Proceedings of the Twenty-Eighth Annual ACM Symposium on Theory of Computing* p 212
[6] Harrow A W et al 2009 *Phys. Rev. Lett.* **103** 150502
[7] Cai X D et al 2013 *Phys. Rev. Lett.* **110** 230501
[8] Raussendorf R and Briegel H J 2001 *Phys. Rev. Lett.* **86** 5188
[9] Menicucci N C et al 2006 *Phys. Rev. Lett.* **97** 110501
[10] Su X L et al 2007 *Phys. Rev. Lett.* **98** 070502
[11] Yukawa M et al 2008 *Phys. Rev.* A **78** 012301
[12] Chen M et al 2014 *Phys. Rev. Lett.* **112** 120505
[13] Su X L et al 2012 *Opt. Lett.* **37** 5178
[14] Yokoyama S et al 2013 *Nat. Photon.* **7** 982
[15] Ukai R et al 2011 *Phys. Rev. Lett.* **106** 240504
[16] Hao S H et al 2014 *Phys. Rev.* A **89** 032311
[17] Wang Y et al 2010 *Phys. Rev.* A **81** 022311
[18] Ukai R et al 2011 *Phys. Rev. Lett.* **107** 250501
[19] Su X L et al 2013 *Nat. Commun.* **4** 2828
[20] Kunz H O 1979 *IEEE Trans. Comput.* **C-28** 267
[21] Tipsmark A et al 2011 *Phys. Rev.* A **84** 050301
[22] Podoshvedov S A 2013 *Phys. Rev.* A **87** 012307
[23] Laing A et al 2012 *Phys. Rev. Lett.* **108** 260505
[24] Zhang J and Braunstein S L 2006 *Phys. Rev.* A **73** 032318
[25] van Loock P et al 2007 *Phys. Rev.* A **76** 032321
[26] Gu M et al 2009 *Phys. Rev.* A **79** 062318
[27] Reid M D 1989 *Phys. Rev.* A **40** 913
[28] Shen H et al 2009 *Phys. Rev.* A **80** 042320
[29] Johnson D H 2006 *Scholarpedia* **1** 2088
[30] Bachor H A and Ralph T C A 2004 *Guide to Experiments in Quantum Optics* (New York: Wiley-VCH)