# Destroying a Peldan Electrostatic Solution Black Hole *

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*By throwing a particle with electric charge and angular momentum into the black holes, much evidence shows that the naked singularity of some (3+1)-dimensional black holes might be seen, which is not allowed in the weak cosmic censorship conjecture. In this study, we consider a (2+1)-dimensional Peldan black hole and find that it could be destroyed under certain conditions in both extreme and near-extreme cases.*

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According to the weak cosmic censorship conjectures (WCCC) proposed by Penrose,[1] all physical singularities are hidden behind an event horizon. Many investigations on the WCCC violation have been carried out [2–12] by putting a test particle with angular momentum and electric charge into a black hole without considering the radiative and self-force effects. Those works find that under some harsh conditions, the particle can pass through the horizon and can cause the horizon to disappear, then a naked singularity could be seen by an observer outside the horizon. We choose a Peldan electrostatic solution (2+1)D black hole [13,14] to check whether it could be destroyed or not.

For the Peldan electrostatic solution, the metric is [13,14]

$$ds^2 = -N(\rho)^2 dt^2 + \frac{1}{L(\rho)^2} d\rho^2 + \rho^2 d\phi^2, \tag{1}$$

where

$$L(\rho) = N(\rho) = \sqrt{\frac{\rho^2}{l^2} - 2b^2 \ln \rho - M}, \tag{2}$$

and $\rho$ is the radius of the black hole, $b$ stands for electric field, $l$ is related to the cosmological constant, and $M$ is the mass of the black hole. Then we have

$$g_{tt} = -N(\rho)^2, \ g_{\rho\rho} = \frac{1}{L(\rho)^2}, \ g_{\phi\phi} = \rho^2, \ g_{t\phi} = 0. \tag{3}$$

The vector potentials $A_t$ and $A_\phi$ are [15]

$$A_t = b \ln \rho, \quad A_\phi = 0. \tag{4}$$

Let us consider a test particle with mass $m$ and charge $q$ moving along a geodesic with $\dot{\phi} = 0$. The conserved energy $E$ of the test particle is given by [9]

$$E = \frac{g_{t\phi} q A_\phi}{g_{\phi\phi}} - q A_t + \left\{ \left( \frac{g_{t\phi}^2 - g_{\phi\phi} g_{tt}}{g_{\phi\phi}^2} \right) [(-q A_\phi)^2 + m^2 g_{\phi\phi} (1 + g_{\rho\rho} \dot{\rho}^2)] \right\}^{1/2}. \tag{5}$$

Calculating $E$ at the event horizon $\rho_{\rm h}$ indicated by $L(\rho_{\rm h}) = 0$ by substituting Eqs. (3) and (4) into Eq. (5), we obtain

$$E = -bq \ln \rho_{\rm h} + m|\dot{\rho}_{\rm h}|. \tag{6}$$

Obviously, the minimum energy for a free particle to reach the event horizon is

$$E_{\rm min} = -bq \ln \rho_{\rm h}. \tag{7}$$

This gives the lower bound on the energy of a test particle to be absorbed into a black hole.

For the extreme case, the roots of $L(\rho)^2 = 0$ are equal. Then we could use the first derivative of $L(\rho)^2$ to find the root. We let

$$F(\rho) = L(\rho)^2 = \frac{\rho^2}{l^2} - 2b^2 \ln \rho - M. \tag{8}$$

Then the event horizon $\rho_{\rm h}$ indicated by $F'(\rho_{\rm h}) = 0$ is

$$\rho_{\rm h} = \rho_{\rm e} \equiv |bl|. \tag{9}$$

Putting Eq. (9) into $F(\rho_{\rm h}) = 0$ we obtain

$$M_{\rm e} = b^2 - 2b^2 \ln \rho_{\rm e} = b^2 - 2b^2 \ln(|bl|). \tag{10}$$

This is the mass of the extreme Peldan black hole. Before the test particle falls into the black hole, we define $F(\rho)$ from Eq. (8) as

$$F_{\rm be-e}(\rho) = \frac{\rho^2}{l^2} - 2b^2 \ln \rho - M_{\rm e}. \tag{11}$$

From Eq. (7), one can obtain the minimum energy for a particle to pass through the event horizon of the extreme Peldan black hole is

$$E_{\rm min-e} = -qb \ln(|bl|). \tag{12}$$

As the mass in Eq. (10) should be positive, we need $|bl| < \sqrt{e}$. For simplicity, we just consider $bl > 0$. On the other hand, the energy of a test particle entering the black hole shall be positive, which gives a

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constraint condition among $b$, $l$ and $q$. This means that, for $b > 0$, we have $bl < 1$ if $q > 0$, or $bl > 1$ if $q < 0$. Here we just consider the case of $q > 0$, since it is easy to prove that for $q < 0$ the similar result could be obtained.

From Eq. (8) we could see that if $F(\rho) > 0$ for any value of $\rho$, there is no black hole. A naked singularity could be seen. Then we have

$$M < \frac{\rho^2}{l^2} - 2b^2 \ln \rho. \tag{13}$$

By capturing a massive and charged test particle, the extreme black hole will change into a new system with the mass $M \to M_{\rm e} + E_{\rm e}$ and the charge $b \to b + q$, respectively, where $E_{\rm e}$ represents the energy of the test particle in the extreme case. One can find the extreme of Eq. (13) located in $\rho = (b+q)l$. Then, a naked singularity could be seen if

$$M_{\rm e} + E_{\rm e} < (b+q)^2 - 2(b+q)^2 \ln[(b+q)l]. \tag{14}$$

Thus we could obtain an upper bound on the particle's energy

$$E_{\rm max-e} = (b+q)^2 - 2(b+q)^2 \ln[(b+q)l] - M_{\rm e}. \tag{15}$$

Thus if the energy of the particle is between $E_{\rm min-e}$ and $E_{\rm max-e}$, the black hole will become a naked singularity.

We now set $b = 1$, and choose the coordinates to let $2l = 1$, i.e., $l = 0.5$. Thus $bl = 0.5 < 1$ satisfies the requirement. The mass in Eq. (10) is $M_{\rm e} = 2.38629$, and the radius of an extreme black hole is 0.5. Then Fig. 1 is obtained.

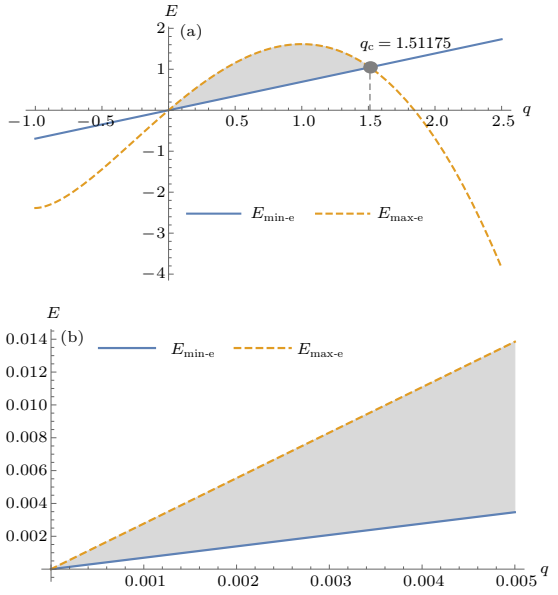


**Fig. 1.** Plot of $E$–$q$. The shaded area is the intersection of $E_{\rm min-e} < E$ and $E < E_{\rm max-e}$.

According to Fig. 1, if $q > q_{\rm c}$, the particle entering into the black hole does not destroy the black hole. Here we choose $q = 0.001 \ll b = 1$, then $E_{\rm min-e} = 0.000693147$ and $E_{\rm max-e} = 0.00277197$. We choose $E_{\rm e} = 0.001$, which is between $E_{\rm min-e}$ and $E_{\rm max-e}$. After the particle jumps into the black hole, $b$ changes into $b + q$ and $M_{\rm e}$ changes into $M_{\rm e} + E_{\rm e}$. Then from Eq. (8) we have

$$F_{\rm af-e}(\rho) = \frac{\rho^2}{l^2} - 2(b+q)^2 \ln \rho - (M_{\rm e} + E_{\rm e}). \tag{16}$$

The function $F_{\rm af-e}(\rho)$ with the values of parameters as mentioned above is shown in Fig. 2(a). From Fig. 2(a), we find that the real solution of $F_{\rm af-e}(\rho) = 0$ does not exist anymore. This implies that the horizon of the black hole vanishes and the naked singularity appears.

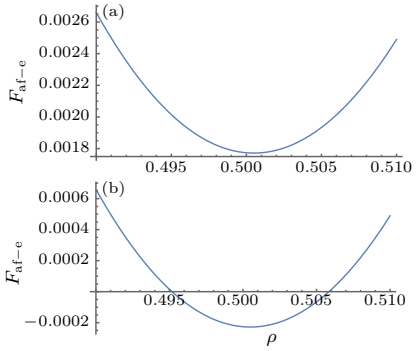


**Fig. 2.** The plots of $F_{\rm af-e}(\rho)$. (a) There is no horizon anymore. (b) There are two points of intersection on the horizontal axis. The left point stands for the inner horizon, and the right one stands for the outer horizon. The extreme black hole becomes a common black hole.

On the other hand, if we consider $E_{\rm e} = 0.003 > E_{\rm max-e}$ and keep other parameters with the same values, we obtain Fig. 2(b). One can see that there are two different solutions, the black hole becomes a common black hole. This implies that as long as the energy of the test particle locates in the shaded area in Fig. 1, the black hole will be destroyed.

The corresponding effective potential for the test particle is obtained from the relation [10,12]

$$V(\rho) = -\dot{\rho}^2. \tag{17}$$

Then, from Eq. (5), by replacing $\dot{\rho}^2$ with $-V(\rho)$, we can obtain the effective potential for an extreme black hole

$$V_{\rm e}(\rho) = \frac{\rho^2}{l^2} - 2b^2 \ln \rho - M_{\rm e} - \frac{(E_{\rm e} + qb \ln \rho)^2}{m^2}. \tag{18}$$

Then, by choosing the same parameters, we can obtain Fig. 3.

In Fig. 3, the plot $E_1$ describes a particle with $E_{\rm e} = m = 0.0002$ which is smaller than $E_{\rm min-e}$, and the plot $E_2$ for a particle with $E_{\rm min-e} < E_{\rm e} = m = 0.001 < E_{\rm max-e}$, which is in the proper range. We can see that if the energy is smaller than $E_{\rm min-e}$, there exists a potential barrier near the horizon which will stop the particle falling into the black hole. If the energy is between $E_{\rm min-e}$ and $E_{\rm max-e}$, the curve is descending and it is easy for the particle to fall into the black hole.

For a near-extreme Peldan electrostatic solution black hole, the radii of the horizons are quite near the

radius of the extreme case. We use a parameter $\delta$ to denote the mass difference between an extreme black hole and a near-extreme black hole. Thus we consider the mass from Eq. (10) as

$$M_{\rm ne} = M_{\rm e} + \delta = b^2 - 2b^2\ln(bl) + \delta, \tag{19}$$

where $bl > 0$, and $\delta$ is positive and much smaller than the mass of an extreme black hole $M_{\rm e}$ in Eq. (10), i.e., $\delta \ll M_{\rm e}$. Similar to Eq. (11), before the test particle falls into the black hole we have

$$F_{\rm be-ne}(\rho) = \frac{\rho^2}{l^2} - 2b^2\ln\rho - M_{\rm ne} = F_{\rm be-e}(\rho) - \delta, \tag{20}$$

where $F_{\rm be-e}(\rho)$ is given in Eq. (11). It is clear that if $\delta > 0$, the function $F_{\rm be-ne}(\rho) = 0$ would have two different solutions. If we still choose the same parameters, i.e., $b = 1$, $l = 0.5$, $M_{\rm e} = 2.38629$ and let $\delta = 0.0001 \ll M_{\rm e}$, one can obtain the inner and outer horizons by solving the equation $F_{\rm be-ne}(\rho) = 0$ as

$$\rho_- = 0.496469, \quad \rho_+ = 0.50354, \tag{21}$$

where $\rho_+$ corresponds to the event (outer) horizon of the black hole. Putting the outer horizon into Eq. (7), one has

$$E_{\rm min-ne} = -qb\ln(\rho_+). \tag{22}$$

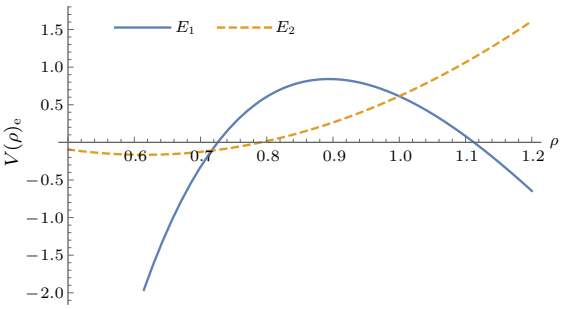


**Fig. 3.** Plot of the effective potential $V_{\rm e}(\rho)$ of the test particle with different energies outside the event horizon with $0.5 \le \rho \le 1.2$. Here $E_1 < E_{\rm min-e}$ and $E_{\rm min-e} < E_2 < E_{\rm max-e}$.

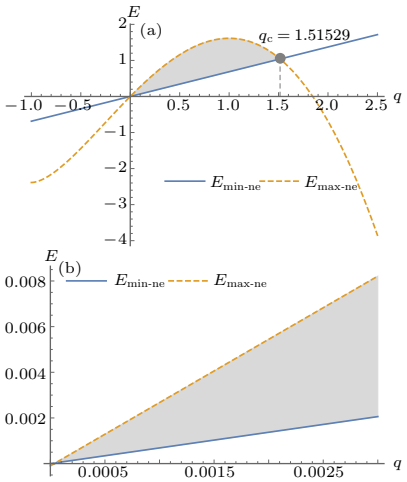


**Fig. 4.** Plot of $E$–$q$ on the near-extreme case with $\delta = 0.0001$. The shaded area is the intersection of $E_{\rm min-ne} < E$ and $E < E_{\rm max-ne}$.

Accordingly, the near-extreme Peldan electrostatic black hole turns into a naked singularity if

$$M_{\rm ne} + E_{\rm ne} < (b+q)^2 - 2(b+q)^2\ln[(b+q)l], \tag{23}$$

where $E_{\rm ne}$ is the energy of a test particle in the near-extreme case. Then we have

$$\begin{aligned} E_{\rm max-ne} &= (b+q)^2 - 2(b+q)^2\ln[(b+q)l] - M_{\rm ne} \\ &= E_{\rm max-e} - \delta, \end{aligned} \tag{24}$$

where $E_{\rm max-e}$ is given in Eq. (15). Similarly, we need to find the range of the parameter $q$ to make sure that $E_{\rm max-ne} > E_{\rm min-ne}$, which is plotted in Fig. 4. The numerical values are the same as those in obtaining Fig. 1, i.e., $b = 1$, $l = 0.5$, $M_{\rm e} = 2.38629$, with $\delta$ chosen as 0.0001. From Fig. 4, one can find that if $q > q_{\rm c}$, the particle entering into the black hole does not destroy the black hole. Then choosing $q = 0.001 \ll b$, we could obtain $E_{\rm min-ne} = 0.000686093$ and $E_{\rm max-ne} = 0.00267197$, respectively. We choose $E_{\rm ne} = 0.001$, which is between $E_{\rm min-ne}$ and $E_{\rm max-ne}$. Thus from Eq. (16), we can obtain $F(\rho)$ of the near-extreme black hole after capturing a test particle,

$$F_{\rm af-ne}(\rho) = \frac{\rho^2}{l^2} - 2(b+q)^2\ln\rho - (M_{\rm e} + E_{\rm ne}) - \delta. \tag{25}$$

From Fig. 2(a) we can see that $\delta$ will shift the curve graph which means that $\delta$ needs an upper bound. Putting the location of the extreme of $F_{\rm af-ne}(\rho)$ into Eq. (25), one can obtain $F_{\rm af-ne}(\rho) = 0.00177359 - \delta$. This means that to guarantee $F_{\rm af-ne}(\rho) > 0$, we need $\delta < 0.00177359$. Thus the above chosen $\delta = 0.0001$ is appropriate. Then, by plotting $F_{\rm af-ne}(\rho)$ in Fig. 5(a), we can see that there are no solutions. The black hole has been destroyed as well.

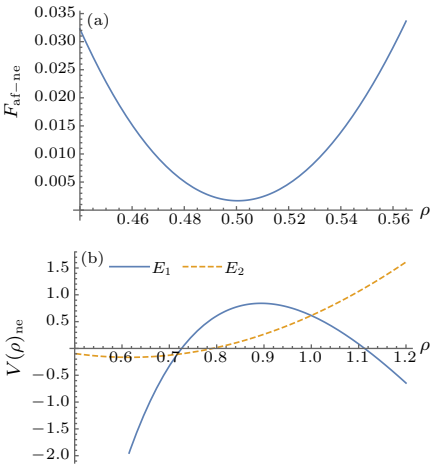


**Fig. 5.** The graphs of $F_{\rm af-ne}(\rho)$ and effective potential $V_{\rm ne}(\rho)$ with $\delta = 0.0001$, $q = 0.001$ and $\rho_+ = 0.50354 \le \rho \le 1.2$. (a) The plot of $F_{\rm af-ne}(\rho)$. There are no horizons anymore. (b) The plot of the effective potential $V_{\rm ne}(\rho)$ of the test particle with different energies outside a near outer event horizon of the near-extreme black hole, where $E_1 < E_{\rm min-ne}$ and $E_{\rm min-ne} < E_2 < E_{\rm max-ne}$.

With the same method, the corresponding effective potential for the test particle in the near-extreme

case is obtained from Eqs. (5) and (17),

$$V_{\rm ne}(\rho) = \frac{\rho^2}{l^2} - 2b^2 \ln\rho - M_{\rm ne} - \frac{(E_{\rm ne} + qb\ln\rho)^2}{m^2}, \quad (26)$$

which is plotted in Fig. 5(b).

The numeric values are the same as those in Fig. 4, the test particle energy is chosen as $E_1 = m = 0.0002 < E_{\rm min-ne}$, and $E_2 = m = 0.001$ which is between $E_{\rm min-ne}$ and $E_{\rm max-ne}$. If the energy is smaller than $E_{\rm min-ne}$, there exists a potential barrier near the horizon which will stop the particle falling into the black hole. However, if the energy is between $E_{\rm min-ne}$ and $E_{\rm max-ne}$, the curve is descending and the particle will easily fall into the black hole as well.

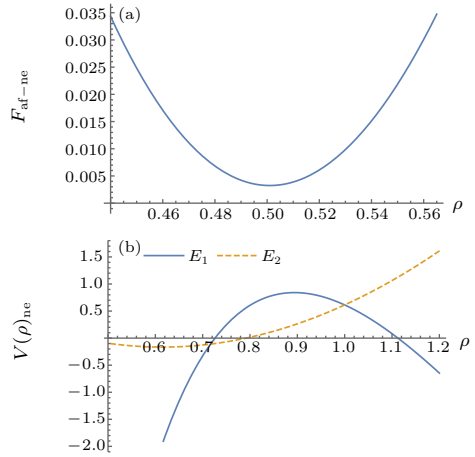


**Fig. 6.** The graphs of $F_{\rm af-ne}(\rho)$ and effective potential $V_{\rm ne}(\rho)$ with $\delta = 0.0003$, $q = 0.002$ and $\rho_+ = 0.501 \leq \rho \leq 1.2$. (a) The plot of $F_{\rm af-ne}(\rho)$. There are no horizons anymore as well. (b) The plot of the effective potential $V_{\rm ne}(\rho)$ of the test particle with different energies outside a near-extreme black hole, where $E_1 < E_{\rm min-ne}$ and $E_{\rm min-ne} < E_2 < E_{\rm max-ne}$.

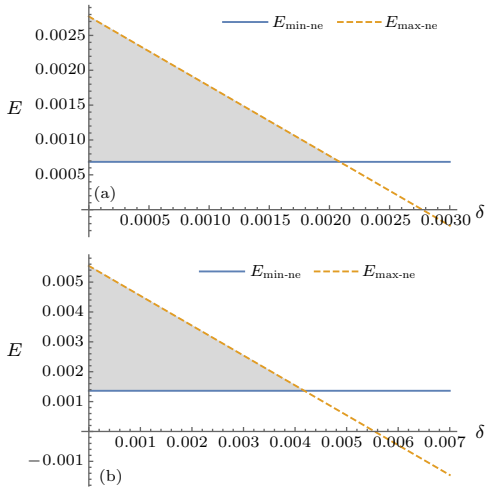


**Fig. 7.** The graphs of $E$–$\delta$ on different values of $q$. The shaded area is the energy that one could choose which satisfies $E_{\rm max-ne} > E_{\rm min-ne}$. Both the figures show that if $\delta \to 0$, we always have $E_{\rm max-ne} > E_{\rm min-ne}$. This is different from the result in Ref. [10]. The values of $\delta$ chosen above are proper.

To make sure of the result, we choose another value of $\delta = 0.0003$. By carrying out the same calculations, we obtain Fig. 6. From Fig. 6(a), we can see that the black hole is also destroyed. The corresponding effective potential is shown in Fig. 6(b). In Fig. 6(b), the plot $E_1$ describes a particle with $E_{\rm ne} = m = 0.0004$ which is smaller than $E_{\rm min-ne}$, and the plot $E_2$ for a particle with $E_{\rm ne} = m = 0.002$ which is between $E_{\rm min-ne}$ and $E_{\rm max-ne}$. This means that as long as $\delta$ is much smaller than $M_{\rm e}$, different values of $\delta$ will lead to similar results, and the near-extreme black hole could be destroyed as well.

To support our result that the near-extreme black hole could become a naked singularity, we discuss the range of $\delta$ which leads to $\Delta E > 0$ numerically. The graphs are given in Fig. 7 with $M_{\rm e} = 2.38629$ and $q = 0.001$ (Fig. 7(a)) and $q = 0.002$ (Fig. 7(b)). One can see that in this shadowed area the black hole can be destroyed even if the black hole is extreme, since $E_{\rm max-ne}$ keeps larger than $E_{\rm min-ne}$ when $\delta \to 0$.

In summary, without taking into account the radiative and self-force effects, we have shown that a test particle may destroy the horizons of the extreme and near-extreme Peldan electrostatic black holes, which provides evidence of violation of the WCCC in (2+1)D black hole case.

We also calculate the minimum and maximum energies of the test particle and find the ranges of the permitted energies on both extreme and near-extreme cases. However in our work, the ranges of the permitted energies in both the cases are slightly larger, which are different from other works. This means that the Peldan black hole is easier to destroy. The differences might come from the different methods of the calculations. In other works, they take analyticity and contain some high-order terms, while we sacrifice the analyticity and use numerical calculation in our work. We would like to study this issue in the future work. All of those results come from the precondition that we ignore the radiative and self-force effects. The question whether we could also obtain the similar results by considering the two effects is also worth discussing in the future work.

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