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Localization of Vector Field on Pure Geometrical Thick Brane *

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We investigate the localization of a five-dimensional vector field on a pure geometrical thick brane. By introducing two types of interactions between the vector field and the background scalar field, we obtain a typical volcano potential for the first type of coupling and a Pöschl–Teller potential for the second one. These two types of couplings guarantee that the vector zero mode can be localized on the pure geometrical thick brane under certain conditions.

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In the last two decades, the theory of higher-dimensional spacetime has provided a new insight for solving some relevant physics problems (e.g., the gauge hierarchy problem and the cosmological constant problem).[1−3] However, in the original Randall–Sundrum (RS) model,[2] the brane is very ideal because its thickness is ignored, and subsequently, more realistic thick branes with length scale have been investigated.[4−9] It is assumed that the zero modes of all matter fields (electromagnetic, Yang–Mills, fermions etc.) are confined to the 3-brane without conflict with experiments. The assumption leads to an important question how to realize the localization of various bulk fields on a brane. It is well known that scalar field can be localized on the RS brane or its generalized branes.[10−15] For a fermion field, without introducing the scalar–fermion coupling[16−22] or fermion–gravity coupling,[23] it does not have normalizable zero modes in five-dimensional RS-like brane models. However, for the vector field, when only considering the coupling between the vector field and the background metric, the zero mode can be localized on the de Sitter brane[24,25], AdS brane,[26] Bloch Brane,[27] and some pure geometrical branes.[8,28−30] However, in some other pure geometrical brane cases,[11,19,31] the zero mode of the vector field cannot be localized on the branes. To obtain the localized vector zero mode on these pure geometrical branes, we need a new localization mechanism by introducing the interaction between the vector field and the Weyl scalar.

We start with a five-dimensional pure geometrical Weyl brane, which is based on the following action[11]

$$S_5^{\rm W}=\int_{M_5^{\rm W}}\frac{d^5x\sqrt{-g}}{16\pi G_5}e^{\frac{3}{2}\omega}[R+3\tilde{\xi}(\nabla\omega)^2+6U(\omega)],\quad(1)$$

where $M_5^{\rm W}$ is a five-dimensional Weyl-integrable spacetime specified by the pair (g_{MN},ω) with a five-dimensional metric g_{MN} and a Weyl scalar ω. The parameter $\tilde{\xi}$ is a coupling constant, and $U(\omega)$ is a self-interaction potential of ω. In the Weyl frame, the affine connections Γ^P_{MN} are defined as

$$\Gamma^P_{MN}=\{^P_{MN}\}-\frac{1}{2}(\nabla_M\omega\delta^P_N+\nabla_N\omega\delta^P_M-g_{MN}\nabla^P\omega),$$

where $\{^P_{MN}\}$ is the Christoffel symbols. The Weyl rescaling

$$g_{MN}\to\Omega^2 g_{MN},\quad \omega\to\omega+\ln\Omega^2,$$
$$\tilde{\xi}\to\tilde{\xi}/(1+\partial_\omega\ln\Omega^2)^2,\quad(2)$$

will break the invariance of the action Eq. (1), where Ω^2 is a smooth function of ω on M_5^W. To keep the invariance of the Weyl action, we set $U(\omega)=\lambda e^{\omega}$ with λ being a constant parameter.

In this study, we consider the five-dimensional flat brane solution in the pure geometrical Weyl gravity Eq. (1). The metric that respects the four-dimensional Poincaré invariance is

$$ds_5^2=e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu+dy^2,\quad(3)$$

where $e^{2A(y)}$ is the warp factor, and y stands for the extra coordinate.

By making the conformal transformation $\hat{g}_{MN}=e^{\omega}g_{MN}$, the Weyl affine connection becomes the Christoffel symbol, and the Weyl–Ricci tensor turns into the Riemannian–Ricci tensor.[19] Thus the Weyl frame Eq. (1) can be mapped into a Riemannian form, where the five-dimensional gravity is coupled to a scalar field with a self-interaction potential $\hat{U}(\omega)=e^{-\omega}U(\omega)$,

$$S_5^{\rm R}=\int_{M_5^{\rm R}}\frac{d^5x\sqrt{-\hat{g}}}{16\pi G_5}[\hat{R}+3\xi(\hat{\nabla}\omega)^2+6\hat{U}(\omega)],\quad(4)$$

where $\xi=\tilde{\xi}-1$. It should be noted that the hatted magnitudes and operators refer to the Riemann frame. After the conformal transformation, the line element in the Riemann frame reads

$$d\hat{s}_5^2=e^{2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu+e^{\omega(y)}dy^2,\quad(5)$$

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where $2\sigma = 2A+\omega$. In the case of Z_2-symmetric thick brane with infinite extra dimension ($-\infty < y < +\infty$), the solutions for the warp factor and the scalar field read[19]

$$e^{2A(y)} = [\cosh(ay)]^b,\ \omega(y) = kb\ln\cosh(ay), \quad (6)$$

where the parameters a and b are given by

$$a = \sqrt{\frac{4+3k}{1+k}}2\lambda,\ b = \frac{2}{4+3k},\ \left(\lambda > 0, k < -\frac{4}{3}\right). \quad (7)$$

The parameter b is negative, and the warp factor is concentrated near the origin $y = 0$. Although both quantities $e^{2\sigma(y)}$ and $e^{\omega(y)}$ diverge at $\pm\infty$, the curvature scalar approaches zero ($R(y \to \pm\infty) = 0$).

In Ref. [11], Liu *et al.* investigated the localization of a free massless vector field on the pure geometrical brane. The action for such a vector field is given by[11]

$$S_1 = -\frac{1}{4}\int d^5x\sqrt{-g}g^{MN}g^{RS}F_{MR}F_{NS}, \quad (8)$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength for the vector field A_M. For convenience, one changes the metric Eq. (3) into a conformal from

$$ds_5^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2). \quad (9)$$

The relationship between the new coordinates z and y is $dz = e^{-A(y)}dy$, $e^{2A(z)} = (1+a^2z^2)^{\frac{b}{2}}$, and $\omega(z) = \frac{bk}{2}\ln(1+a^2z^2)$. For the above brane configuration, the zero mode $\widetilde{\rho_0}$ in the conformal flat coordinates defined in Eq. (9) is obtained,[11]

$$\widetilde{\rho_0}(z) \propto 1/(1+3\lambda z^2)^{1/4}, \quad (10)$$

where the parameter has been set to $k = -5/3$. Unfortunately, the integration related to the vector zero mode in the KK reduction is not convergent, which means that the vector zero mode (Eq. (10)) is non-normalized. This results in the vector zero mode that cannot be localized on the thick brane, just as the RS brane case.

To obtain a localized vector zero mode, we introduce the interaction between the vector field and the background scalar field ω. The general action for the vector field coupled with the background scalar is

$$S_1 = -\frac{1}{4}\int d^5x\sqrt{-g}G(\omega)g^{MN}g^{RS}F_{MR}F_{NS}, \quad (11)$$

where $G(\omega)$ is a function of the background scalar field ω. Considering the background geometry Eq. (9), the equations of motion read

$$e^A G(\omega)\partial_\nu(\eta^{\mu\nu}F_{\mu 4}) = 0, \quad (12)$$

$$e^A G(\omega)\partial_\nu\left(\eta^{\nu\rho}\eta^{\mu\lambda}F_{\rho\lambda}\right) + \eta^{\mu\lambda}\partial_z\left(e^A G(\omega)F_{4\lambda}\right) = 0. \quad (13)$$

According to Ref. [11] we suppose that A_M satisfies the Z_2 symmetry with respect to the extra dimension z, which requires that A_4 has no zero mode in the effective four-dimensional theory. Moreover, for being consistent with the gauge invariant ($\oint dzA_4 = 0$), we set $A_4 = 0$ using the gauge freedom. Under the KK decomposition of the vector field, $A_\mu(x^\nu, z) = \sum_n a_\mu^n(x^\mu)\tilde{\rho}_n(z)$, and the orthonormality conditions

$$\int_{-\infty}^{\infty} dze^A G(\omega)\tilde{\rho}_m(z)\tilde{\rho}_n(z) = \delta_{mn}, \quad (14)$$

the action Eq. (11) can be reduced into an effective four-dimensional form

$$S_1 = \sum_n \int d^4x\Big(-\frac{1}{4}\eta^{\mu\lambda}\eta^{\nu\rho}f_{\mu\nu}^{(n)}f_{\lambda\rho}^{(n)} - \frac{1}{2}m_n^2\eta^{\mu\nu}a_\mu^{(n)}a_\nu^{(n)}\Big), \quad (15)$$

where $f_{\mu\nu}^{(n)} = \partial_\mu a_\nu^{(n)} - \partial_\nu a_\mu^{(n)}$ is the four-dimensional field strength, and m_n is the mass of the nth vector KK mode.

It can be shown that the extra dimensional part $\tilde{\rho}_n(z)$ satisfies the following differential equation

$$-\partial_z(e^A G(\omega)\partial_z\tilde{\rho}_n) = m_n^2 e^A G(\omega)\tilde{\rho}_n. \quad (16)$$

For convenience, we rewrite $G(\omega)$ as $G(\omega) = e^{g(\omega)}$ with $g(\omega)$ a function of ω. By defining $\rho_n = e^B\tilde{\rho}_n$ with $B = \frac{1}{2}(A + g(\omega))$, Eq. (16) can be rewritten as the following Schrödinger-like equation

$$[-\partial_z^2 + V(z)]\rho_n(z) = m^2\rho_n(z), \quad (17)$$

where the effective potential $V(z)$ reads

$$V(z) = \partial_z^2 B + (\partial_z B)^2. \quad (18)$$

Precisely this kind of potential can be found in Ref. [11]. In fact, the Schrödinger-like equation (17) can also be written as $\mathcal{H}\rho_n = m_n^2\rho_n$ with the Hamiltonian operator given by $\mathcal{H} = Q^\dagger Q = (\partial_z + \partial_z B)(-\partial_z + \partial_z B)$. As the operator $\mathcal{H}$ is positive definite, there is no tachyonic vector mode with negative m_n^2.

For the zero mode ρ_0, it can be solved from the Schrödinger-like equation (17) by setting $m^2 = 0$,

$$\rho_0 = c_0 e^B = c_0 e^{(A+g(\omega))/2}, \quad (19)$$

where c_0 is the normalization constant. We know from Eq. (14) that, to obtain the effective action of the four-dimensional massless vector field $a_\mu^{(0)}$, the zero mode needs to satisfy the following normalization condition

$$\int_{-\infty}^{\infty} dz|\rho_0(z)|^2 = c_0^2\int_{-\infty}^{\infty} dze^{A+g(\omega)} = 1, \quad (20)$$

which can be transformed into the original y coordinate

$$\int_{-\infty}^{\infty} dye^{-A}\rho_0(y)^2 = c_0^2\int_{-\infty}^{\infty} dye^{g(\omega)} = 1. \quad (21)$$

Now it is clear that without the non-minimal coupling function $G(\omega) = e^{g(\omega)}$, the vector zero mode cannot be

localized on the pure geometrical brane.[11] To obtain the normalized zero mode, we need to choose the specific expression of $g(\omega)$. In the following we introduce a simple expression $g(\omega) = \tau\omega$ and then put forward a specific form $g(\omega) = \theta e^{\frac{3}{10}\omega} + \frac{3}{10}\omega$.

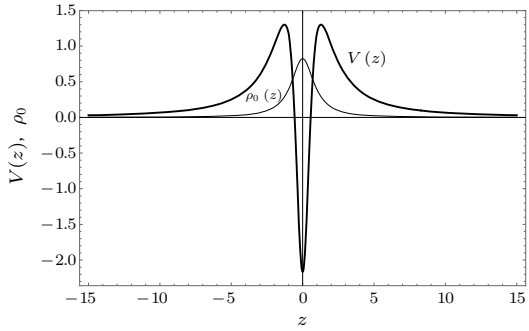

Fig. 1. The shapes of the effective potential $V(z)$ (thick line) of the vector KK modes and the vector zero mode $\rho_0(z)$ (thin line) for coupling I ($g(\omega) = \tau\omega$). The parameters are set to $k = -5/3$, $\lambda = 1/3$, and $\tau = -1$.

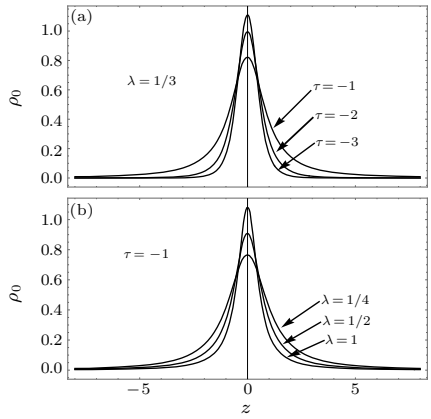

Fig. 2. The effect of the parameters τ and λ on the vector zero mode $\rho_0(z)$ for coupling I. The parameter k is set to $k = -5/3$.

We first assume the simple function $g(\omega) = \tau\omega$ with τ being a constant. Such a coupling between the background scalar field and the vector field is called the dilaton coupling.[32,33] The effective potential $V(z)$ in Eq. (18) is

$$V(z) = \frac{\lambda(10\tau - 3)[\lambda(10\tau - 9)z^2 + 2]}{4(3\lambda z^2 + 1)^2}, \tag{22}$$

which has an asymptotic behavior that $V(z \to \pm\infty) \to 0$ and $V(z = 0) = \frac{1}{2}\lambda(10\tau - 3)$. The effective potential in this case is a typical volcano potential.[34] It means that the effective potential provides a continuum gapless mass spectrum of the vector KK modes with positive $m_n^2 > 0$. The expression of the zero mode $\rho_0(z)$ is

$$\rho_0 = c_0(1 + 3\lambda z^2)^{\frac{5\tau}{6} - \frac{1}{4}}. \tag{23}$$

The normalization condition Eq. (20) requires

$$\tau < 0. \tag{24}$$

With the above condition, the integration in Eq. (20) is convergent for infinite extra dimension. The normalization constant is given by

$$c_0^2 = \sqrt{\frac{3\lambda}{\pi}}\Gamma\Big(\frac{1}{2} - \frac{5\tau}{3}\Big)/\Gamma\Big(-\frac{5\tau}{3}\Big). \tag{25}$$

Figure 1 shows the shapes of the effective potential $V(z)$ and the vector zero mode $\rho_0(z)$ with a set of parameters $k = -5/3$, $\lambda = 1/3$, and $\tau = -1$. Figure 2 shows the effect of the parameters τ and λ on the vector zero mode. It can be seen that, with the increase of $|\tau|$ or λ, the vector zero mode becomes higher but thinner and hence it is localized at a narrower region around the origin of the extra dimension $z = 0$.

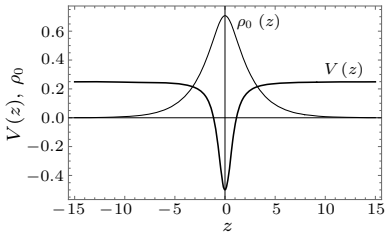

Fig. 3. The shapes of the effective potential $V(z)$ (thick line) and the vector zero mode $\rho_0(z)$ (thin line) for coupling II. The parameters are set to $k = -5/3$, $\lambda = 1/3$, and $\theta = -1$.

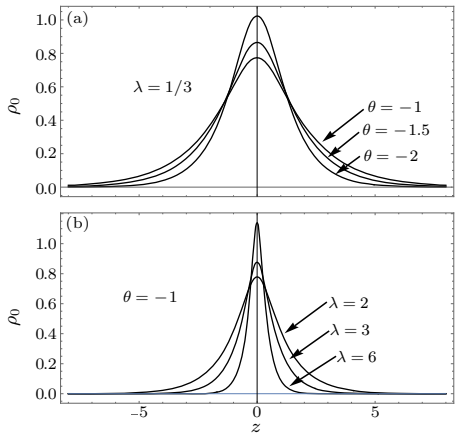

Fig. 4. The effect of the parameters θ and λ on the vector zero mode $\rho_0(z)$ for coupling II. The parameter k is set to $k = -5/3$.

Next, we consider the case of $g(\omega) = \theta e^{\frac{3}{10}\omega} + \frac{3}{10}\omega$ with a dimensionless constant θ. The expression of the effective potential $V(z)$ in Eq. (18) is now given by

$$V(z) = \frac{3\lambda\theta(3\lambda\theta z^2\sqrt{3\lambda z^2 + 1} + 2)}{4(3\lambda z^2 + 1)^{3/2}}, \tag{26}$$

which has the asymptotic behavior of a PT potential[35] $V(z \to \pm\infty) \to (\frac{3\lambda\theta}{2})^2$ and $V(z = 0) = \frac{3\lambda\theta}{2}$. It means that the effective potential provides a mass gap to separate the zero mode from the KK modes. The expression of the zero mode $\rho_0(z)$ in this case reads

$$\rho_0(z) = c_0 e^{\frac{1}{2}\theta\sqrt{3\lambda z^2 + 1}}. \tag{27}$$

The integration in Eq. (20) is convergent for infinite extra dimension if

$$\theta < 0, \tag{28}$$

and the normalization constant is

$$c_0^2 = -\sqrt{3\lambda}\ \theta/(2e^{\theta/\sqrt{3\lambda}}). \tag{29}$$

The shapes of the effective potential $V(z)$ and the zero mode $\rho_0(z)$ are shown in Fig. 3 with the parameters $k = -5/3$, $\lambda = 1/3$, and $\theta = -1$. Figure 4 gives a detailed description of the effect of the parameters θ and λ on the vector zero mode. The vector zero mode has a similar phenomenon as case I in which it becomes higher but thinner and is localized in a narrower region around $z = 0$ with the increase of $|\theta|$ or λ.

In summary, we have investigated the localization of a bulk vector field on a pure geometrical flat thick brane. The interaction between the bulk vector field and the background scalar field, i.e., $-\frac{1}{4}e^{g(\omega)}F_{MN}F^{MN}$, is introduced. We give two forms of $g(\omega)$ to ensure that the zero mode of the vector field can be localized on the thick brane. For the first case $g(\omega) = \tau\omega$, we obtain a typical volcano potential. The localization condition turns out to be $\tau < 0$. The vector zero mode will be localized in a narrower region around $z = 0$ with the increase of the coupling parameter $|\tau|$ or the model parameter λ. The effective potential for the second coupling function, i.e., $g(\omega) = \theta e^{\frac{3}{10}\omega} + \frac{3}{10}\omega$, has the asymptotic behavior of a PT potential. The coupling parameter θ should be negative ($\theta < 0$) to localize the vector zero mode on the brane. The increasing $|\theta|$ or λ can also make the vector zero mode close to a narrower region. If we consider that the brane is the time t function, just like S-brane,[36] there may be some new features about the localization of vector field.

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