CHIN. PHYS. LETT. Vol. 34, No. 4 (2017) 040302

Continuous Variable Quantum Teleportation in Beam-Wandering Modeled Atmosphere Channel *

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(Received 5 July 2016)

We investigate the continuous variable quantum teleportation in atmosphere channels. The beam-wandering model is employed to analyze the teleportation of the unknown single-mode coherent state. Two methods, one is deterministic by increasing the aperture size of the detecting device and one is probabilistic by entanglement distillation, are proposed to improve the teleportation fidelity in the presence of atmosphere noises.

PACS: 03.67.Bg, 42.50.Ex, 03.67.Hk, 03.67.Mn DOI: 10.1088/0256-307X/34/4/040302

Quantum entanglement is an important quantum resource, and quantum teleportation is one of the most important forms of quantum information processing.[1] Quantum teleportation allows for faithful quantum state transmission via pre-shared two-party entanglement and single-party local operations. Since its first proposal, quantum teleportation has been theoretically and experimentally studied in both discrete variable [2] and continuous variable [3] quantum systems. Experimental quantum teleportation has been implemented in many kinds of physical systems such as quantum optics, atomic ensemble [4,5] and even hybrid quantum systems.[6,7] The records of long-distance quantum teleportation are quickly updated: desktop demonstration in 1997,[2] 13 km in 2005,[8] 16 km in 2010,[9] and 143 km in 2012.[10]

To implement the global quantum teleportation, the quantum channel of free space, which is always filled with atmospheric turbulence, is one of the most important problems researchers should mainly focus on. Actually, the atmosphere channel is a quite complex channel. For one thing, the optic signal may be attenuated by the fog, rain, dust, snow, smoke, and other aerosol particulate matter in the atmosphere and for the other, random fluctuation in turbulence could cause unstable distribution of index-of-reflection, which will finally result in random beam positioning at the receiving plane. To overcome the former problem, one could minimize the absorption by the atmosphere with appropriate selection of the optical wavelength. However, to overcome the latter, one should carefully study the beam wandering at the receiving plane.

The beam-wandering model for studying conventional optic communication has been widely studied in the 1970s,[11] but it cannot be directly applied to analyzing quantum dynamics evolution under realistic conditions.[12] Recently, a model with an optic beam splitter and a first-principle study of the probability distribution of the transmission coefficient (PDTC) has been obtained by Vasylyev *et al.*[12] This model gives a realistic description of quantum light transmission in the atmosphere and fits very well under weak turbulence conditions. In this Letter, we give an investigation of the quantum teleportation of a continuous variable coherent state via the atmospherically disturbed two-mode squeezed vacuum state (TMSS). More precisely, we consider the single-side loss scenario where only one mode is sent through the atmospheric turbulence channel. We use the teleportation fidelity as a figure of merit to evaluate the performance of teleportation. Our result shows that improving teleportation with increasing the size of detecting aperture is not always useful and the method of probabilistic entanglement distillation is more useful in eliminating the influence of decoherence.

The most general continuous variable (CV) quantum teleportation scheme is shown in Fig. 1. We have three optic modes, labeled by A, B and C. Here A and B are in two-mode entanglement which plays the role of the quantum channel, and mode C contains the unknown coherent state one intends to teleport. To implement the teleportation, the optic mode C and one mode in the two-mode entanglement, e.g., A mode, is mixed with a 50:50 beam splitter. Then two conjugated quadratures of the outgoing modes, $x_{\rm C}$ and $p_{\rm A}$, are measured by balanced homodyne measurements. These measuring results are then used as the reference information for subsequent local displacement operation. Here we assume that the teleportation with unit gain is applied, which means that the displacement by the amount of $-\beta$ with $\beta = x_{\rm C} + ip_{\rm A}$ is applied to the optic mode B.

Let $\rho_{\rm C} = |\alpha\rangle\langle\alpha|$ be the density matrix of mode C. The 50:50 beam splitter coupling and subsequent homodyne measurement can be equivalent to a generalized heterodyne detection on the optic mode A,[13,14]

$$\Pi_{\rm A}(\beta) = \frac{1}{\pi} D(\beta)\rho^{\rm T} D^{\dagger}(\beta), \tag{1}$$

where $D(\beta) = \exp(\beta a^{\dagger} - \beta^* a)$ is the optic displacement operator, and the superscript T stands for the

*Supported by the National Natural Science Foundation of China under Grant Nos 11574400, U1304613, 11204197, 11204379 and 11074244.
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matrix transposition.

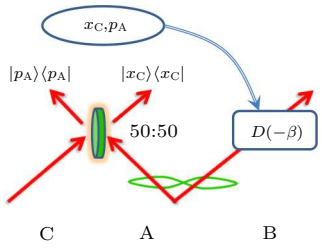

Fig. 1. (Color online) Scheme diagram of continuous variable quantum teleportation of coherent state. A and B constitute the continuous variable two-mode entanglement state, and C mode is the coherent state to be teleported. The optic beam splitter of transmittance 1/2 and relevant homodyne measurement of $|x_C\rangle\langle x_C|, |p_A\rangle\langle p_A|$ implements the joint quantum measurement. Quantum teleportation is completed when a quadrature displacement operation $D(-\beta)$ (depending on the homodyne measurement) is applied to the optic mode B.

We assume that the A–B mode entanglement is an arbitrary state with the density matrix given by $\widetilde{\rho}_{\rm AB}$. The probability of detecting $(x_{\rm C}, p_{\rm A})$ in conjugated homodyne measurement can be given by $P(\beta) = {\rm Tr}[(\Pi_{\rm A}(\beta) \otimes I)\widetilde{\rho}_{\rm AB}]$. The state after homydyne measurement follows $\rho_{\rm B} = \frac{1}{P(\beta)}{\rm Tr}_{\rm A}[(\Pi_{\rm A} \otimes I)\widetilde{\rho}_{\rm AB}]$, with ${\rm Tr}_{\rm A}$ being the partial trace over the optic mode A. Thus after the final quadrature displacement on the optic mode B, the final state (conditioned on measurement result $(x_{\rm C}, p_{\rm A})$) follows $\rho_{\rm B}(\beta) = D(\beta)\rho_{\rm B}D^{\dagger}(\beta)$. We can define the fidelity $F(\beta) = \langle\alpha|\rho_B(\beta)|\alpha\rangle$ to evaluate how the output state resembles the original one. Since the measurement result $(x_{\rm C}, p_{\rm A})$ occurs probabilistically, the average fidelity weighted by probability $P(\beta)$ is given by

$$\begin{aligned} F &= \int d^2\beta P(\beta)F(\beta) \\ &= \frac{1}{\pi}\int d^2\beta {\rm Tr}[\widetilde{\rho}_{\rm AB}D(\beta)\rho_{\rm C}^{\rm T}D(\beta)^{\dagger} \\ &\quad \otimes D(\beta)|\alpha\rangle\langle\alpha|D^{\dagger}(\beta)] \\ &= {\rm Tr}[\widetilde{\rho}_{\rm AB}O_F], \end{aligned} \tag{2}$$

with $O_F = \frac{1}{\pi}\int d^2\beta D(\beta) \otimes D(\beta)(\rho_C^{\rm T} \otimes |\alpha\rangle\langle\alpha|)D^{\dagger}(\beta) \otimes D^{\dagger}(\beta)$. For any α, we have the invariance $d^2\beta = d^2(\beta + \alpha)$. Using the equality in Ref. [15], i.e., $\int d^2\gamma \exp[-s|\gamma|^2]\gamma^n\gamma^{*m} = (\pi n!/s^{n+1})\delta_{nm}$, (Re$s > 0$), we obtain $O_F = \sum_{K=0}^{\infty}\sum_{i,j=0}^{K}\frac{1}{2^{K+1}}\sqrt{\binom{K}{i}\binom{K}{j}}|i,j\rangle\langle K-j, K-i|$.

Thus we have now obtained the explicit formula for evaluating the average fidelity with the arbitrary CV two-mode entanglement state with density matrix $\widetilde{\rho}_{\rm AB}$. For example, if the two-mode entanglement is the pure TMSS $\widetilde{\rho}_{\rm AB} = |\psi_{\rm TMSS}\rangle\langle\psi_{\rm TMSS}|$, with $|\psi_{\rm TMSS}\rangle = \cosh^{-1}s\sum_{n=0}^{\infty}\tanh^n(s)|n\rangle|n\rangle$, the average fidelity is given by $F = (1 + \tanh(s))/2$, which coincides with the result given by Ref. [16].

In this work we consider the two-mode CV entanglement obtained by the transmitting pure TMSS state via a single-side atmosphere channel. Actually, the free space in which the CV entanglement may be transmitted is always filled with atmospheric turbulence, which is unstable and highly fluctuating. Many models for atmospheric turbulence have been proposed, and the beam-wandering model is more suitable in simulating weak turbulence. The beam-wandering model fits very well and has been widely used to analyze the Bell-inequality violation,[17] satellite-based entanglement distribution[18] and entanglement distillation in realistic scenarios.[19]

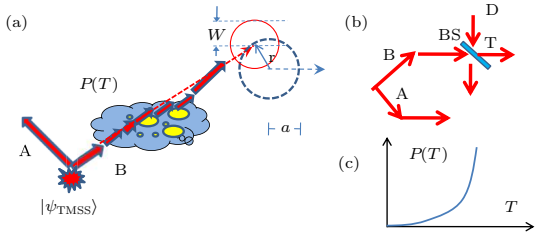

Fig. 2. (Color online) (a) Entanglement distribution over a single-side atmosphere channel. Here W is the radius of optic beam at the receiving plane, a is the radius of the detector aperture, and r is the deflection between the two centers. (b) Beam splitter model of single-side transmission of two-mode entanglement. BS denotes the beam splitter of transmittance T. (c) Probabilistic distribution $P(T)$ as a function of the transmittance T.

The beam wandering model fits very well under a weak turbulence condition, where the main effect on the beam propagation is the movement of the optic intensity centroid at the receiver plane. Suppose that the beamspot radius is W at the aperture plane, and the aperture radius is a. A figure depicting the geometric relation among r, W and a is shown in Fig. 2.

Due to the atmosphere disturbance, the beam center will be deflected by the distance r from the aperture center. Moreover, r is not a constant but a random variable, following the Rice distribution with parameter $d = \langle r\rangle$ and variance σ.[12] The beam-splitter model is clearly shown in Fig. 1(b). Let $|\psi_{\rm TMSS}\rangle$ be the two-mode pure state of optic modes A and B, $|0\rangle_{\rm C}$ be the vacuum state of the optic mode C. The beam splitter coupling between mode B and mode C is given by $|\psi_{\rm ABC}\rangle = I \otimes U_{\rm BC}(|\psi_{\rm TMSS}\rangle\langle\psi_{\rm TMSS}| \otimes |0\rangle\langle 0|)$, with $U_{\rm BC}$ being the unitary operator for the beam splitter between B and C: $U_{\rm BC}(T) = \exp[\arctan\sqrt{(1-T)/T}(a_{\rm B}a_{\rm C}^{\dagger} - a_{\rm B}^{\dagger}a_{\rm C})]$. Finally, the fixed single-side loss in the optic mode B can be mathematically formulated by $\rho_{\rm AB}(T) = {\rm Tr}_{\rm C}[|\psi_{\rm ABC}\rangle\langle\psi_{\rm ABC}|]$.

To obtain a good approximation, the probability distribution of T will be given by the log-negative generated Rice distribution $P(T) = \frac{R^2}{\sigma^2\lambda T}[\ln\frac{T^*}{T}]^{\frac{2}{\lambda}-1}I_0(\frac{Rd}{\sigma^2}[\ln\frac{T^*}{T}]^{1/\lambda})\exp\{-\frac{1}{2\sigma^2}[R^2(\ln\frac{T^*}{T})^{\frac{2}{\lambda}} + d^2]\}$, with $T = T^*\exp[-(\frac{r}{R})^{\lambda}]$, and $T^* = 1 - \exp[-2\tau^2]$, $\tau = a/W$, $\lambda = \frac{8\tau^2\exp[-4\tau^2]I_1(4\tau^2)}{1-\exp[-4\tau^2]I_0(4\tau^2)} / \ln(\frac{2T^*}{1-\exp[-4\tau^2]I_0(4\tau^2)})$, $R = a[\ln(\frac{2T^*}{1-\exp[-4\tau^2]I_0(4\tau^2)})]^{-1/\lambda}$, in which $I_n(x)$ is the modified Bessel function of order n. Particularly, if the beam fluctuates around the aperture center, $d = 0$, the distribution reduces to the log-negative Wibull func-

tion, $P(T) = \frac{R^2}{\sigma^2\lambda T}[\ln\frac{T^*}{T}]^{\frac{2}{\lambda}-1}\exp[-\frac{1}{2\sigma^2}R^2(\ln\frac{T^*}{T})^{\frac{2}{\lambda}}]$. After the single side quantum state transmission via the atmosphere channel, the final two-mode entanglement is given by $\rho_{\rm AB} = \int \rho_{\rm AB}(T)P(T)dT$.

To study the performance of teleportation in a realistic atmosphere channel, let us first identify the relevant parameters of a, W and σ. For a given aperture size a in the detection plane, the atmosphere index-of-reflection structure constant C_n^2 and the transmission distance are the key factors that determine the beam wandering σ and the beam size W. Assuming that the coefficient C_n^2 is constant along the path and W_0 is the beamspot radius at the radiation source, the variance in the detecting plane can be given by $\sigma^2 = 1.305C_n^2(2W_0)^{-1/3}L^3$.[11,20–22] Due to the atmosphere turbulence, the chief ray of the beam is randomly moving in the transverse plane, making the beam spread. The beam size in the detecting plane can be given by [22] $W = \sqrt{W_0^2 + 2(\frac{L\lambda_0}{\pi r_0})^2}$, with λ_0 being the wavelength and $r_0 = [0.5476(\frac{2\pi}{\lambda_0})^2C_n^2L]^{-3/5}$ the coherent length.

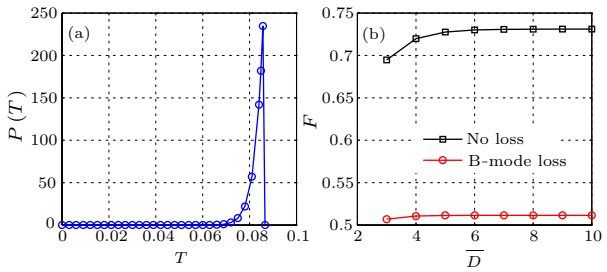

Fig. 3. (Color online) (a) The probability distribution $P(T)$ of transmission coefficient T at $W_0 = a = 0.10$ m, $C_n^2 = 10^{-15}\,\mathrm{m}^{-2/3}$, $\lambda_0 = 850$ nm, and $L = 10$ km. (b) Numerical simulation of teleportation of the coherent state with atmospherically disturbed TMSS with $\overline{D}$-dimensional few-photon subspace. The squared lines indicate the fidelity of teleportation with an ideal pure TMSS state. The circled lines represent the fidelity of teleportation with the B-mode damped TMSS state $\rho_{\rm AB}$.

We now take the wavelength $\lambda_0 = 850$ nm as an example and numerically evaluate the teleportation via $L = 10$ km atmosphere transmission. It is assumed that the original squeezing parameter is $s = 0.5$ and teleportation takes place under the weak turbulence condition $C_n^2 = 10^{-15}\,\mathrm{m}^{-2/3}$. For convenience, we can assume that both the transmitter and the receiver use the telescopes of the same size to send and collect the beam: $W_0 = a = 0.10$ m. The probability distribution of $P(T)$ is clearly shown in Fig. 3. The average value of transmittance is $\langle T\rangle = \int P(T)dT = 0.083$. Generally, in numerical simulation we cannot treat the continuous variable quantum entanglement in an infinite-dimensional space and the relevant optic mode is conveniently truncated within few-photon subspace. Here each mode of $\rho_{\rm AB}$ is truncated with the $\overline{D}$-dimensional subspace spanned by $\{|0\rangle, |1\rangle, \ldots, |\overline{D}-1\rangle\}$. The fidelity is evaluated by using Eq. (2) for the increasing size of $\overline{D}$. Our numerical simulation quickly converges at $\overline{D} = 6$. The fidelity of teleportation of the coherent state is substantially decreased from $F = 0.7310$ to $F = 0.5115$ after 10 km atmospheric transmission.

Increasing the aperture size in the detecting plane is one of the most direct ways to combat against atmosphere-induced decoherence and to improve the fidelity in teleportation. We study a 10-km teleportation scheme with a pair of telescopes of larger size. Again we assume that the telescopes in the transmitter and the receiver are the same as $W_0 = a$. In Fig. 4(a), we plot the average transmittance $\langle T\rangle$ for the increasing value of a, in units of meters. In Fig. 4(b), we evaluate the corresponding teleportation fidelity. Our first insight that a detecting aperture of large size will improve the teleportation is confirmed. However, the extent of improvement is limited. Even in the regime of a very large telescope $a = 2$ m, the teleportation fidelity $F = 0.7040$ still cannot beat the value $F = 0.7310$ in the ideal and no-loss case.

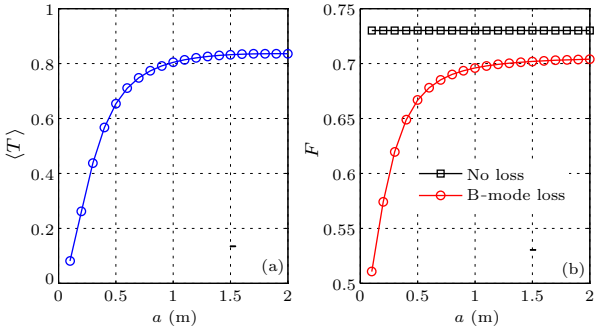

Fig. 4. (Color online) Probability distribution $P(T)$ as a function of the aperture size $W_0 = a$. (b) The corresponding teleportation fidelity for the increasing value of a. The squared lines indicate the fidelity of teleportation with an ideal pure TMSS state $|\psi_{\rm TMSS}\rangle$. The circled lines represent the fidelity of teleportation with the B-mode damped TMSS state $\rho_{\rm AB}$. Other parameters are $s = 0.50$, $C_n^2 = 10^{-15}\,\mathrm{m}^{-2/3}$, $\lambda_0 = 850$ nm, and $L = 10$ km.

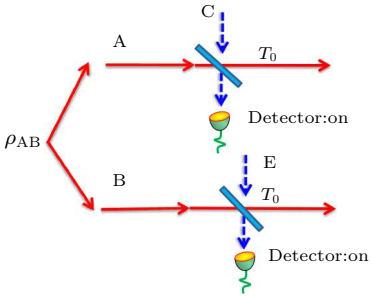

Fig. 5. (Color online) Scheme for distillation of atmospheric-turbulence-disturbed entanglement through bi-side photon subtraction. Here T_0 represents the transmittance coefficient of the relevant beam splitter. The inputs of C- and E-mode are assumed to be vacuum states.

To completely combat against the decoherence in the atmosphere channel, an alternative method would be the CV entanglement distillation, first proposed by Opatrný *et al.*[23] and later extensively studied in Refs. [14,16,23–28]. We consider the case that bi-side photon subtraction (PS) is applied to an atmospherically disturbed state $\rho_{\rm AB}$. We study the typical photon subtraction implemented with beam splitter of transmittance T_0 and on-off photonic detectors,[28] as shown in Fig. 5. The entangled state to be dis-

tilled is now the atmosphere disturbed state $\rho_{\rm AB}$. In both the beams, a beam splitter with transmittance T_0 and an on-off photon detector are employed to implement the photon subtraction-based distillation. The distillation succeeds when both detectors register non-zero counts. Expressed in the Fock state space, the distilled state can be given by $\rho_{\rm AB}^{\rm dist} = {\rm Tr}_{\rm CE}[\rho_{\rm ABCE}(\Pi_{\rm on}^{\rm (C)} \otimes \Pi_{\rm on}^{\rm (E)})]/P_{\rm dist}$, with $\rho_{\rm ABCE} = \mathbb{U}(\rho_{\rm AB} \otimes |00\rangle_{\rm (CE)}\langle 00|)\mathbb{U}^\dagger$, $\mathbb{U} = U_{\rm AC}(T_0) \otimes U_{\rm BE}(T_0)$, $\Pi_{\rm on}^{(j)} = I - \Pi_{\rm off}^{(j)} = \sum_{k=1}^{\infty} |k\rangle_{(j)}\langle k|$, $j = {\rm C}, {\rm E}$, and $P_{\rm dist} = {\rm Tr}[\rho_{\rm ABCE}(\Pi_{\rm on}^{\rm (C)} \otimes \Pi_{\rm on}^{\rm (E)})]$.

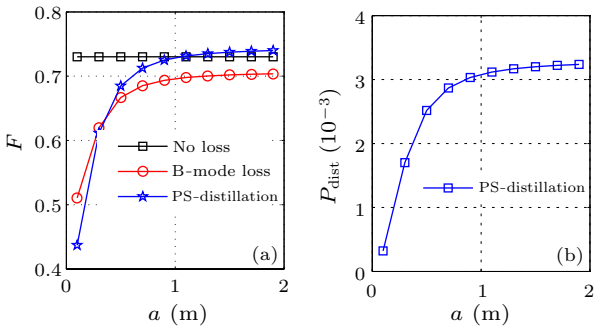

Fig. 6. (Color online) (a) Teleportation fidelity with bi-side PS-distilled $\rho_{\rm AB}$. (b) Corresponding probability of successful distillation. The squared lines indicate the fidelity of teleportation with an ideal pure TMSS state $|\psi_{\rm TMSS}\rangle$. The circled lines represent the fidelity of teleportation with the B-mode damped TMSS state $\rho_{\rm AB}$. The lines marked with stars indicate the improved fidelity of teleportation with both increased detecting aperture and probabilistic entanglement distillation. Other parameters are $s = 0.50$, $W_0 = a$, $C_n^2 = 10^{-15}\,{\rm m}^{-2/3}$, $\lambda_0 = 850\,{\rm nm}$, $L = 5\,{\rm km}$, and $T_0 = 0.95$.

The distilled state is then used as an entanglement channel for quantum teleportation in Fig. 1. The fidelity with the distilled entanglement is shown by the lines marked with stars in Fig. 6(a). Note that the channel of distance $L = 5\,{\rm km}$ is considered. Numerical results show that the PS-based distillation is useful for complete elimination of influence of the atmospheric turbulence from the point of view of teleportation fidelity. However, the distillation is not always efficient for all channels. There should be some thresholds for the channel and only the channel with sufficient quality allows for distillation. For the bi-side PS distillation with $T_0 = 0.95$, the threshold for the aperture is $a_{\rm th} = 0.30$. In addition, the elimination of influence of atmosphere is fulfilled by sacrificing the probability of success, as clearly shown in Fig. 6(b).

In summary, atmosphere fills all the space above the earth. We choose the beam-wandering model as an example and investigate the possibility of enhancing of entanglement distribution via atmosphere channel with bi-side photon subtraction or bi-side photon addition. Our results represent a trial of entanglement pre-processing to enhance more practical entanglement distribution among free space channels. On the other hand, our result, once again, proves the significance of non-Gaussian operations such as photon subtractions in continuous variable quantum communication.

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