Negative Refraction Index Manipulated by a Displaced Squeezed Fock State in the Mesoscopic Dissipative Left-Handed Transmission Line *

Hong-Wei Guo(郭宏伟), Shun-Cai Zhao(赵顺才)**, Xiao-Jing Wei(魏晓菁), Xin Li(李昕)

Department of Physics, Faculty of Science, Kunming University of Science and Technology, Kunming 650500

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Negative refractive index (NRI) of the mesoscopic dissipative left-handed transmission line (LHTL) is manipulated by the displaced squeezed Fock state and the dissipation presented by the resistance and conductance. Compared with the classical LHTL, some novel characteristics of NRI are shown in the LHTL because of the quantum effect, which will be significant for its miniaturization application in microwave frequency.

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The earliest concept of the left-handed materials was theoretically considered by Veselago[1] in 1968, in which the electric field $\boldsymbol{E}$, magnetic field $\boldsymbol{H}$ and the wavevector $\boldsymbol{k}$ form a left-handed system, which is opposite to the ordinary right-handed media. Many special physical phenomena such as inverse Doppler effects,[2–4] reverse Cerenkov radiation,[5] perfect lens[6–9] occur because of the counterintuitive physical properties of negative refractive index (NRI).[10–13] Negative-index metamaterial (NIM) transmission lines, also called left-handed transmission lines (LHTL) existing at microwave frequency band[14,15] are special microwave transmission lines that have been utilized in many applications.[16–19] Instead of the conventional equivalent circuit model with series inductance and shunt capacitance distributed, an LHTL has series capacitance and shunt inductance distributed which leads to opposite signs of phase and group velocities. For a realistic LHTL, loss occurring in series resistance and shunt conductance is needed in the equivalent circuit model, as shown in Figs. 1(a) and 1(b).

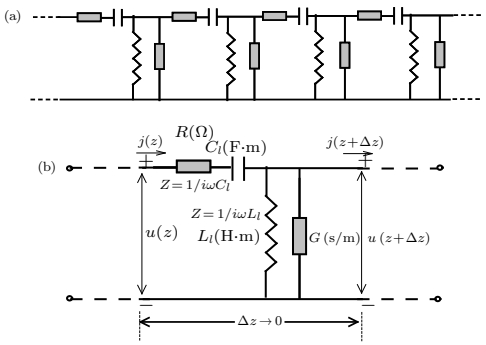

Fig. 1. (a) The equivalent circuit model of the dissipative left-handed transmission line. (b) The equivalent unit cell model of the dissipative left-handed transmission line.

There is increasing interest in the nanoelectronics. When the devices or circuits are so small that the transport dimension approaches the charge-carrier inelastic coherent length and the charge carrier confinement dimension reaches the Fermi wavelength, the quantum effects of the devices and circuits must be taken into account.[20–22] With the miniaturization applications of LHTL in mind, we propose a promising new feature in our previous work,[23] in which the quantum influences on NRI of the ideal LHTL are discussed in the thermal Fock state. Here we investigate the mesoscopic dissipative LHTL in a displaced squeezed Fock state[24,25] which has wide applications in the field of quantum optics,[27] nonlinear problems[28–30] and electromagnetic wave.[31] Some novel characteristics of NRI will be shown and will be significant to its miniaturization applications in microwave transmission lines.

We consider the LHTL equivalent circuit composed as in Fig. 1, which is distributed by series capacitance and shunt inductance, and in which the present R and G (resistance and conductance) represent the dissipation. The classical differential equations can be rewritten as

$$u(z,t) = j(z,t)\left[R + \frac{1}{i\omega C_l}\right]\Delta z + u(z+\Delta z,t), \quad (1)$$

$$j(z,t) = u(z,t)\left[G + \frac{1}{i\omega L_l}\right]\Delta z + j(z+\Delta z,t), \quad (2)$$

where L_l and C_l are the inductance and capacitance of the unit length dissipative LHTL, respectively, $u(z,t)$ is the voltage, $j(z,t)$ is the current, and ω is the angle frequency. Considering the separated current between the inductance L_l and conductivity G, and the relation between the voltage of the inductance, the solutions to the classical equations of motion corresponding to Eqs. (1) and (2) as a consequence of Kirchhoff's law are as follows:

$$j(z,t) = e^{-\sigma z}[Ae^{(i\beta z - i\omega t)} + A^* e^{(i\omega t - i\beta z)}], \quad (3)$$

$$u(z,t) = \frac{i\omega L_l}{\omega L_l G + 1} e^{-\sigma z}[A^* e^{(i\omega t + i\beta z)} - Ae^{(i\beta z - i\omega t)}], \quad (4)$$

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**Corresponding author. Email: zhaosc@kmust.edu.cn

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where σ and β are the displacement damping coefficient and wave number, respectively. Their expressions are as follows:

$$\sigma = \Big[\frac{1}{2}\Big[(x-y) + \Big[(x-y)^2 + \Big(\frac{G}{\omega C_l} + \frac{R}{\omega L_l}\Big)^2\Big]^{\frac{1}{2}}\Big]\Big]^{\frac{1}{2}},$$

$$\beta = \Big[\frac{1}{2}\Big[-(x-y) + \Big[(x-y)^2 + \Big(\frac{G}{\omega C_l} + \frac{R}{\omega L_l}\Big)^2\Big]^{\frac{1}{2}}\Big]\Big]^{\frac{1}{2}},$$

where $x = RG$ and $y = \frac{1}{\omega^2 L_l C_l}$. According to the quantization scheme of Louisell,[32] we introduce two variables η and ν. They are defined as

$$j(z,t) = e^{-\sigma z}\eta(z,t), \tag{5}$$

$$u(z,t) = \frac{\omega^2}{z_0}e^{-\sigma z}\nu(z,t), \tag{6}$$

where z_0 is the length of per unit cell circuit. It is easy to verify

$$\frac{\partial}{\partial\eta}\Big(\frac{\partial\eta}{\partial t}\Big) + \frac{\partial}{\partial\nu}\Big(\frac{\partial\nu}{\partial t}\Big) = 0. \tag{7}$$

Thus η and ν are the normal variables. As a result, the Hamiltonian of the equivalent unit cell model of dissipative LHTL is

$$H = \frac{\omega^2\nu^2(\omega L_l G+1)}{2L_l z_0} + \frac{L_l z_0}{2(\omega L_l G+1)}\eta^2. \tag{8}$$

We assume that $\hat{\eta}$ and $\hat{\nu}$ satisfy the following commutation relation

$$[\hat{\eta},\hat{\nu}] = i\hbar = \frac{2iL_l z_0}{\omega_l(\omega L_l G+1)}[\hat{A},\hat{A}^*]. \tag{9}$$

Then we obtain the quantization of the mesoscopic dissipative LHTL with

$$[\hat{A},\hat{A}^*] = \frac{\hbar\omega}{2L_l z_0}, \tag{10}$$

where $\hat{A}$ and $\hat{A}^*$ satisfy

$$\hat{A} = \hat{a}\sqrt{\frac{\hbar\omega(\omega L_l G+1)}{2L_l z_0}}, \tag{11}$$

$$\hat{A}^* = \hat{a}^*\sqrt{\frac{\hbar\omega(\omega L_l G+1)}{2L_l z_0}}, \tag{12}$$

and $[\hat{a},\hat{a}^*] = 1$. Thus $\hat{a}$ and $\hat{a}^*$ are the creation and annihilation operators, respectively, and $\hat{a}^*$ should be written as $\hat{a}^\dagger$. The corresponding operators for $\eta(z,t)$ and $\nu(z,t)$ can be written as

$$\hat{\eta}(z,t) = \sqrt{\frac{\hbar\omega(\omega L_l G+1)}{2L_l z_0}}[\hat{a}e^{(i\beta z-i\omega t)} + \hat{a}^\dagger e^{(i\omega t-i\beta z)}], \tag{13}$$

$$\hat{\nu}(z,t) = i\sqrt{\frac{\hbar L_l z_0}{2\omega(\omega L_l G+1)}}[\hat{a}^\dagger e^{(i\omega t-i\beta z)} - \hat{a}e^{(i\beta z-i\omega t)}]. \tag{14}$$

The displaced squeezed Fock state can be defined as[33–35]

$$|z,\xi,n\rangle = \hat{D}(z)\hat{S}(\xi)|n\rangle, \tag{15}$$

where the operators $\hat{D}$ and $\hat{S}$ satisfy $\hat{D}(z) = e^{(z\hat{a}^\dagger - z^*\hat{a})}$, $\hat{S}(\xi) = e^{(\frac{1}{2}\xi\hat{a}^{\dagger 2} - \frac{1}{2}\xi^*\hat{a}^2)}$, with $z = |z|e^{i\theta}(|z| > 0, 0 \le \theta < 2\pi)$ being a displaced parameter, and $\xi = |\xi|e^{i\phi}(|\xi| > 0, 0 \le \phi < 2\pi)$ being a squeezed parameter with the squeezed direction ϕ. Using the formula

$$e^{\lambda\hat{A}}\hat{B}e^{-\lambda\hat{A}} = \hat{B} + \lambda[\hat{A},\hat{B}] + \frac{\lambda^2}{2}[\hat{A},[\hat{A},\hat{B}]] + \ldots \tag{16}$$

we can achieve the following transformations

$$\hat{D}^\dagger(z)\hat{a}\hat{D}(z) = \hat{a} + z, \tag{17}$$

$$\hat{D}^\dagger(z)\hat{a}^\dagger\hat{D}(z) = \hat{a}^\dagger + z^* \tag{18}$$

$$\hat{S}^\dagger(\xi)\hat{a}\hat{S}(\xi) = \hat{a}\cosh|\xi| + \hat{a}^\dagger e^{i\phi}\sinh|\xi|, \tag{19}$$

$$\hat{S}^\dagger(\xi)\hat{a}^\dagger\hat{S}(\xi) = \hat{a}^\dagger\cosh|\xi| + \hat{a}e^{-i\phi}\sinh|\xi|. \tag{20}$$

Therefore, the mean value of the current operator in the Heisenberg picture in the displaced squeezed Fock state (DSFS) can be obtained

$$\begin{aligned}\langle\hat{j}(z)\rangle &= \langle n,\xi,z|\hat{j}(z)|z,\xi,n\rangle \\ &= Fe^{-\sigma z}[|z|e^{i\theta}e^{i\beta z} + |z|e^{-i\theta}e^{-i\beta z}],\end{aligned} \tag{21}$$

where $F = \sqrt{\frac{\hbar\omega(\omega LG+1)}{2Lz_0}}$, and its corresponding mean square value can be written as

$$\begin{aligned}\langle\hat{j}(z)^2\rangle &= \langle n,\xi,z|\hat{j}(z)^2|z,\xi,n\rangle \\ &= F^2e^{-2\sigma z}\{[(2n+1)\cosh|\xi|\sinh|\xi|e^{i\phi} \\ &\quad + z^2]e^{2i\beta z} + [(2n+1)\cosh|\xi|\sinh|\xi|e^{-i\phi} \\ &\quad + z^{*2}]e^{-2i\beta z} + 2(n+1)\cosh^2|\xi| \\ &\quad + 2n\sinh^2|\xi| + 2zz^*\}.\end{aligned} \tag{22}$$

Then the quantum fluctuation of the current should be

$$\begin{aligned}\langle(\Delta\hat{j})^2\rangle &= \langle\hat{j}(z)^2\rangle - \langle\hat{j}(z)\rangle^2 \\ &= (2n+1)F^2e^{-2\sigma z}\Big[\frac{1}{2}\sinh|2\xi|\Big[e^{i\phi}e^{2i\beta z} \\ &\quad + e^{-i\phi}e^{-2i\beta z}\Big] + \cosh|2\xi| + 1\Big].\end{aligned} \tag{23}$$

It is noticed that βz is infinitesimal when z is dimensionless, and using the following transformation relations[11–13]

$$\beta = \omega\sqrt{\mu\epsilon}, \quad n_r = \frac{c_0\beta}{\omega}, \tag{24}$$

and Eq. (23), we can obtain the refraction index of the mesoscopic dissipative LHTL with respect to the fluctuation of the current as follows:

$$\begin{aligned}n_r = &-\frac{c_0}{\omega z\sin(\phi)}\Big[\frac{2Lz_0\langle(\Delta\hat{j})^2\rangle e^{2\sigma z}}{(2n+1)\hbar\omega(\omega L_l G+1)\sinh|2\xi|} \\ &- \frac{\cosh|2\xi|}{\sinh|2\xi|} - \frac{\cos(\phi)}{2}\Big].\end{aligned} \tag{25}$$

As mentioned above, its value is negative within the microwave frequency band.[14–16]

In our model, the resistance R and conductance G represent the dissipation in the mesoscopic dissipative LHTL, and we set the relation $G = \frac{1}{R} \times 10^{-2}$ in Fig. 2. The NRI dependent resistance is controlled by the squeezed parameter in Fig. 2. In general, the resistance plays a destructive role in electricity because of Ohm's Law. However, a different feature of R in the mesoscopic dissipative LHTL appears in the interval of [0, 0.4Ω]. It shows that NRI increases when the resistance R has a fainter increase in the gray block of [0, 0.4Ω]. However, in the other interval of [0.5Ω, 2Ω], NRI almost remains an invariable value with the same squeezed parameter. At the same time, the effect of the squeezed parameter on NRI is prominent. We notice that the squeezed parameters increase by 0.05π, and NRI decreases sharply from -1.9 to -0.8. Compared with the classic LHTL, the counterintuitive characteristic of the resistance only takes place in the mesoscopic dimension of LHTL.

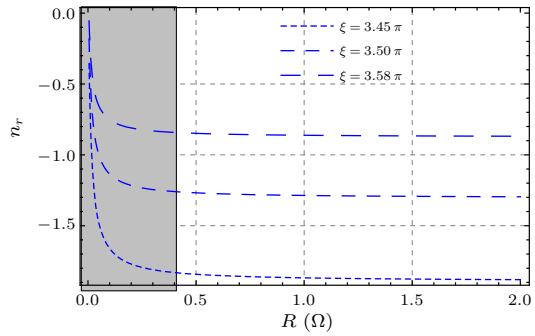

Fig. 2. (Color online) The dependence of the negative refraction index on the resistance R of the unit cell circuit with different squeezed parameters. Other parameters are $\omega = 3\,\mathrm{GHz}$, $\langle(\Delta\hat{j})^2\rangle = 10$, $\phi = \frac{\pi}{3}$, $n = 2$, $z_0 = 4z = 4\,\mu\mathrm{m}$, $L = 398\,\mu\mathrm{H}$, $C = 995\,\mathrm{pF}$.

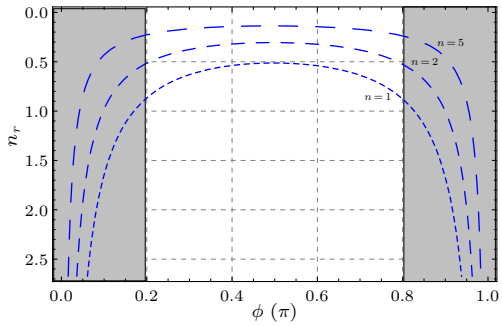

Fig. 3. (Color online) The dependence of the negative refraction index on the squeezed direction ϕ with different photon numbers $n = 1$, 2 and 15. Here $\xi = 2.8\pi$, $G = 0.2\,\mathrm{S/m}$, $R = 0.2\,\Omega$, and other parameters are the same as Fig. 2.

Another interesting regulatory parameter is the squeezed parameter. Figure 3 shows the influence of squeezed direction on NRI in DSFS, i.e., NRI versus the squeezed direction of [0,π] with different photon numbers 1, 2 and 15. The two gray blocks correspond to two angular intervals [0, 0.2π] and [0.8π, 1.0π], respectively, which means two opposite squeezed directions. The interval of [0, 0.2π] shows the forward squeezed direction along the transmission line, and an inverse squeezed direction happens in the interval of [0.8π, 1.0π]. In the forward squeezed direction, NRI decreases when the direction angular enhances, while NRI increases in the reverse squeezed direction. Utilizing Eqs. (8), (13) and (14), the quantum Hamiltonian of the dissipative LHTL reaches $\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$. Therefore, the energy of the electromagnetic wave in LHTL is proportional to the photon numbers. Figure 3 shows that the amount of the energy of the electromagnetic wave reduces NRI, which is a novel quantum behavior in the mesoscopic LHTL.

In the classic LHTL, NRI is a constant with the circuit when the circuit is in the steady status. However, something may be different in the mesoscopic LHTL. Figure 4 plots NRI with the circuit within a unit cell z_0, and NRI shows some attractive features because of the quantum effect. We note that NRI decreases with the circuit within a unit cell z_0, and it reaches asymptotically to a constant in spite of the different squeezed parameters. The larger squeezed parameter leads to a smaller NRI. That NRI with the circuit within the unit cell has a gradient value is distinctive to the LHTL in the macro scale. This feature may be significant and interesting to miniature design and practical applications for LHTL.

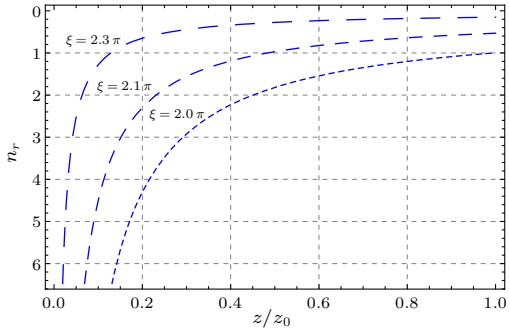

Fig. 4. (Color online) The dependence of the negative refraction index on the displacement squeezed parameters with different squeezed parameters. Here $n = 5$, $G = 0.02\,\mathrm{S/m}$, $R = 0.02\,\Omega$ $\phi = \frac{\pi}{5}$, and other parameters are the same as Fig. 3.

In conclusion, the quantum characteristic of NRI has been manipulated by the DSFS and dissipation in the mesoscopic dissipative LHTL. Along the transmission line, NRI is not homogeneous but decreases to an asymptotic value with different squeezed parameters. The forward and inverse squeezed directions result in the decreasing or increasing NRI along the transmission line. The tiny dissipation, i.e., the resistance or conductance, leads to the increasing NRI, while the increasing resistance brings out an asymptotic constant of NRI. The effect of DSFS and dissipation caused by quantum effect will be significant to NRI in its miniaturization applications in the future.

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