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Nonlinear Propagation of Positron-Acoustic Periodic Travelling Waves in a Magnetoplasma with Superthermal Electrons and Positrons

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The nonlinear propagation of positron acoustic periodic (PAP) travelling waves in a magnetoplasma composed of dynamic cold positrons, superthermal kappa distributed hot positrons and electrons, and stationary positive ions is examined. The reductive perturbation technique is employed to derive a nonlinear Zakharov–Kuznetsov equation that governs the essential features of nonlinear PAP travelling waves. Moreover, the bifurcation theory is used to investigate the propagation of nonlinear PAP periodic travelling wave solutions. It is found that kappa distributed hot positrons and electrons provide only the possibility of existence of nonlinear compressive PAP travelling waves. It is observed that the superthermality of hot positrons, the concentrations of superthermal electrons and positrons, the positron cyclotron frequency, the direction cosines of wave vector k along the z-axis, and the concentration of ions play pivotal roles in the nonlinear propagation of PAP travelling waves. The present investigation may be used to understand the formation of PAP structures in the space and laboratory plasmas with superthermal hot positrons and electrons.

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Recently, the investigations of a nonlinear wave propagation in electron–positron–ion (e-p-i) plasmas have attracted considerable interests from many researchers,[1−16] because they have great significance in understanding the fundamental features of astrophysical plasmas, viz. solar atmosphere, pulsar environments, supernovas, interplanetary space and active galactic nuclei,[1,2,10,16] as well as in laboratory plasmas such as intense laser solid-density plasma experiments.[9] However, propagation of positron acoustic waves depends on the existence of two distinct temperature positron components (i.e., cold positrons and hot positrons). Therefore, the cold positron mass supplies the inertia, whereas the restoring force of a positron acoustic wave is provided by both hot electrons and hot positrons. On the other hand, the ions are considered stationary for forming a neutralizing background. During the past many years, many researchers studied the nonlinear characteristics of positron waves in various models of plasmas.[17−29] Moreover, in a four-component plasma system composed of cold positron fluid, stationary positive ions, and Boltzmann distributed electrons and positrons, Tribeche *et al.*[18,19] examined the positron acoustic solitary waves and double layers, respectively, Sahu[20] discussed the planar and nonplanar positron acoustic shock waves, and El-Shamy *et al.*[21] theoretically studied the characteristics of the head-on collision between two positron acoustic solitary waves. Recently, Mamun *et al.* also studied the structure of nonlinear positron acoustic waves in different models of plasmas.[22−29] For instance, Uddin *et al.*[27] studied the nonplanar positron-acoustic Gardner solitary waves in an unmagnetized plasma system consisting of immobile positive ions, mobile cold positrons, and superthermal hot positrons and electrons. However, one may ask what are superthermal particles? Actually, high energy or superthermal particles have velocity distributions with a high-energy tail, then they may deviate from the Maxwellian distribution, and hence obey a kappa distribution. Moreover, fast or superthermal particles may arise due to the interactions between waves and particles or the effects of external forces on the natural space plasma. Furthermore, many researchers[30−33] solved the self-consistent problem of the electron kappa distribution in recent years by numerically solving the wave kinetic equation for Langmuir turbulence. However, most of these works related to the nonlinear positron acoustic waves (such as solitary waves, double layers, and planar and nonplanar shock waves) are concentrated on the Maxwellian electrons and positrons, except the two by Uddin *et al.*[27−29] who used superthermal hot positrons and electrons to study, respectively, the nonplanar positron acoustic Gardner solitary waves and instability analysis of obliquely propagating positron acoustic solitary waves. It is clear that all of the above-mentioned investigations[17−29] are limited only to the nonlinear positron acoustic waves without nonlinear positron acoustic periodic (PAP) travelling waves. Therefore, the goal of this work is to examine the propagation of nonlinear PAP travelling waves in a magnetoplasma with superthermal hot positrons and electrons by using the bifurcation theory.[34] It should be mentioned here that the bifurcation theory[34] is a suitable method to discuss the propagation of periodic travelling waves. By the bifurcation theory, the Zakharov–Kuznetsov (ZK) equation has been studied in different models to investigate solitary waves and periodic travelling waves structures

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in magnetized plasmas.[35–38] In general, as a result of the nonlinear ZK equation, the dynamical system has two equilibrium points. Based on the physical parameters in the proposed model, one can obtain phase portraits with qualitatively different trajectories. Moreover, because of two equilibrium points in the presented system, we can find one homoclinic orbit at the saddle point and a family of periodic orbits about the center. In fact, nonlinear periodic waves are highly significant to examine real phenomena in various branches of physics and have many applications in the nonlinear transport phenomenon and defocusing region of ionospheric plasma.[39] To the best of our knowledge, the fundamental features of PAP travelling waves in three dimensions in a superthermal magnetoplasma system through the bifurcation analysis are previously unexamined.

We consider a four-component superthermal plasma consisting of cold positron fluid, superthermal hot positrons and electrons, and immobile positive ions, in the presence of an external static magnetic field $\boldsymbol{B} = B_0\hat{e}_z$ along the z-axis (where $\hat{e}_z$ is the unit vector in the z-axis direction). At equilibrium, $n_{\rm e}^{(0)} = n_{\rm cp}^{(0)} + n_{\rm hp}^{(0)} + Z_{\rm i}n_{\rm i}^{(0)}$, where $n_{\rm e}^{(0)}$, $n_{\rm cp}^{(0)}$, $n_{\rm hp}^{(0)}$ and $n_{\rm i}^{(0)}$ are the unperturbed number densities of superthermal electrons, cold positrons, hot superthermal positrons, and ions, respectively. Let us assume that there appears a singly ionized plasma, i.e., $Z_{\rm i} = 1$. The normalized basic equations governing the dynamics of nonlinear positron acoustic waves propagating in a superthermal magnetoplasma are presented as follows:[29]

$$\frac{\partial n_{\rm cp}}{\partial t} = \nabla\cdot(n_{\rm cp}\boldsymbol{u}_{\rm cp}) = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}_{\rm cp}}{\partial t} + (\boldsymbol{u}_{\rm cp}\cdot\nabla)\boldsymbol{u}_{\rm cp} = -\nabla\phi + (\boldsymbol{u}_{\rm cp}\times\hat{e}_z), \tag{2}$$

$$\nabla^2\phi = -(n_{\rm cp} + \mu_{\rm hp}n_{\rm hp} - \mu_{\rm e}n_{\rm e} + \mu_{\rm i}), \tag{3}$$

and the number densities of hot superthermal electrons and positrons can be described by the normalized equations[16,29]

$$n_{\rm e} = \Big(1 - \frac{\phi}{(\kappa_{\rm e} - 3/2)}\Big)^{(-\kappa_{\rm e}+1/2)}, \tag{4}$$

$$n_{\rm hp} = \Big(1 + \frac{\sigma\phi}{(\kappa_{\rm p} - 3/2)}\Big)^{(-\kappa_{\rm p}+1/2)}, \tag{5}$$

where the variables $n_{\rm cp}$, $n_{\rm hp}$ and $n_{\rm e}$ are densities of cold positrons, hot superthermal positrons and electrons, respectively, $\boldsymbol{u}_{\rm cp}$ is the cold positron fluid velocity, ϕ is the electrostatic potential, t is the time variable, and $\nabla = (\partial/\partial x' + \partial/\partial y' + \partial/\partial z')$ with x', y' and z' being the space coordinates. The variables in Eqs. (1)–(5) have been scaled as follows: $n_{\rm cp} \to n_{\rm cp}/n_{\rm cp}^{(0)}$, $n_{\rm hp} \to n_{\rm hp}/n_{\rm hp}^{(0)}$, $n_{\rm e} \to n_{\rm e}/n_{\rm e}^{(0)}$, $u_{{\rm cp}(x,y,z)} \to u_{{\rm cp}(x,y,z)}/C_{\rm cp}$, $\phi \to e\phi/K_{\rm B}T_{\rm e}$, $t \to t\omega_{\rm cp}$ and $\nabla \to \nabla\lambda_{\rm D}$, where $C_{\rm cp} = \sqrt{K_{\rm B}T_{\rm e}/m_{\rm p}}$ is the cold positron acoustic speed, $\omega_{\rm cp} = \sqrt{4\pi e^2 n_{\rm cp}^{(0)}/m_{\rm p}}$ is the cold positron plasma frequency, and $\lambda_{\rm D} = \sqrt{K_{\rm B}T_{\rm e}/4\pi e^2 n_{\rm cp}^{(0)}}$ is the Debye length. Furthermore, $\Omega_{\rm cp} = (eB_0/m_{\rm p}c)/\omega_{\rm cp}$ is the cold positron cyclotron frequency. We introduce the parameters, $\sigma = T_{\rm e}/T_{\rm hp}$, $\mu_{\rm hp} = n_{\rm hp}^{(0)}/n_{\rm cp}^{(0)}$, $\mu_{\rm e} = n_{\rm e}^{(0)}/n_{\rm cp}^{(0)}$ and $\mu_{\rm i} = n_{\rm i}^{(0)}/n_{\rm cp}^{(0)}$, where $T_{\rm e}$ ($T_{\rm hp}$) is the temperature of electrons (hot positrons), and $\kappa_{\rm e}$ ($\kappa_{\rm p}$) is the superthermality of hot electrons (positrons).

Now, let us describe the nonlinear propagation of positron acoustic waves in a magnetized superthermal plasma, using the well-known reductive perturbation method through the stretched coordinates[40] $x' = \epsilon^{1/2}x$, $y' = \epsilon^{1/2}y$, $z' = \epsilon^{1/2}(z - \lambda t)$, $t' = \epsilon^{3/2}t$, where λ is the phase velocity of the wave to be determined later, and ϵ is a smallness parameter measuring the strength of the nonlinearity. The dependent variables are expanded as follows: $n_{\rm cp} = 1 + \epsilon n_{\rm cp}^{(1)} + \epsilon^2 n_{\rm cp}^{(2)} + \ldots$, $u_{{\rm cp}(x,y)} = \epsilon^{3/2}u_{{\rm cp}(x,y)}^{(1)} + \epsilon^2 u_{{\rm cp}(x,y)}^{(2)} + \ldots$, $u_{{\rm cp}(z)} = \epsilon u_{{\rm cp}(z)}^{(1)} + \epsilon^2 u_{{\rm cp}(z)}^{(2)} + \ldots$, and $\phi = 1 + \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \ldots$. Substituting the stretched variables x', y', z' and t', and dependent variables $n_{\rm cp}$, $u_{{\rm cp}x}$, $u_{{\rm cp}y}$, $u_{{\rm cp}z}$ and ϕ into Eqs. (1)–(5), we can obtain the lowest nonzero order in ϵ, $n_{\rm cp}^{(1)} = \phi^{(1)}/\lambda^2$, $u_{{\rm cp}x}^{(1)} = -(\partial\phi^{(1)}/\partial y)/\Omega_{\rm cp}$, $u_{{\rm cp}y}^{(1)} = (\partial\phi^{(1)}/\partial x)/\Omega_{\rm cp}$, $u_{{\rm cp}z}^{(1)} = \phi^{(1)}/\lambda^2$ and $\lambda = \sqrt{1/(\Theta_{\rm p} + \Theta_{\rm e})}$, where $\Theta_{\rm p} = \sigma\mu_{\rm hp}(\kappa_{\rm p} - 1/2)/(\kappa_{\rm p} - 3/2)$, and $\Theta_{\rm e} = \mu_{\rm e}(\kappa_{\rm e} - 1/2)/(\kappa_{\rm e} - 3/2)$. Following the same strategy, with the aid of the nonzero order, and after algebraic manipulations, we obtain the nonlinear ZK equation as

$$\frac{\partial\phi}{\partial t} + A\phi\frac{\partial\phi}{\partial z} + B\frac{\partial^3\phi}{\partial z^3} + C\Big(\frac{\partial^3}{\partial z\partial x^2} + \frac{\partial^3}{\partial z\partial y^2}\Big)\phi = 0, \tag{6}$$

where $\phi = \phi^{(1)}$, and $A = B((3/\lambda^4) + (\Theta_{\rm e}\chi_{\rm e} - \Theta_{\rm p}\chi_{\rm p}))$, $B = \lambda^3/2$, and $C = B(1 + (1/\Omega_{\rm cp}^2))$ with $\chi_{\rm e} = \mu_{\rm e}(\kappa_{\rm e}+1/2)/(2\kappa_{\rm e}-3)$, and $\chi_{\rm p} = \sigma(\kappa_{\rm p}+1/2)/(2\kappa_{\rm p}-3)$.

To transform Eq. (6) into an ordinary differential equation, one can use the transformation $\xi = \alpha(\ell_x x + \ell_y y + \ell_z z - u_0 t)$, where ℓ_x, ℓ_y and ℓ_z are, respectively, the direction cosines of the propagation vector $\boldsymbol{k}$ with respect to x, y, and z axes, u_0 is the velocity of the moving frame normalized by $C_{\rm cp}$, and α is a constant. Putting $\phi(\xi) = \phi(x, y, z, t)$ integrating Eq. (6) with respect to ξ and using the boundary conditions $\phi(\xi) \to 0$, and $\phi'(\xi) \to 0$ for $\xi \to \pm\infty$, one can obtain $\frac{\partial^2\phi}{\partial\xi^2} = \delta\phi - \rho\phi^2$, where $\delta = u_0/\alpha^2\ell_z(\ell_z^2(B - C) + C)$ and $\rho = A/2\alpha^2(\ell_z^2(B - C) + C)$. By Eq. (7), the planar Hamiltonian system of travelling wave equations can be written as[37]

$$\frac{d\phi}{d\xi} = \Psi, \quad \frac{d\Psi}{d\xi} = (\delta - \rho\phi)\phi. \tag{7}$$

Therefore, the Hamiltonian function is defined by $H(\phi, \Psi) = \frac{\Psi^2}{2} - (\frac{\delta}{2} - \frac{\rho}{3}\phi)\phi^2 = h$. It is clear that the planar Hamiltonian system, Eq. (7), depends on the physical parameters $\mu_{\rm hp}$, $\mu_{\rm e}$, $\mu_{\rm i}$, $\kappa_{\rm e}$, $\kappa_{\rm p}$, $\Omega_{\rm cp}$ and

ℓ_z. It is essential to mention here that the phase portrait defined by the vector fields of Eq. (7) will describe all nonlinear positron-acoustic travelling wave solutions to Eq. (6). Based on the above-mentioned physical parameters, let us examine the bifurcation of phase orbits of Eq. (7) in the (ϕ, Ψ) phase plane. Consequently, a positron-acoustic solitary wave solution to Eq. (6) identifies to a homoclinic orbit of Eq. (7). On the other hand, a phase orbit of Eq. (7) corresponds to a PAP traveling wave solution to Eq. (6). It is worth noticing that, at $\delta \neq 0$ and $\rho \neq 0$, the planar Hamiltonian system Eq. (7) has two equilibrium points $Q_0(\phi_0, 0)$ and $Q_1(\phi_1, 0)$, where $\phi_0 = 0$ and $\phi_1 = 2u_0/A\ell_z$. If $M(\phi_i, 0)$ is the coefficient matrix of the linearized system of the system (8) at an equilibrium point $E_i(\Phi_i, 0)$, one can obtain $J = \det M(\phi_i, 0) = -\delta + 2\rho\phi_i$. Using the theory of the planar dynamical system, an equilibrium point $E_i(\Phi_i, 0)$ of the planar Hamiltonian system (7) is a saddle (center) point when $J < 0$ ($J > 0$). Applying the bifurcation analysis, the various phase orbits of Eq. (7) depending numerically on the concentration of superthermal positrons (i.e., μ_{hp}) are displayed in Figs. 1 and 2. Therefore, with superthermal positrons μ_{hp} (i.e., $0 < \mu_{hp} < 1$) and λ ($\lambda > 0$), Fig. 1 verifies the condition $3/\lambda^4 + \Theta_e\chi_e > \Theta_p\chi_p$. Of course, without superthermal positrons $3/\lambda^4 + \Theta_e\chi_e > 0$ is satisfied, which gives the phase orbit as depicted in Fig. 2. Thus we obtain that A, B, and C are always positive. Accordingly, Figs. 1 and 2 show that there exist a homoclinic portrait and a family of periodic portraits at the equilibrium point $Q_0(\phi_0, 0)$ ($Q_0(\phi_0, 0)$ is a saddle point) and $Q_1(\phi_1, 0)$ ($Q_1(\phi_1, 0)$ is a center point), respectively. Thus under the relationship $3/\lambda^4 + \Theta_e\chi_e > \Theta_p\chi_p$, one can observe the presence of both positron acoustic solitary waves and periodic traveling waves in the present model. The nonlinear ZK Eq. (6) admits a compressive positron acoustic sech-shaped soliton solution, $\phi = \phi_0 \mathrm{sech}^2(\xi/\Delta)$, where $\phi_0 = 3u_0/A\ell_z$ and $\Delta = \sqrt{4\alpha^2\ell_z(\ell_z^2(B-C)+C)/u_0}$ are the amplitude and the width of the positron acoustic solitary wave, respectively. It is important to mention here that the study of positron acoustic solitary waves is beyond the scope of the present work. However, the present investigation is in agreement with the findings of Uddin *et al.*[29] In the following let us focus our attention on the characteristics of nonlinear PAP travelling waves in a magnetized superthermal plasma. The nonlinear PAP travelling wave solution to Eq. (6) in terms of the Jacobian elliptic functions is given by[41]

$$\phi(\xi) = \phi_3 + (\phi_2 - \phi_3)\mathrm{sn}^2(D\xi, k_1), \tag{8}$$

where sn is the Jacobian elliptic function, $(\phi_2 - \phi_3) > 0$ is the amplitude of the PAP wave, and the condition $\phi_1 > \phi_2 > \phi_3$ should also be satisfied. The elliptic parameter k_1 ($0 < k_1 \leqslant 1$), and the parameter D are written as $\sqrt{(\phi_2 - \phi_3)/(\phi_1 - \phi_3)}$ and $\sqrt{[u_0(\phi_1 - \phi_3)]/[12\alpha^2\ell_z(\ell_z^2(B-C)+C)]}$, respectively.

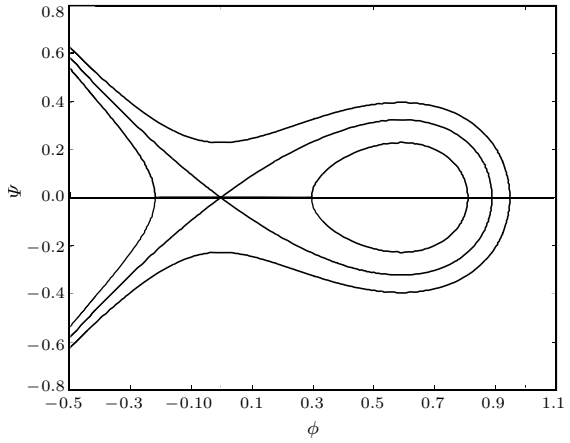

Fig. 1. Phase curves with $\alpha = 4.32$, $\kappa_e = 1.6$, $\kappa_p = 3.5$, $\mu_i = 0.2$, $\mu_{hp} = 0.3$, $\sigma = 1.1$, $\Omega_{cp} = 0.3$ and $\ell_z = 0.5$.

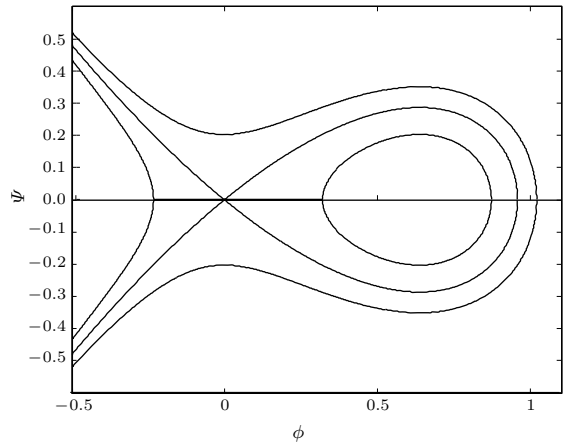

Fig. 2. Phase curves with $\alpha = 4.32$, $\kappa_e = 1.6$, $\kappa_p = 3.5$, $\mu_i = 0.2$, $\mu_{hp} = 0.0$, $\sigma = 1.1$, $\Omega_{cp} = 0.3$ and $\ell_z = 0.5$.

Now, let us study numerically the properties of nonlinear PAP travelling waves in the existence of physical parameters μ_{hp}, μ_e, μ_i, κ_e, Ω_{cp} and ℓ_z. Figure 3 illustrates the propagation of nonlinear PAP travelling waves in a certain range of the space coordinate for different values of the superthermal parameter κ_e. As displayed in Fig. 3, the amplitude and the width of PAP travelling waves increase with the increase of κ_e. This means, physically, that the increase in κ_e leads to an enhancement of the nonlinearity of the system and concentrates a valuable amount of energy, which makes the PAP travelling wave taller. The electrostatic potential profiles associated with the PAP travelling waves are demonstrated in Fig. 4 for varying the concentration of superthermal positrons μ_{hp}. It is found that an increase in the values of μ_{hp} leads to a decrease in the amplitude and the width of the nonlinear PAP periodic wave. It is vitally important to examine the dependence of the nonlinear PAP waves on Ω_{cp}. To this end, the structures of PAP travelling waves are displayed in Fig. 5 for different values of Ω_{cp}. It is observed that the width of the nonlinear PAP travelling waves decreases with the increase of Ω_{cp}. Moreover, the height remains fixed, which indicates that the strength of the electrostatic potential also remains constant for various values of Ω_{cp}. Fig-

ure 6 corresponds to the variation of PAP profiles with the direction cosines of the wave vector k along the z-axis ℓ_z. It is essential to notice that the profile of the periodic travelling wave reduces with the increase of ℓ_z. On the contrary, it is clear that an increase in the values of ℓ_z leads to an increase in the width of the nonlinear PAP wave. Figure 7 is plotted to check the influence of the concentration of stationary ions $\mu_{\rm i}$ on the nonlinear PAP waves. It is obvious that the presence of stationary ions leads to a decrease in the restoring force of the nonlinear PAP travelling waves which in turn leads the nonlinearity to become shorter.

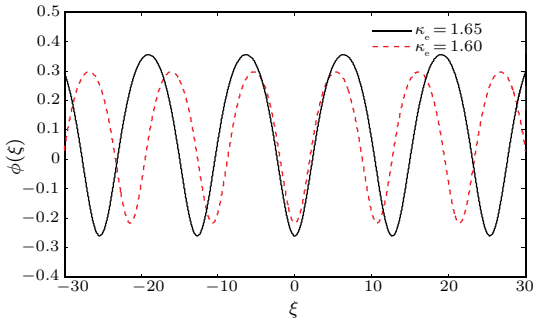

Fig. 3. The variation of PAP travelling wave $\phi(\xi)$ for $\alpha = 4.32$, $\kappa_{\rm p} = 3.6$, $\mu_{\rm i} = 0.2$, $\mu_{\rm hp} = 0.3$, $\Omega_{\rm cp} = 0.3$, $\ell_z = 0.5$, $\kappa_{\rm e} = 1.65$ (solid curve) and $\kappa_{\rm e} = 1.6$ (dashed curve).

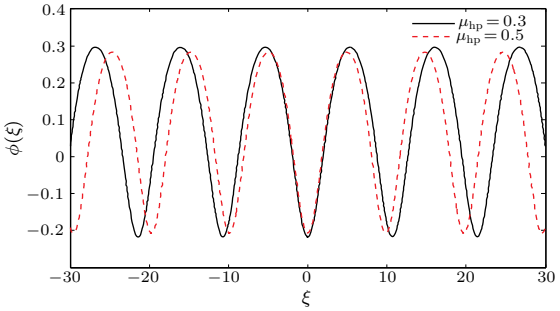

Fig. 4. The variation of PAP travelling wave $\phi(\xi)$ for $\alpha = 4.32$, $\kappa_{\rm e} = 1.6$, $\kappa_{\rm p} = 3.6$, $\mu_{\rm i} = 0.2$, $\sigma = 1.1$, $\Omega_{\rm cp} = 0.3$, $\ell_z = 0.5$, $\mu_{\rm hp}$=0.3 (solid curve) and $\mu_{\rm hp} = 0.5$ (dashed curve).

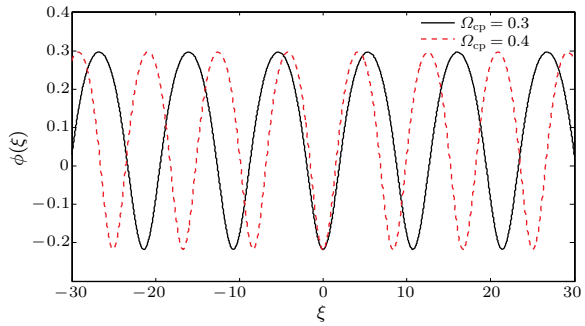

Fig. 5. The variation of PAP travelling wave $\phi(\xi)$ for $\alpha = 4.32$, $\kappa_{\rm e} = 1.6$, $\kappa_{\rm p} = 3.6$, $\mu_{\rm i} = 0.2$, $\mu_{\rm hp} = 0.3$, $\sigma = 1.1$, $\ell_z = 0.5$, $\Omega_{\rm cp} = 0.3$ (solid curve) and $\Omega_{\rm cp} = 0.4$ dashed curve).

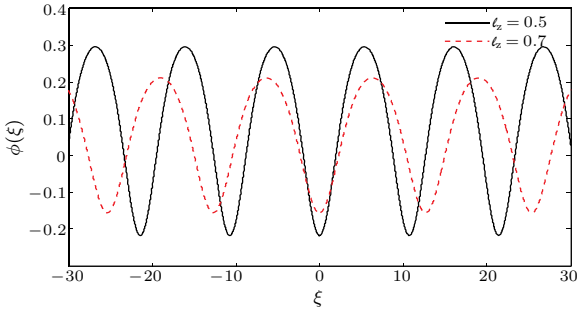

Fig. 6. The variation of PAP travelling wave $\phi(\xi)$ for $\alpha = 4.32$, $\kappa_{\rm e} = 1.6$, $\kappa_{\rm p} = 3.6$, $\mu_{\rm i} = 0.2$, $\mu_{\rm hp} = 0.3$, $\sigma = 1.1$, $\Omega_{\rm cp} = 0.3$, $\ell_z = 0.5$ (solid curve) and $\ell_z = 0.7$ (dashed curve).

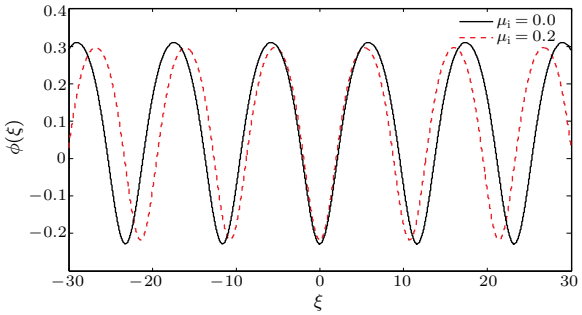

Fig. 7. The variation of PAP travelling wave $\phi(\xi)$ for $\alpha = 4.32$, $\kappa_{\rm e} = 1.6$, $\kappa_{\rm p} = 3.6$, $\sigma = 1.1$, $\mu_{\rm hp} = 0.3$, $\Omega_{\rm cp} = 0.3$, $\ell_z = 0.5$, $\mu_{\rm i} = 0.0$ (solid curve) and $\mu_{\rm i} = 0.2$ (dashed curve).

In summary, we have applied the bifurcation method to study the properties of nonlinear PAP travelling waves in superthermal magnetoplasmas composed of cold positrons, superthermal hot electrons and positrons, and stationary positive ions. Numerically, it is found that the ZK equation supports only compressive PAP travelling waves with distinct physical structures. It is important to note that $\mu_{\rm hp}$, $\mu_{\rm e}$, $\mu_{\rm i}$, $\kappa_{\rm e}$, $\Omega_{\rm cp}$ and ℓ_z play important roles in the fundamental characteristics of nonlinear PAP travelling waves. In addition, we believe that the present results will help us to understand the localized structures that may occur in space plasma environments such as solar atmosphere, interplanetary space, and the Earth's innermost magnetosphere, where the superthermality effect of high energy particles in magnetoplasmas is significant.[17,21]

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