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Theoretical Modeling of Luminous Efficacy for High-Power White Light-Emitting Diodes *

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The photometric characteristics of high-power white light-emitting diode (LED) devices are investigated. A theoretical model for the luminous efficacy of high-power white LED devices and LED systems is proposed. With the proposed theoretical model, the mechanism of the luminous efficacy decrease is explained. Meanwhile, the model can be used to estimate the luminous efficacy of LEDs under general operation conditions, such as different operation temperatures and injection currents. The wide validity of the luminous efficacy model is experimentally verified through the measurements of different types of LEDs. The experimental results demonstrate a high estimation accuracy. The proposed models not only can be applied to estimate the LED photometric performance, but also is helpful for reliability research of LEDs.

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High-power white light-emitting diodes (LEDs) have attracted more and more attention. This kind of solid-state lighting source is recognized as the most promising lighting candidate. It has great potential to replace traditional gas-discharge lamps. With the sustained increase of the luminous efficacy, nowadays LEDs have been widely used in automotive headlights, displays, electric hand torches, and other low power applications.[1–3] However, for the general lighting applications, LEDs are still unable to replace fluorescent lamps successfully. Some LED lamps which were used to replace fluorescent lamps can only produce 50–60% of the light output. One possible reason for such a large gap between the actual light output and the claimed light output is the lack of understanding of the luminous efficacy of LED systems operated under practical conditions. It is valuable to investigate the mechanism of the luminous efficacy decrease.

One work issued by U.S. Department of Energy, compared the relative power conversion for several kinds of popular light sources and summarized a power conversion table for 'white' light sources. Incandescent lamps can convert 8% electric power into visible light, 73% electric power into infrared (IR), and 19% electric power into heat. Fluorescent and metal halide sources convert a higher proportion of electric power into visible light, around 20–30%, but also emit IR, ultraviolet (UV), and heat. LEDs generate slight IR and UV. The electric power dissipated by LEDs is divided into two parts. One is heat, and the other is visible light. If an LED converts 20% electric power into visible light, it means 80% electric power will be dissipated as heat. Such a large amount of heat is a hidden trouble for solid state devices.[4,5] Junction temperature is one critical impact factor of the decrease of the luminous efficacy of LEDs.[6–8]

In addition to the junction temperature, the injection current also contributes to the decrease of the luminous efficacy. Schubert *et al.* reports that the efficacy drop, which occurs under high injection conditions, is unrelated to junction temperature.[9] Meanwhile, even driving the blue/green LEDs under short and low-duty cycle pulses to minimize the self-heating effect, the drop in quantum efficiency remains nearly identical.[10,11] The fundamental cause of efficiency drop with current is identified as the auger recombination and polarization fields.[9,10,12] Generally, it is observed from Refs. [13–17] that the maximum luminous efficacy usually happens at very low current, typically a few milliamps. As the injection current beyond this threshold current, the efficacy begins to decrease monotonically. This monotonic decrease of efficacy with the increasing injection current becomes another critical restriction for the usage of LEDs in high-power applications.[18,19]

LED application manufacturers and academic research groups have been making efforts to develop the most convenient techniques for luminous efficacy and junction temperature measurements. Schubert *et al.* provides the effect of temperature on the emission intensity (I_{E}) of LEDs.[20] Hui *et al.* investigated the effect of junction temperature on the luminous efficacy.[21] In addition, some research presents the measurement results of internal quantum efficiency and external quantum efficiency variation with the injection current.[22–26]

This study proposes a theoretical model of the luminous efficacy. Simultaneous effects of the injection current and the junction temperature on the light output are expressed in the models. With the proposed models, the mechanism of the luminous efficacy decrease is demonstrated and the value of the luminous efficacy under different operation conditions can be calculated accurately. Each new parameter in the model has definite physical meaning and is easily measured. Meanwhile, the proposed model is not

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only applicable to LED devices but also applicable to LED systems under different possible operation conditions. Overall, the proposed model can be used as a tool to estimate the photometric performance of LED devices and LED systems under any operation condition. They are helpful for LED reliability research. The validity of the proposed luminous efficacy model has been experimentally verified.

The modeling is started with the physical meaning of the luminous efficacy. The luminous efficacy is defined as the total luminous flux emitted by an LED over the electrical power applied to the LED. The unit of the luminous efficacy is lm/W.

According to Ref. [20], the effect of temperature on the emission intensity ($I_{\rm E}$) of an LED can be expressed as

$$I_{\rm E} = I_{\rm E}|_{300\,\rm K} \exp\Big(-\frac{T - 300\,\rm K}{T_1}\Big), \tag{1}$$

where T_1 is the characteristic temperature, and T is the junction temperature.

According to Ref. [21], the luminous efficacy as a function of the junction temperature of an LED device can be expressed as

$$E = E_{\rm o}[1 + k_{\rm e}(T_{\rm j} - T_{\rm o})], \tag{2}$$

where $E_{\rm o}$ is the rated efficacy at the rated temperature $T_{\rm o}$, $k_{\rm e}$ is relative rate of reduction of efficacy with increasing temperature, $T_{\rm j}$ is the junction temperature, and $T_{\rm o}$ is the typical temperature 25℃.

Equations (1) and (2) describe the variation of the luminous efficacy with the junction temperature, under a constant injection current level.

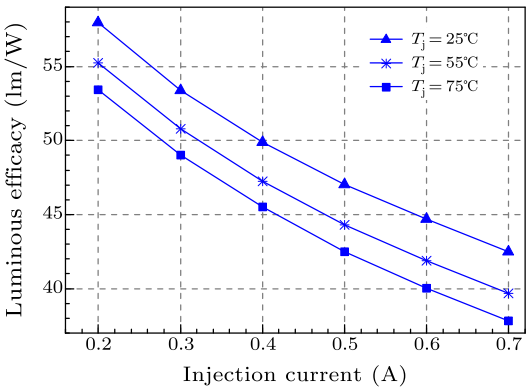

Fig. 1. Luminous efficacy versus injection current at a constant junction temperature.

To find out the effect of the injection current on the luminous efficacy, the luminous efficacy under different injection currents was measured. Figure 1 shows the degradation of the luminous efficacy with the increasing current. In Fig. 1, data points on each curve were measured at the same junction temperature, which eliminated the self-heating effect. From Fig. 1, it is found that the luminous efficacy decreases with the injection current. Figure 2 shows the degradation of the luminous efficacy with the increasing junction temperature. In Fig. 2, the luminous efficacy has different decay curves for different current levels.

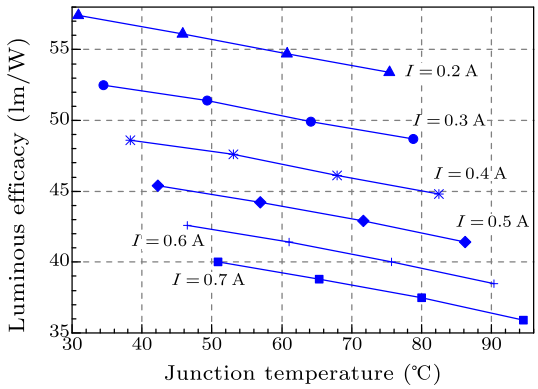

Fig. 2. Luminous efficacy versus junction temperature under different current levels.

In Fig. 2, at each current level, the luminous efficacy decreases linearly with the junction temperature. Table 1 lists the data of the slope of the E–$T_{\rm j}$ curve for two different types of LED devices. In Table 1, we use k_t to represent the slope of the E–$T_{\rm j}$ curve, $E_{\rm o}k_{\rm e}$. From data in Table 1, we find that the trend of k_t ($E_{\rm o}k_{\rm e}$) variation is unpredictable, and the range of the k_t variation is not large for the same type of the LED device. Meanwhile, from the curves in Fig. 2, it can be observed that these curves are nearly in parallel, for high power application. Therefore, in this work, we consider the slopes of each E–$T_{\rm j}$ line to be equal to each other, and $E_{\rm o}k_{\rm e}$ is assumed to be a constant value for the same type of LED device. Practically, $E_{\rm o}k_{\rm e}$ is related to chip materials and light color of LEDs.

In addition, from Fig. 2, it can be observed that intercepts of each line with the vertical axis are not identical. The difference in the intercepts of each line is significant so that it needs to be considered into luminous efficacy equation. In Fig. 2, the intercept is the value of the luminous efficacy at a typical temperature of 25℃. Meanwhile, we can observe that in Fig. 2 the intercepts of the extrapolated lines with the vertical axis can reproduce the decay curve ($T_{\rm j} = 25$℃) in Fig. 1. Based on these phenomena, the luminous efficacy equation is expressed as

$$E = E_{\rm o}k_{\rm e}(T_{\rm j} - T_{\rm o}) + E|_{25℃}, \tag{3}$$

where $E|_{25℃}$ is not a constant value, and $T_{\rm o} = 25$℃.

Parameters in Eq. (3) have their respective physical meanings, $E_{\rm o}$ is rated efficacy at temperature 25℃, $k_{\rm e}$ is the relative rate of reduction of efficacy with increasing temperature,[21] and $E_{\rm o}k_{\rm e}$ expresses the slope of the E–$T_{\rm j}$ line, which indicates that the speed of efficacy decreases with the junction temperature.

In Eq. (3), $E|_{25℃}$ represents the luminous efficacy at the junction temperature of 25℃. Figure 1 gives the curves of the luminous efficacy at three different junction temperatures, $E|_{25℃}$, $E|_{55℃}$ and $E|_{57℃}$. In Fig. 2, $E|_{25℃}$ is the intercepts of E–$T_{\rm j}$ lines with the vertical axis. From Figs. 1 and 2, it can be observed

that the value of $E|_{25℃}$ varies with the injection current. To find out the relationship between $E|_{25℃}$ and injection current I, we plot the curve of $E|_{25℃}$ of Fig. 1 into a new semi-logarithmic coordinate system, as shown in Fig. 3, in which the x-axis is common axis divided evenly, and the y-axis is the logarithmic coordinate axis. It can be observed that the curve of $E|_{25℃}$ in Fig. 1 becomes a straight line in Fig. 3, which indicates that $E|_{25℃}$ decreases exponentially with the injection current. Thus $E|_{25℃}$ can be expressed as

$$E|_{25℃} = E_{io} \exp(-k_i I), \tag{4}$$

where E_{io} represents the maximum efficacy at junction temperature of 25℃, k_i represents the decay coefficient of the luminous efficacy with the increasing current at junction temperature of 25℃, and k_i characterizes the decrease speed of the luminous efficacy with the increasing current.

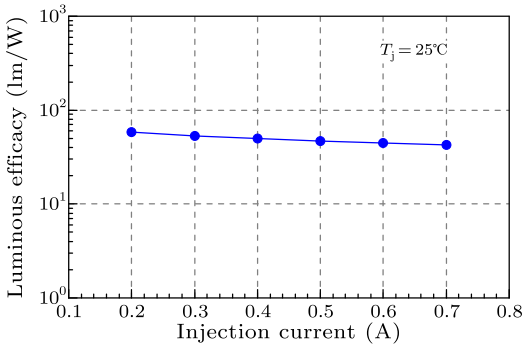

Fig. 3. Semi-logarithmic coordinate systems: luminous efficacy versus injection current at a constant junction temperature 25℃.

Table 1. The values of k_t of E–T_j lines of the two tested devices.

LED1		LED2	
I (A)	k_t (lm/(W℃))	I (A)	k_t (lm/(W℃))
0.2	−0.1701	0.2	−0.0903
0.3	−0.1743	0.3	−0.0874
0.4	−0.1742	0.4	−0.0876
0.5	−0.1821	0.5	−0.0908
0.6	−0.1802	0.6	−0.0938
0.7	−0.1813	0.7	−0.0935

InGaN/GaN LEDs exhibits the maximum quantum efficiency at very low current, typically a few milliamps.[19,22–26] As the current increases beyond that value, a monotonic drop in quantum efficiency will happen, even under constant junction temperature. In Eq. (4), E_{io} represents the maximum luminous efficacy, occurring at initial small current at junction temperature of 25℃. For example, one typical current for InGaN LEDs to exhibit the highest efficacy of E_{io} is 0.6 mA, provided by Ref. [19].

With the proposal of Eqs. (3) and (4), the complete luminous efficacy equation as a function of both the injection current and the junction temperature is proposed as

$$E = E_o k_e (T_j - T_o) + E_{io} \exp(-k_i I), \ I > 0, \tag{5}$$

which is the proposed luminous efficacy model of a single LED device, and E_o is the rated efficacy at temperature 25℃. For example, for a 1 W LED device, E_o should take the value at $I = 350$ mA. Here E_{io} is the highest efficacy at temperature 25℃, and happens at the initial several milliamps of current.

In practice, it is not easy to apply Eq. (5) to estimate the luminous efficacy, because the junction temperature of LED devices cannot be measured directly. To simplify the estimation procedure, the parameter of T_j in Eq. (5) is expressed by external parameters.

For an LED device, the junction temperature is expressed as

$$T_j = T_c + R_{jc} k_h P_d, \tag{6}$$

where k_h is the heat dissipation coefficient, $k_h P_d$ is the dissipated heat power that is a part of the electrical power P_d, R_{jc} is the thermal resistance of LED package, and T_c is the case temperature that can be measured directly.

With Eq. (6), the luminous efficacy equation of an LED device becomes

$$E = E_o k_e (T_c - T_o + R_{jc} k_h P_d) + E_{io} \exp(-k_i I). \tag{7}$$

For an LED system that contains N numbers of LED devices mounted on a heatsink, with the assumption of identical distribution of heatsink surface temperature, the junction temperature of each device reads

$$T_j = T_a + (R_{jc} + N R_{hs}) k_h P_d, \tag{8}$$

where N is the number of LED devices in an LED system.

By substituting Eq. (8) into Eq. (5), the luminous efficacy equation of an LED system is obtained as

$$E = E_o k_e [T_a - T_o + k_h (R_{jc} + N R_{hs}) P_d] + E_{io} \exp(-k_i I). \tag{9}$$

In Eqs. (7) and (9), k_h can be calculated by the method proposed in Ref. [27], in which k_h is derived from the wall plug efficiency.

Equations (5) and (7) can be applied to estimate the luminous efficacy for a high-power white LED device. Meanwhile, Eqs. (5) and (9) can be used to estimate the luminous efficacy for an LED system.

In the following the validity of the proposed luminous efficacy model is verified by testing. Three types of LED devices from different manufacturers are used to verify the validity of the proposed model. Type-one LED is CREE XLamp XR-E LED (Cree XREWHT-L1-0000-00C01). Type-two LED is SEOUL N42180 LED (N42180-EC01). Type-three LED is OSRAM-LUM (W5AM-KYLX-4C8E) LED device. The measurements of LED device parameters, including thermal parameters and photometric parameters, were conducted with the TeraLED-T3ster equipment,[28,29] which integrates the function of temperature control, junction temperature measurement and photometric measurement of semiconductor devices.

The luminous efficacy of the type-1 LED device was measured under different operation conditions. Parameters $E_o k_e$, E_{io} and k_i of the LED device were also obtained with the TeraLED-T3ster equipment, which are $-0.175\,\mathrm{lm/(W℃)}$, $0.481\,\mathrm{A^{-1}}$, and $110\,\mathrm{lm/W}$, respectively. Parameter $E_o k_e$ is read from the slope of the E–T_j curve. Here E_{io} is the maximum efficacy at a junction temperature of 25℃, which can be directly measured by the TeraLED-T3ster equipment, with the setting of injection current in milliamps. The value of k_i can be read from the E–I curve at a junction temperature 25°C.

By substituting the measured parameters into Eq. (5), the luminous efficacy equation becomes

$$E = -0.175(T_j - 25) + 110\exp(-0.481I). \quad (10)$$

The luminous efficacy variation with the junction temperature was measured at different current levels, as shown in Fig. 4. The calculated curves of the luminous efficacy versus the junction temperature from Eq. (10) are also displayed in Fig. 4. The red curves are measured data, and blue curves are the calculated values. The calculated values are generally consistent with the measurement results.

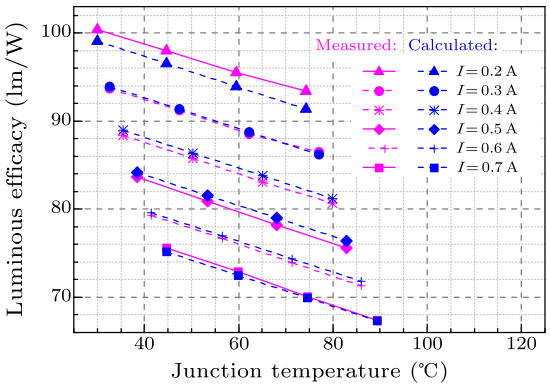

Fig. 4. Calculated and measured luminous efficacy versus junction temperature for the type-1 LED device.

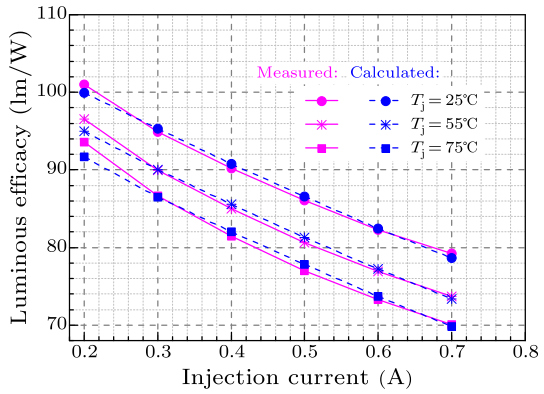

Fig. 5. Calculated and measured luminous efficacy versus injection current for the type-1 LED device.

Meanwhile, the luminous efficacy variation with the injection current was measured at different constant junction temperatures, as shown in Fig. 5. The calculated values from Eq. (10) are also displayed in Fig. 5. Good agreement between the calculated and measured values is obtained. Both Figs. 4 and 5 indicate that not only the junction temperature but also the injection current can lead to a decrease of the luminous efficacy.

A different type of LED device from another manufacturer was tested. The tested values for parameters $E_o k_e$, k_i and E_{io} for the type-2 LED device are $-0.091\,\mathrm{lm/(W℃)}$, $0.612\ \mathrm{A^{-1}}$ and $64.5\,\mathrm{lm/W}$, respectively. Fitting these parameters into Eq. (5), the luminous efficacy equation of this kind of device becomes

$$E = -0.091(T_j - 25) + 64.5\exp(-0.612I). \quad (11)$$

The calculated luminous efficacy versus the junction temperature using Eq. (11) is plotted in Fig. 6. The practical measurements are also plotted in Fig. 6. The calculated and measured luminous efficacy versus the injection current is plotted in Fig. 7. It can be observed that the calculated results and measured results are in good agreement.

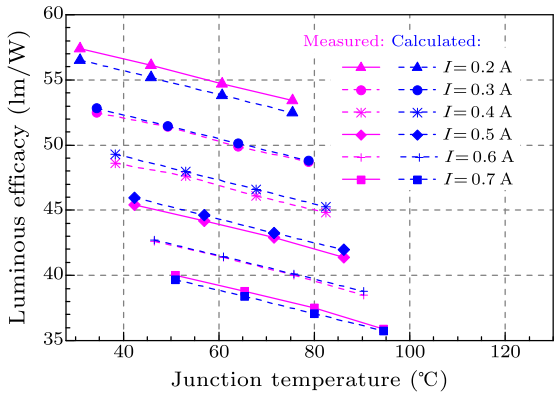

Fig. 6. Calculated and measured luminous efficacy versus junction temperature for the type-2 LED device.

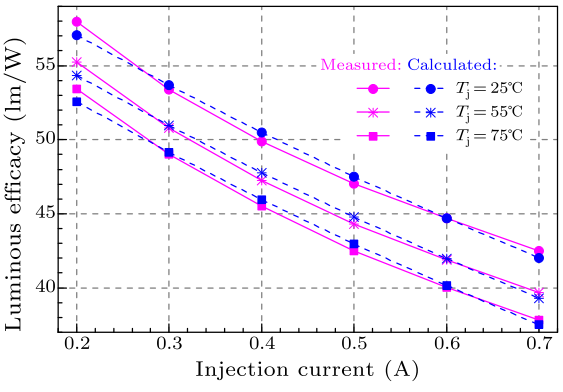

Fig. 7. Calculated and measured luminous efficacy versus injection current for the type-2 LED device.

To verify that the luminous efficacy equation can be applied to every high-power white LED, the third-type of LED device from another manufacturer was used for the evaluation. The tested parameter values required in Eq. (5) include $E_o k_e$ of $-0.117\,\mathrm{lm/(W℃)}$, k_i of $0.93\,\mathrm{A^{-1}}$ and E_{io} of $124.9\,\mathrm{lm/W}$. The calculation

formula of luminous efficacy of the type-3 LED device is

$$E = -0.117(T_{\rm j} - 25) + 124.9\exp(-0.93I). \quad (12)$$

The calculated and measured luminous efficacy under different operation conditions is plotted in Figs. 8 and 9, respectively. In general, the calculated and measured results are in good agreement.

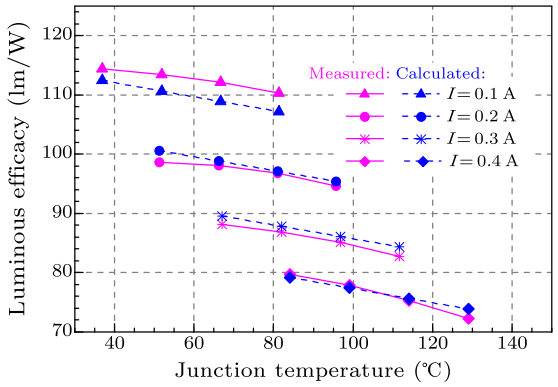

Fig. 8. Calculated and measured luminous efficacy versus junction temperature for the type-3 LED device.

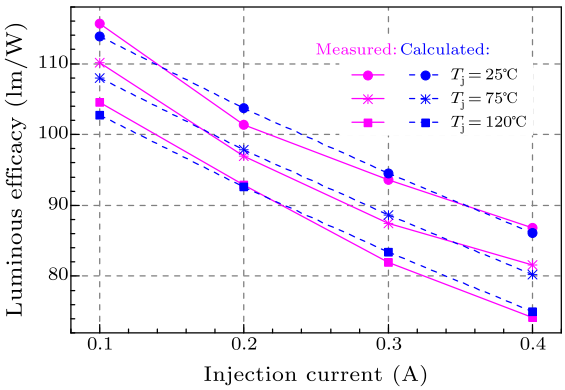

Fig. 9. Calculated and measured luminous efficacy versus injection current for the type-3 LED device.

In summary, a mathematic and physical model of luminous efficacy for high-power white LED devices has been proposed. The proposed model provides simultaneous effect of the junction temperature and the injection current on degradation of the luminous efficacy. Each parameter in the model is new, and reasonable explanation of the meaning of the parameters is provided. The model describes the degradation mechanism of the luminous efficacy more fully. The experimental tests show that the estimation accuracy of the luminous efficacy model is high. Meanwhile, it is not only applied to LED devices, but also applied to LED systems, under any operation condition. It is valuable for the prediction of photometric performance and reliability research of LED devices and LED systems.

References

[1] Chiu H J, Lo Y K, Lee T P, Mou S C and Huang H M 2009 *IEEE Trans. Ind. Electron.* **56** 2793
[2] Lv X, Loo K H, Lai Y M and Tse C K 2014 *IEEE Trans. Ind. Electron.* **61** 4665
[3] Chiu H J, Lo Y K, Chen J T, Cheng S J, Lin C Y and Mou S C 2010 *IEEE Trans. Ind. Electron.* **57** 735
[4] Arik M, Petroski J and Weaver S 2002 *IEEE 8th Intersoc. Conf. Thermal Thermomech. Phenomena Electron. Syst.* (Orlando, Florida 28–31 May 2008) p 113
[5] Osvald J, Kuzmik J, Konstantinidis G, Lobotka P and Georgakilas A 2005 *Microelectronic Eng.* **81** 181
[6] Tan S C 2010 *IEEE Trans. Ind. Electron.* **57** 1342
[7] Biber C 2008 *IEEE Semicond. Therm. Meas. Manage. Symp.* (San Jose, California 16–20 March 2008) p 180
[8] Yu Y J, Zhang F H and Ni J J 2014 *IEEE Trans. Ind. Electron.* **61** 2423
[9] Kim M H, Schubert M F, Dai Q, Kim J K and Schubert E F 2007 *Appl. Phys. Lett.* **91** 183507
[10] Ling S C, Lu T C, Chang S P, Chen J R, Kou H C and Wang S C 2010 *Appl. Phys. Lett.* **96** 231101
[11] Yang Y, Cao X A and Yan C 2008 *IEEE Trans. Electron. Devices* **55** 1771
[12] Shen Y C, Mueller G O, Watanabe S, Gardner N F, Munkholm A and Krames M R 2007 *Appl. Phys. Lett.* **91** 141101
[13] Kim A Y, Götz W, Seigerwald D A, Wierer J J, Gardner N F, Sun J, Stockman S A, Martin P S, Krames M R, Kern R S and Steranka F M 2001 *Phys. Status Solidi* A **188** 15
[14] Akita K, Kyono T, Yoshizumi Y, Kitabayashi H and Katayama K 2007 *J. Appl. Phys.* **101** 033104
[15] Efremov A A, Bochkareva N I, Gorbunov R I, Larinovich D A and Shreter Yu G 2006 *Phys. Semicond. Devices* **40** 605
[16] Rozhansky V and Zakheim D A 2006 *Phys. Status Solidi* C **3** 2160
[17] Rozhansky V and Zakheim D A 2007 *Phys. Status Solidi* A **204** 227
[18] Krames M R, Shchekin O B, Mueller-Mach R and Craford M G 2007 *J. Display Technol.* **3** 160
[19] Mukai T, Yamada M and Nakamura S 1999 *Jpn. J. Appl. Phys.* **38** 3976
[20] Schubert E F 2006 *Light-Emitting Diodes* 2nd edn (Cambridge: Cambridge University Press) chap 5 p 98
[21] Hui S Y R and Qin Y X 2009 *IEEE Trans. Power Electron.* **24** 1967
[22] Cai J X, Sun H Q, Zheng H, Zhang P J and Guo Z Y 2014 *Chin. Phys.* B **23** 058502
[23] Liu W, Zhao D G, Jiang D S, Chen P, Liu Z S et al 2015 *Chin. Phys.* B **24** 127801
[24] Zhao Y K, Li Y F, Yun F et al 2015 *Chin. Phys.* B **24** 056806
[25] Zhou X J, Zhang J, Li S T et al 2014 *Chin. Phys.* B **23** 068502
[26] Rajabi K, Cao W Y, Hu X D et al 2015 *Chin. Phys. Lett.* **32** 027802
[27] Chen H T, Tao X H and Hui S Y R 2012 *IEEE Trans. Power Electron.* **27** 2176
[28] Farkas G, Vader Q V V, Poppe A and Bogn'ar G 2005 *IEEE Trans. Compon. Packag. Technol.* **28** 45
[29] Sz'ekely V, Poppe A, Rencz M, Rosental M and Tesz'eri T 2000 *Microelectron. Rel.* **40** 517