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Controllably Coupling Superconducting Charge and Flux Qubits by Using Nanomechanical Resonator

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We propose an effective mechanism to couple superconducting charge and flux qubits by using a quantized nanomechanical resonator. The coupling between the charge and flux qubits can be controlled by the external flux of the charge qubit. Under the strong coupling limit, an iSWAP gate can be generated by this scheme. The experimental feasibility in our scheme is also presented.

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In recent years, many works have extensively studied mesoscopic superconducting circuits, because they have broad application prospects in the physical realization of quantum information processing.[1−4] In Ref. [5], Nakamura *et al.* demonstrated for the first time the coherent oscillations of a single superconducting charge qubit. After that, many researches have been carried out on the characteristics of a single qubit, especially for a single qubit coherent time.[6] It is commonly accepted that the flux and the charge qubits at the optimum point are not sensitive to fluctuations in the bias parameters, which corresponds to a longer coherence time. For this type of system, the multiple-qubit schemes have also been studied.[7−11] However, between the superconducting qubits, how to achieve the best controllable couplings is still an important issue.[12−15] Various theoretical protocols on controllable coupling schemes have been proposed,[16−27] while some of them are not working at the optimal point. In Ref. [22], Rigetti *et al.* proposed a technique to solve this problem, which was a double resonance similar to nuclear magnetic resonance.[28] A few years later, this scheme was developed to the so-called parametric coupling,[25,27] and was verified by the time domain experiment.[25] A very efficient way to make the superconducting qubit work at the best point is to utilize a capacitance coupled flux qubit or an inductive coupling charge qubit. At these points, the energy level of the qubit was directly affected by the coupling term.

On the other hand, the nanomechanical resonator (NAMR) has recently attracted considerable interest due to their inherent characteristics of small mass, low dissipation and high frequency. Systems[29−32] using NAMR coupled superconducting qubit have been extensively studied, such as non-demolition measurement of qubits,[33] and the preparation of two-qubit entangled gates.[34]

In this Letter, we present a novel method to couple the superconducting flux qubit and the charge qubit with the quantized mode of the NAMR,[35] where the NAMR is used as a bosonic data bus and it can virtually exchange bosons with the qubits. The flux qubit operates at the optimal point with a quite long decoherence time. Between the flux and charge qubits, we can produce a switchable $\sigma_z\sigma_x$ type interaction through a controllable external coupling magnetic field. After selecting appropriate parameters for the hybrid quantum circuit, an iSWAP gate can be implemented. The maximal advantage of our scheme is that the magnetic flux passing through the charge qubit can be adjusted so that the coupling between the flux qubit and the charge qubit becomes controllable.

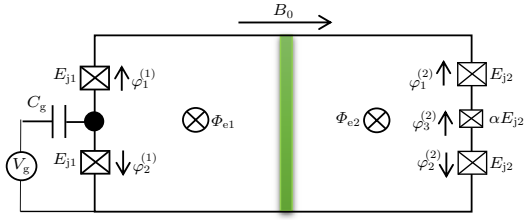

Fig. 1. Structure diagram of the circuit. A charge qubit and a flux qubit are connected to an NAMR (the middle narrow box). The perpendicular oscillation of the NAMR is controlled by a transverse magnetic field B_0, which is parallel to the coplanarity of the NAMR and the qubits. The charge qubit consists of two identical Josephson junctions, where the Josephson coupling energy and capacitance are E_{j1} and C_1, respectively. The flux qubit contains two identical Josephson junctions and a smaller one. The coupling energy and capacitance of the two identical Josephson junctions are E_{j2} and C_2, while the smaller ones are αE_{j2} and αC_2, where $0.5 < \alpha < 1$. The directions of the phase difference on each side of the Josephson junction are indicated by the arrows in the figure. Here Φ_{e1} and Φ_{e2} represent the magnetic flux passing through the left and right qubits, respectively, and V_g is the bias voltage of the charge qubit.

We propose an effective mechanism to couple the superconducting charge qubit with the flux qubit by using an NAMR (see Fig. 1). The distance between the flux and charge qubits is about a few micrometers considering the non-dissipative phase correlation of the superconducting circuit.[36]

The superconducting charge qubit consists of two identical Josephson junctions with Josephson energy E_{j1} and capacitance C_1. Here V_g is the biased voltage of the superconducting charge qubit through a gate

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capacitance $C_{\rm g}$. To adjust the Josephson coupling, an external magnetic flux $\Phi_{\rm e1}$ is applied through the ring. The Hamiltonian of the charge qubit contains the charge energy, and the Josephson energy reads

$$H_1 = E_{\rm c1}(n-n_{\rm g})^2 - E_{\rm j1}(\cos\varphi_1^{(1)} + \cos\varphi_2^{(1)}), \quad (1)$$

where $n_{\rm g} = C_{\rm g}V_{\rm g}/2e$ represents the gate charges induced by $V_{\rm g}$, $E_{\rm c1} = 2e^2/C_1$ is the charging energy, and n is the number of the extra Cooper pairs on the superconducting island. The quantization condition of the flux gives the limiting relationship between the phase difference of the Josephson junctions,

$$\varphi_1^{(1)} - \varphi_2^{(1)} + 2\pi f_1 = 0, \quad (2)$$

where $f_1 = \Phi_{\rm e1}/\Phi_0$ is the reduced external flux, and $\Phi_0 = h/2e$ represents the magnetic flux quantum. Then the Hamiltonian of the charge qubit can be written as

$$H_1 = E_{\rm c1}(n-n_{\rm g})^2 - 2E_{\rm j1}\cos(f_1\pi)\cos\varphi_{\rm A}, \quad (3)$$

where $\varphi_{\rm A} = (\varphi_1^{(1)} + \varphi_2^{(1)})/2$.

Here we consider that the bias voltage of the charge qubit is close to the optimal point so that the charge qubit can be regarded as an ideal two-level system of $|0\rangle$ and $|1\rangle$, which are corresponding to the ground state and the excited state, respectively. Based on the two charge states, the Hamiltonian of Eq. (3) is reduced to[16,17]

$$H_1 = \varepsilon_1(V_{\rm g})\sigma_z^{(1)} - E_{\rm j1}\cos(f_1\pi)\sigma_x^{(1)}, \quad (4)$$

where $\varepsilon_1(V_{\rm g}) = \frac{1}{2}E_{\rm c1}(C_{\rm g}V_{\rm g}/e-1)$, $\sigma_z^{(1)}$ and $\sigma_x^{(1)}$ are Pauli matrices.

The circulating supercurrent I_1 of the charge qubit is $I_1 = 2I_{\rm c1}\sin(\pi f_1)\cos\varphi_{\rm A}$, which can be rewritten as $I_1 = I_{\rm c1}\sin(\pi f_1)\sigma_x^{(1)}$ using the basis states $|0\rangle$and $|1\rangle$, where $I_{\rm c1} = 2\pi E_{\rm j1}/\Phi_0$.

The flux qubit is composed of three different Josephson junctions inserted into a superconducting ring, where two Josephson junctions are exactly the same, while the third is much smaller than them. Here $E_{\rm j2}$ and C_2 are the coupling energy and the capacitance of the two identical Josephson junctions, while for the third, they are $\alpha E_{\rm j2}$ and αC_2 with $0.5 < \alpha < 1$. Here $\varphi_2^{(2)}$, $\varphi_1^{(2)}$, $\varphi_3^{(2)}$ express the phase differences of the Josephson junctions, respectively (see Fig. 1). The Hamiltonian of the flux qubit with Josephson energy and the charge energy reads[37]

$$H_2 = \frac{P_{\rm p}^2}{2M_{\rm p}} + \frac{P_{\rm q}^2}{2M_{\rm q}} + 2E_{\rm j2}(1-\cos\varphi_{\rm p}\cos\varphi_{\rm q}) + \alpha E_{\rm j2}[1-\cos(2\varphi_{\rm q}-2\pi f_2)], \quad (5)$$

where $P_n = -i\hbar\partial/\partial\varphi_n$, $(n = p,q)$, $M_{\rm p} = 2(\Phi_0/2\pi)^2C_2$, $M_{\rm q} = M_{\rm p}(1+2\alpha)$, and $f_2 = \Phi_{\rm e2}/\Phi_0$. The redefined phases are

$$\varphi_{\rm p} = \frac{1}{2}(\varphi_1^{(2)} + \varphi_2^{(2)}), \quad (6)$$

$$\varphi_{\rm q} = \frac{1}{2}(\varphi_1^{(2)} - \varphi_2^{(2)}). \quad (7)$$

The phase difference of the flux qubit is also restricted by the flux quantization

$$\varphi_1^{(2)} + \varphi_3^{(2)} - \varphi_2^{(2)} - 2\pi f_2 = 0. \quad (8)$$

Here $|\uparrow\rangle$ and $|\downarrow\rangle$ can be used to indicate the states of the maximum clockwise and counter clockwise persistent supercurrents of superconducting flux qubit, thus Eq. (5) can be rewritten as[38]

$$H_2 = \varepsilon_2(\Phi_{\rm e2})\sigma_z^{(2)} - \Delta_2\sigma_x^{(2)}, \quad (9)$$

where $\varepsilon_2(\Phi_{\rm e2}) = \Phi_0 I_{\rm P}(f_2-0.5)$, $I_{\rm p} \approx 2\pi\alpha E_{\rm j2}/\Phi_0 = \alpha I_{\rm c}$ is the maximal persistent supercurrent of the flux qubit,[38] $I_{\rm c}$ is the critical current of the two identical Josephson junctions, $\sigma_x^{(2)}$ and $\sigma_{\rm z}^{(2)}$ are Pauli matrices, and Δ_2 is the tunneling splitting between the two states.

The circulating supercurrent I_2 of the flux qubit is $I_2 = 2\pi\alpha E_{\rm j2}\sin(-\varphi_1^{(2)} + \varphi_2^{(2)} + 2\pi f_2)/\Phi_0$, which can be rewritten as $I_2 = I_{\rm P}\sigma_z^{(2)}$ using the basis states $|\uparrow\rangle$ and $|\downarrow\rangle$.

The bending mode of NAMR can be explained by the Euler–Bernoulli equation in Ref. [39]. In our scheme only basic bending modes are considered, while other modes are negligible due to a smaller coupling with qubits.[40] Thus the NAMR can be used as a harmonic oscillator with a high-Q mode. Thus the non-dissipative Hamiltonian can be expressed as[41]

$$H_3 = \frac{p_y^2}{2m} + \frac{1}{2}m\omega_{\rm d}^2 y^2, \quad (10)$$

where $\omega_{\rm d}$ and m represent the frequency and mass of the NAMR, respectively. The canonical momentum p and coordinate y satisfy the commutation relation $[y,p] = i$ (hereafter assuming $\hbar = 1$). They can be indicated by the annihilation operator a and the creation operator a^+ as $y = 1/\sqrt{2m\omega_{\rm d}}(a^+ + a)$ and $p = i\sqrt{m\omega_{\rm d}/2}(a^+ - a)$, respectively. Then Eq. (10) reduces to

$$H_3 = \omega_{\rm d}\Big(a^+a + \frac{1}{2}\Big). \quad (11)$$

As we know, when a current passes through a beam coated with superconducting material (see Fig. 1), the perpendicular arrangement of an external magnetic field and the direction of the current will generate a Lorentz force in the plane of the beam, which is just the actuation part of the magnetomotive technique. At the same time, the resulting displacement of the beam under the Lorentz force generates an electromotive force, or voltage, which serves as measurement. Then, if the doubly-clamped nano-beam of conducting material is incorporated in the superconducting qubit loop, the persistent current induces a Lorentz force

with opposite directions for counterclockwise and clockwise currents. The oscillation of the NAMR is modulated by these Lorentz forces.[42] With this method, we couple the quantized harmonic oscillation mode of the beam to the quantum state of the qubit system.

As discussed above, the circulating suppercurrents I_1 and I_2 under the magnetic field generate the Lorentz forces in the perpendicular direction of the plane composed of the NAMR and the qubits, the magnitudes of the forces are $F_1 = B_0 I_1 L$ and $F_2 = B_0 I_2 L$, respectively, where $L = \beta L_0$ is the effective length of the resonator, L_0 is the actual length of the NAMR, and β is a parameter that depends on the oscillation mode.[43] The basic oscillation mode for a double clamping beam reads $\beta \approx 0.8$.

These forces cause two forced terms in the Hamiltonian, which read $H_I^1 = F_1 z = B_0 I_1 L z$ and $H_I^2 = F_2 z = B_0 I_2 L z$. With the singlemode boson approximation and the two-level approximation of the qubits, we have $z = \delta_z(a + a^+)$, and $\delta_z = \sqrt{1/2m\omega_{\rm d}}$ is the amplitude of zero point motion of the NAMR. Then the above coupling Hamiltonians can be written as $H_I^1 = g_1\sigma_x^{(1)}(a^+ + a)$ and $H_I^2 = g_2\sigma_z^{(2)}(a^+ + a)$, respectively, where $g_1 = B_0(t) I_{\rm c1} L \sin(\pi f_1)\delta_z$ and $g_2 = B_0(t) I_{\rm p} L \delta_z$ represent the coupling coefficient for the NAMR with the charge qubit and flux qubit, respectively.

Combining Eqs. (4), (9) and (11), the total Hamiltonian for coupling two qubits and the NAMR can be written as

$$\begin{aligned} H &= H_1 + H_2 + H_3 + H_I^1 + H_I^2 \\ &= \varepsilon_1(V_{\rm g})\sigma_z^{(1)} - E_{\rm j1}\cos(f_1\pi)\sigma_x^{(1)} \\ &+ \varepsilon_2(\Phi_{\rm e2})\sigma_z^{(2)} - \Delta_2\sigma_x^{(2)} + \omega_{\rm d}a^+a \\ &+ (g_1\sigma_x^{(1)} + g_2\sigma_z^{(2)})(a^+ + a). \end{aligned} \tag{12}$$

We consider that the qubit energy splitting $\omega_{\rm q}$ is much smaller than the plasmon energy splitting $\omega_{\rm d}$. It can be assumed that the harmonic oscillator stays in the ground state. Through the exchange of virtual bosons between the qubit and the NAMR, the charge qubit can efficiently interact with the flux qubit. In the Schödinger picture, neglecting the fast oscillatory terms, the total Hamiltonian of Eq. (12) can be rewritten as[44]

$$\begin{aligned} H =& \varepsilon_1(V_{\rm g})\sigma_z^{(1)} - E_{\rm j1}\cos(f_1\pi)\sigma_x^{(1)} + \varepsilon_2(\Phi_{\rm e2})\sigma_z^{(2)} \\ &- \Delta_2\sigma_x^{(2)} + \eta\sigma_x^{(1)}\sigma_z^{(2)}, \end{aligned} \tag{13}$$

where $\eta = 2g_1g_2/\omega_{\rm d}$ is the strength of the interqubit coupling.

In the following we will study how to generate the iSWAP gate. Considering $\Phi_{\rm e1} = \Phi_{\rm e2} = \Phi_0/2$, in this case $E_{\rm j1}\cos(f_1\pi) = \varepsilon_2 = 0$. By choosing a suitable gate voltage $V_{\rm g}$ to obtain $\varepsilon_1 = -\Delta_2$, the Hamiltonian of Eq. (13) can be reduced to

$$H = -\Delta_2\sigma_z^{(1)} - \Delta_2\sigma_x^{(2)} + \eta\sigma_x^{(1)}\sigma_z^{(2)}. \tag{14}$$

For convenience, we utilize the new basis states $|g\rangle$ and $|e\rangle$ corresponding to the ground state and excited state of the flux qubit, respectively.

Then, using the basis states $\{|0g\rangle, |1g\rangle, |0e\rangle, |1e\rangle\}$, the time-dependent evolution operator $U = \exp(-iHt)$ of the whole system can be obtained in matrix form

$$U = \begin{pmatrix} a^* & 0 & 0 & b \\ 0 & c & d & 0 \\ 0 & d & c & 0 \\ b & 0 & 0 & a \end{pmatrix}, \tag{15}$$

where $a^* = \cos(\delta t) - i2\Delta_2\sin(\delta t)/\delta$, $b = -2i\eta\sin(\delta t)/\delta$, $c = \cos(\eta t)$, $d = -i\sin(\eta t)$, and $\delta = \sqrt{4\Delta_2^2 + \eta^2}$.

It is interesting to note that if η and δ satisfy the conditions

$$\eta t = \Big(2m - \frac{1}{2}\Big)\pi, (m = 1, 2, 3, \ldots), \tag{16}$$

$$\delta t = 2n\pi, \ (n = 1, 2, 3, \ldots), \tag{17}$$

the iSWAP gate can be achieved as

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{18}$$

It should be noted that in our scheme the coupling between the charge qubit and the flux qubit is controllable. The interqubit coupling can be turned off when the flux $\Phi_{\rm e1}$ through the charge is zero.

We now discuss the experimental feasibility of the iSWAP gate. It is noted that recent advances in fabrication techniques have enabled us to product and characterize the nanoscale mechanical resonators.[45] With the latest experimental progress in nanomechanical graphene resonators, the attainable fundamental frequency can reach the order of 200 GHz.[46] Meanwhile, the superconducting qubits have also been extensively studied in recent years, the qubits with the Josephson energy ranging from the order of 10 KHz[47] to 10^3 GHz[48] can be achieved now. Here we take the moderate parameters $\omega_{\rm d} = 32.4$ GHz, $E_{\rm j2} = 24$ GHz and $E_{\rm j1} = 2.163$ GHz. When $\alpha = 0.75$, we could obtain the typical energy splitting of the flux qubit $\Delta_2 \approx 0.02E_{\rm j2} = 480$ MHz, the circulating supercurrent $I_{\rm c1} = 2\pi E_{\rm j1}/\Phi_0 = 6.92 \times 10^{-10}$ A and the maximal persistent supercurrent $I_{\rm p} = 2\pi\alpha E_{\rm j2}/\Phi_0 = 5.76 \times 10^{-9}$ A. Referring to the parameters in Refs. [35,46,49], we choose $B_0 = 65.182$ mT, $L_0 = 3.9$ μm and $\delta_z = 5.2 \times 10^{-10}$ m. Thus the interqubit coupling strength could be derived as $\eta \approx 247.871$ MHz, which is obviously strong enough to realize the operation of the two qubits. Under this condition, the time for implementing the iSWAP gate is $t_1 \approx 0.019$ μs, for $m = 1$ and $n = 3$. The coherence time of the charge qubit is $T \approx 2$ μs,[50] which indicates that it is feasible to prepare the iSWAP gate in our scheme.

Then, we calculate the fidelity of the quantum gate. Based on the parameters implementing the

quantum gate, the total Hamiltonian Eq. (12) reduces to

$$H' = -\Delta_2\sigma_z^{(1)} - \Delta_2\sigma_x^{(2)} + \omega_{\rm d}a^+a + (g_1\sigma_x^{(1)} + g_2\sigma_z^{(2)})(a^+ + a). \quad (19)$$

In the interaction picture, we calculate the evolution operator (up to second order), then convert it to the Schrodinger picture and obtain the evolution operator $u(t,0)$ corresponding to H'. Experimentally, a unitary transformation will not be performed exactly and the actual transformation is a general quantum operation ε. The Kraus operators for ε are $\{M_n\}$ with $M_n = \langle n|u(t,0)|0\rangle$, where $|n\rangle$ corresponds to the state of the harmonic oscillator with n Bosons. Then the average fidelity $\bar{F}_{\varepsilon,U}$ of the quantum gate is given by[51]

$$\bar{F}_{\varepsilon,U} = \frac{\sum_n (\mathrm{Tr}(K_n)(K_n^+)) + D}{D^2 + D}, \quad (20)$$

where $K_n = U^+M_n$, $D = 2^k$ is the dimension of Hilbert space with $k = 2$. After tedious calculations, we derive the average fidelity

$$\bar{F}_{\varepsilon,U} \approx 0.997. \quad (21)$$

Meanwhile, the operation time of the quantum gate operation is much smaller than the coherence time of the qubits, i.e., $t_1 \ll T$, thus the quantum gate operation is robust against the realistic noises.

In conclusion, we have proposed an effective mechanism to couple the superconducting charge and flux qubits by using an NAMR. The coupling between the charge and flux qubit is controllable. Under the strong coupling limit, after selecting appropriate parameters for the hybrid quantum circuit, an iSWAP gate can be generated in our scheme. The experimental feasibility in our scheme is also presented.

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