Destroying Extremal Kerr–Newman-AdS Black Holes with Test Particles *

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Neglecting the self-force, self-energy and radiative effects, we follow the spirit of Wald's gedanken experiment and further discuss whether an extremal Kerr–Newman-AdS (KNA) black hole can turn into a naked singularity when it captures charged and spinning massive particles. It is found that feeding a test particle into an extremal KNA black hole could lead to a violation of cosmic censorship for the black hole.

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According to the well-known 'cosmic censorship conjecture' first proposed by Penrose,[1] all physical singularities due to gravitational collapse are hidden behind an event horizon. Many studies about testing the cosmic censorship have been carried out in recent years.[2–12] In the present study, disregarding the self-force, self-energy and radiative effects, we investigate whether an extremal Kerr–Newman-AdS (KNA) black hole can turn into a naked singularity when it captures charged and spinning massive particles. It is found that a particle could destroy the horizon of a KNA black hole.

In four-dimensional spacetime, the KNA metric takes the following form in Boyer–Lindquist-type coordinates,[13]

$$ds^2 = -\frac{\Delta_r}{\rho^2}\Big(dt - \frac{a\sin^2\theta}{\Xi}d\phi\Big)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta}{\rho^2}\sin^2\theta\Big(adt - \frac{r^2+a^2}{\Xi}d\phi\Big)^2, \tag{1}$$

where

$$\Delta_\theta = 1 - \frac{a^2}{l^2}\cos^2\theta,\ \rho^2 = r^2 + a^2\cos^2\theta,\ \Xi = 1 - \frac{a^2}{l^2},$$
$$\Delta_r = (r^2+a^2)\Big(1+\frac{r^2}{l^2}\Big) - 2Mr + Q^2, \tag{2}$$

with M, a and Q being the mass, angular momentum per unit mass and charge of black hole, respectively, and l being the cosmological constant. Moreover, the electromagnetic field of the KNA black hole has the vector potential

$$A = -\frac{Qr}{\rho^2}\Big(dt - \frac{a\sin^2\theta}{\Xi}d\phi\Big). \tag{3}$$

With regard to the horizons of the KNA black holes, the radii of the horizons are determined by $\Delta_r = 0$. The larger one r_+ and the other one r_- of the two real roots correspond to the radii of the black hole's outer (event) and inner (Cauchy) horizon, respectively. They are given by[12]

$$r_+ = \frac{1}{2}(X+Y),\ r_- = \frac{1}{2}(X-Y), \tag{4}$$

where

$$X = \sqrt{u - l^2 - a^2},$$
$$Y = \sqrt{-u - l^2 - a^2 + \frac{4Ml^2}{X}}, \tag{5}$$
$$u = \frac{l^2+a^2}{3} + \frac{l^{4/3}(M_{1e}^2 - M_{2e}^2)^{2/3}}{(2N^2 - M_{1e}^2 - M_{2e}^2)^{1/3}} + l^{4/3}(2N^2 - M_{1e}^2 - M_{2e}^2)^{1/3}, \tag{6}$$
$$N^2 = M^2 + \sqrt{(M^2 - M_{1e}^2)(M^2 - M_{2e}^2)}. \tag{7}$$
$$M_{1e} = (l/\sqrt{54})\sqrt{\zeta+\eta^3},\ M_{2e} = (l/\sqrt{54})\sqrt{\zeta-\eta^3}. \tag{8}$$

With

$$\zeta = \Big(1+\frac{a^2}{l^2}\Big)\Big[\frac{36(a^2+Q^2)}{l^2} - \Big(1+\frac{a^2}{l^2}\Big)^2\Big],$$
$$\eta = \Big[\Big(1+\frac{a^2}{l^2}\Big)^2 + \frac{12(a^2+Q^2)}{l^2}\Big]^{1/2}. \tag{9}$$

Here M_{1e} and M_{2e} are two extremal mass parameters. The black hole mass parameter M must satisfy the relation

$$M \geq M_{1e}. \tag{10}$$

The equality is satisfied when the black hole is the extremal black hole.[12]

Let us consider a test particle with mass m, angular momentum L and charge q moving along a geodesic. The conserved energy E of the test particle is given as[10]

$$E = \frac{g_{t\phi}}{g_{\phi\phi}}(qA_\phi - L) - qA_t + \Big[\Big(\frac{g_{t\phi}^2 - g_{\phi\phi}g_{tt}}{g_{\phi\phi}^2}\Big)((L - qA_\phi)^2 + m^2 g_{\phi\phi}(1 + g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2))\Big]^{1/2}. \tag{11}$$

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Substituting the components of $g_{\mu\nu}$ in Eq. (1), and using Eq. (3), we obtain the minimum energy which allows the test particle to reach the event horizon,

$$E_{\rm min} = \frac{aL\Xi + qQr_+}{a^2 + r_+^2}. \tag{12}$$

According to Eqs. (4) and (5), the KNA black hole will become extremal if

$$-u - l^2 - a^2 + \frac{4Ml^2}{X} = 0, \tag{13}$$

where the inner and outer horizons coincide with each other. Formally, we define

$$F(a, M, Q, l) = -u - l^2 - a^2 + \frac{4Ml^2}{X}. \tag{14}$$

Now we consider a test particle with energy E, electric charge q, and orbital angular momentum L captured by an extremal KNA black hole. After capturing a massive and charged test particle with E, L and q into the black hole, the black hole is described by the changed parameters,[11]

$$a' = \frac{aM + L}{M + E},\ M' = M + E,\ Q' = Q + q, \tag{15}$$

where the primes are used to describe the changed parameters after capturing a test particle. Then the new extremal condition can be written formally as

$$F(a', M', Q', l) = -u' - l^2 - a'^2 + \frac{4M'l^2}{X'} = 0. \tag{16}$$

In order for the KNA black hole to turn to a naked singularity, the following two conditions must be satisfied,

$$-u' - l^2 - a'^2 + \frac{4M'l^2}{X'} < 0. \tag{17}$$

$$X' > 0. \tag{18}$$

For simplicity, the above condition can be written formally as

$$F(a', M', Q', l) = F'(a, M, Q, l, E, L, q) < 0, \tag{19}$$

$$X(a', M', Q', l) = X'(a, M, Q, l, E, L, q) > 0. \tag{20}$$

Equation (19) means that the event horizon of the black hole is destroyed, and it implies that the test particle has a maximum value $E_{\rm max}$ to destroy the KNA black hole. Equation (20) ensures that Eq. (19) can be used as a physical criterion. To investigate the range of energy to destroy the black hole, we choose the parameters $M = 100$, $a = 0.1$, $q = 0.1$ and $L = 0.1$. Then the radius of the extremal black hole is 3.6384 and charge Q is 23.21758. Based on Eqs. (6)–(10), (15) and (18), we hope to solve Eq. (19), but it is too complex to deal with. Thus we choose the numerical method. By using the above parameters, based on Eqs. (6)–(10), (15) and (18), we can take energy E as an argument range from 0–10.

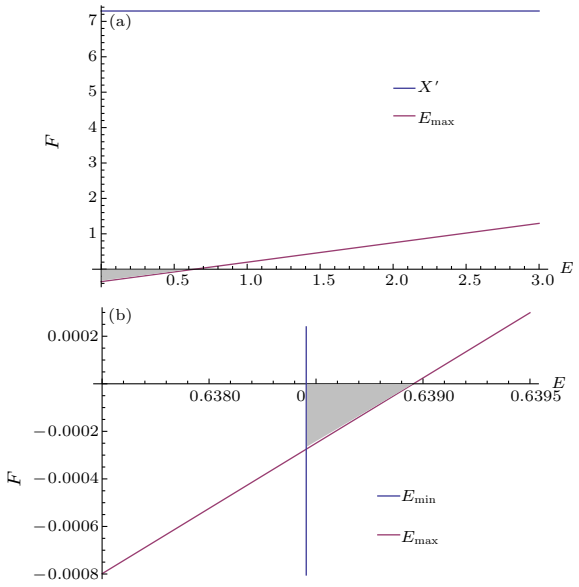

Fig. 1. Plots of $E_{\rm max}$–$E_{\rm min}$, and $E_{\rm max}$–X' on the extremal KNA black hole. (a) Plot of $E_{\rm max}$–X'. Here $X \geq 0$ ensures that the extremal condition of the black hole used as a judgment is meaningful. (b) Plot of $E_{\rm max}$–$E_{\rm min}$. The shaded area is the intersection of $E < E_{\rm max} = 0.638955$ and $E > E_{\rm min} = 0.638402$.

From Fig. 1(a), we can see that when particle's energy E ranges from 0 to 10, Eq. (20) is always satisfied. This means that the extremal condition (19) used as a judgment is meaningful. The value of the corresponding function is negative as E ranges in the interval (0, 0.638955). This means that the event horizon does not exist, and the black hole is destroyed. Considering the minimum energy for the particle to reach the horizon of the black hole, we have allowed the range to destroy the black hole,

$$\Delta E = E_{\rm max} - E_{\rm min}. \tag{21}$$

To show the range of energy for the particle to destroy an extreme KNA black hole, we plot Fig. 1(b). By using the same parameters $M = 100$, $a = 0.1$, $q = 0.1$ and $L = 0.1$, we obtain $E_{\rm min} = 0.638402$. The shaded area shows that the allowed range of the particle's energy E is (0.638402, 0.638955).

To prove that the black hole horizon is destroyed, we choose $E = 0.63860$, which is just between $E_{\rm max}$ and $E_{\rm min}$, and keep the other parameters the same, and obtain the function Δ' in Fig. 2. From it, we find that the real solution does not exist anymore. This implies that after capturing a test particle, the black hole horizon is destroyed. On the other hand, if we choose $E = 0.63950$, which is slightly larger than $E_{\rm max}$, we obtain the function Δ' as shown in Fig. 3, which shows that there are two different solutions of

Δ'. This means that after capturing a test particle, the extremal black hole will have inner and outer horizons. Thus it turns into a common black hole.

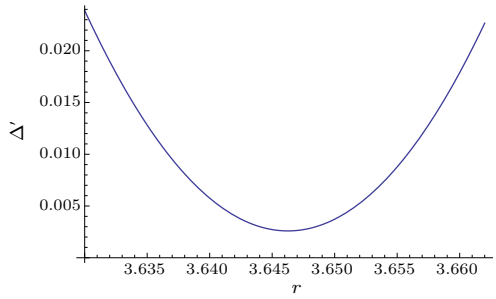

Fig. 2. Plot of Δ'–r. There is no point of the intersection on the horizontal axis.

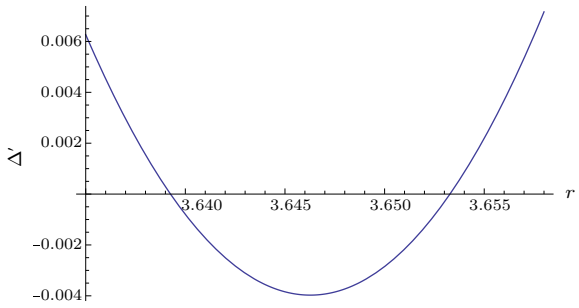

Fig. 3. Plot of Δ'–r. There are two points of the intersection on the horizontal axis after capturing a test particle.

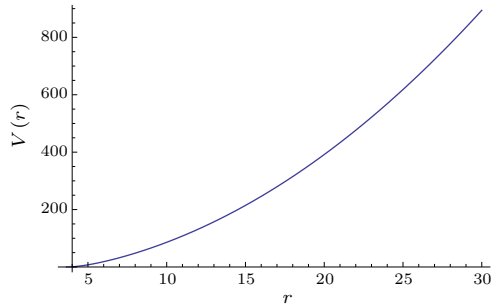

Fig. 4. The value of $V(r)$ of a test particle outside event horizon with $3.6383 < r < 30$. The test particle's energy $E_{\min} < E < E_{\max}$.

Next, we show that a particle far away, whose energy is between $E_{\max}$ and $E_{\min}$, could really fall into the black hole. Since the metric is axisymmetric, there exists an orbit lying entirely in the plane $\theta = \pi/2$. For such an orbit, we define $V(r) = -\dot{r}^2$, one can solve Eq. (11) for $\dot{r}^2$,[5] where the effective potential $V(r)$ is given by

$$\begin{aligned} V(r) = \frac{1}{l^4 m^2 r^4}[&(a^4L^2 - 2a^2l^2L^2 + l^4L^2 \\ &+ l^4m^2r^2)\Delta_r - l^4E^2((a^2+r^2)^2 - a^2\Delta_r) \\ &+ 2l^2E((a^2-l^2)aL\Delta_r \\ &- (a^3L - al^2L - l^2Qqr)(a^2+r^2) \\ &- (al^2L - a^3L + l^2Qqr)^2]. \end{aligned} \tag{22}$$

We still choose $M = 100$, $a = 0.1$, $q = 0.1$ and $L = 0.1$ as above, and $m = E = 0.6385$ such that E is located in the allowed range. Figure 4 shows that the curve is descending and the particle can easily fall into a KNA black hole from infinity.

In summary, we have shown that without taking into account the radiative and self-force effects, a test particle may destroy the horizon of an extremal KNA black hole, resulting in an apparent violation of the cosmic censorship. We can see that the allowed range for the particle's energy E is narrow, which indicates that the energy of the particle must be finely tuned. The fact that ΔE is very small leaves the possibility of radiative and self-force effect considerations to cure the problem of WCCC violation in KNA spacetime.

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