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Evolutionary Games in Two-Layer Networks with the Introduction of Dominant Strategy *

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We study evolutionary games in two-layer networks by introducing the correlation between two layers through the C-dominance or the D-dominance. We assume that individuals play prisoner's dilemma game (PDG) in one layer and snowdrift game (SDG) in the other. We explore the dependences of the fraction of the strategy cooperation in different layers on the game parameter and initial conditions. The results on two-layer square lattices show that, when cooperation is the dominant strategy, initial conditions strongly influence cooperation in the PDG layer while have no impact in the SDG layer. Moreover, in contrast to the result for PDG in single-layer square lattices, the parameter regime where cooperation could be maintained expands significantly in the PDG layer. We also investigate the effects of mutation and network topology. We find that different mutation rates do not change the cooperation behaviors. Moreover, similar behaviors on cooperation could be found in two-layer random networks.

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Evolutionary game theory is a mathematical framework to investigate the emergence and sustainability of cooperation in plenty of fields.[1,2] Prisoner's dilemma game (PDG) and Snowdrift game (SDG) are two widely studied models. For a two-individual-one-shot game, each individual can choose either to cooperate or to defect. If both players choose to cooperate, they receive the reward R, while mutual defection leaves both with the punishment P. A defector exploiting a cooperator receives the highest payoff, the temptation T, whereas the exploited cooperator receives sucker's payoff S. The PDG requires $T > R > P > S$ and $2R > T + S$ while the SDG requires $T > R > S > P$. It has been well known that the population structure which may be represented by complex networks has strong impacts on cooperation.[3–11] In structured populations, cooperators can survive in an evolutionary PDG by forming compact clusters to resist the exploitation from defectors.[12]

The above-mentioned works focused on single-layer networks in which each individual is charted into a network node and all unit-unit interactions are treated on an equivalent footing such as interaction strength. However in real life, it is ubiquitous that one group of individuals may interact with each other in different channels. For example, in social networks, individuals may transfer information through different channels such as facebook and twitter. To model such situations, multilayer networks are required in which each channel (such as relationship, activity, or category) is represented by one layer.[13,14] Among multilayer networks, there are interacting networks, interdependent networks, multiplex networks, and others.[14] In recent years, the evolutionary game theory in multilayer networks has drawn increasing interests from physicists.[15–19] Li *et al.*[15] studied the game theory in two interacting single-layer networks. Wang *et al.*[16] studied the evolutionary dynamics of cooperation on two interdependent networks playing different games, respectively. They found that, with the increase of the network interdependence, the evolution of cooperation is dramatically promoted (or reduced) on the network playing PDG (or SDG). In the work by Wang *et al.*,[17] they introduced the coevolution of strategy and network interdependence and found that the interdependence between networks self-organizes so as to yield optimal conditions for the evolution of cooperation. More works on the evolutionary game theory in interdependent networks may be found in two review works.[20,21]

In this Letter, we study evolutionary games in multiplex networks containing two layers. We consider a population with N individuals. We assume that these individuals interact with each other by playing PDG in one layer (the PDG layer) and playing SDG in the other layer (the SDG layer). For simplicity, we further assume that the two layers are characterized by the same network such as square lattices and Erdös–Rényi networks (ERN). In different layers, an individual may have different neighbors. The strategy, either cooperation or defection, held by an individual in the PDG layer, may be different from that in the SDG layer. The payoff matrices for PDG and SDG are conveniently rescaled depending on a single parameter r. For example, $\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -r \\ 1+r & 0 \end{pmatrix}$ for PDG and $\begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix}$ for SDG with the parameter r in the range of $[0, 1]$.

To establish the correlation between the PDG and the SDG layers, we treat the strategies in the PDG layer ($s_{\rm P}$) and in the SDG layer ($s_{\rm S}$) held by an individual as an strategy pair $(s_{\rm P}, s_{\rm S})$. We introduce two situations, C-dominance where cooperation is the dominant strategy and D-dominance where defection is the dominant strategy, and assume that individuals always perform the same action in the two layers.

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For example, in the situation of C-dominance (or D-dominance), individuals act as defectors (or cooperators) if and only if their strategies in both layers are defection (or cooperation); otherwise, individuals act as cooperators (or defectors). Actually, there are four types of strategy pairs: (C, C), (C, D), (D, C), and (D, D). In the situation of C-dominance, individuals holding the pair (D, D) conduct defection in both layers while individuals with other pairs conduct cooperation. In contrast, in the situation of D-dominance, only individuals hold the pair (C, C) conduct cooperation.

In each time step, the evolution contains three stages: payoff accumulation, mutation, and strategy update. In the first stage, individuals accumulate their payoffs $\Pi_{\rm PDG}$ and $\Pi_{\rm SDG}$ by playing PDG with their neighbors in the PDG layer and SDG with their neighbors in the SDG layer, respectively. In this stage, the action taken by an individual depends not only on his strategy pair but also on the dominant strategy. In the second stage, with a probability ν, individuals spontaneously change their strategies in the PDG layer and the SDG layer. To be noted, the mutation occurs in the two layers independently. In the third stage, each individual x randomly chooses one of his neighbors y in the PDG/SDG layer, and updates his strategy in the PDG/SDG layer by adopting the strategy of the chosen neighbor in the PDG/SDG layer with a probability,

$$W = \frac{1}{1+\exp[(\Pi_{\rm PDG/SDG,}{}_x - \Pi_{\rm PDG/SDG,}{}_y)/\kappa]}, \quad (1)$$

where κ denotes the selection intensity in the strategy update process. In this work, we set $\kappa = 0.1$.

We begin with the model without mutation ($\nu = 0$). We assume that both the PDG layer and the SDG layer are characterized by square lattices of size 100×100. We denote the fraction of individuals holding cooperation in the PDG (or SDG) layer by $\rho_{\rm pdg,c}$ (or $\rho_{\rm sdg,c}$). We also monitor other measures such as $\rho_{\rm CC}$ meaning the fraction of individuals holding the pair (C,C) and $\rho_{\rm c}$ meaning the fraction of individuals acting as cooperators. In the situation of C-dominance, $\rho_{\rm c} = \rho_{\rm CC} + \rho_{\rm CD} + \rho_{\rm DC}$, while $\rho_{\rm c} = \rho_{\rm CC}$ in the situation of D-dominance. These measures are calculated by averaging over 10^3 time steps after 10^4 transient time steps. Unless specified, each data point results from an average of 50 realizations. The synchronous strategy update is used throughout this work.

In the situation of C-dominance, $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ against the game parameter r are presented in Fig. 1. Increasing r reduces the fractions of cooperation in both the layers. The cooperation strategy in the PDG layer becomes extinct at $r = 0.22$ when $\rho_{\rm sdg,c}$ becomes lower than 1. Furthermore, the cooperation strategy in the SDG layer is extinct at around $r = 0.7$. In comparison, we present the fractions of cooperators against r in the insets in Figs. 1(a) and 1(b) for the PDG and the SDG in single-layer square lattices, respectively. It is noted that $\rho_{\rm sdg,c}$ is exactly the same as that for the SDG in a single-layer square lattice while the extinction of $\rho_{\rm pdg,c}$ is greatly postponed in comparison with that for the PDG in a single-layer square lattice. We also present $\rho_{\rm c}$ in the inset in Fig. 1(c). Obviously, the fraction of individuals taking the action of cooperation is the same as that for the SDG in a single-layer square lattice. We have investigated the effects of initial conditions. In Figs. 1(a) and 1(b) (or Figs. 1(c) and 1(d)), an individual in the SDG layer (or the PDG layer) initially takes a strategy of cooperation or defection with equal probability while the fraction of cooperation in the PDG layer (or the SDG layer) is set to be $p_{\rm pdg}$ (or $p_{\rm sdg}$). Five different $p_{\rm pdg}$ (or $p_{\rm sdg}$) are tested. We find that initial conditions strongly influence $\rho_{\rm pdg,c}$ while have no impact on $\rho_{\rm sdg,c}$. In the simulation, we find that the strategy configurations always settle down to absorbing states, for example, the states when $r < 0.22$ where all individuals hold either the strategy pair (C, C) or the pair (D, C) and the states with the coexistence between (D, D) and (D, C) when $r \in (0.22, 0.7)$.

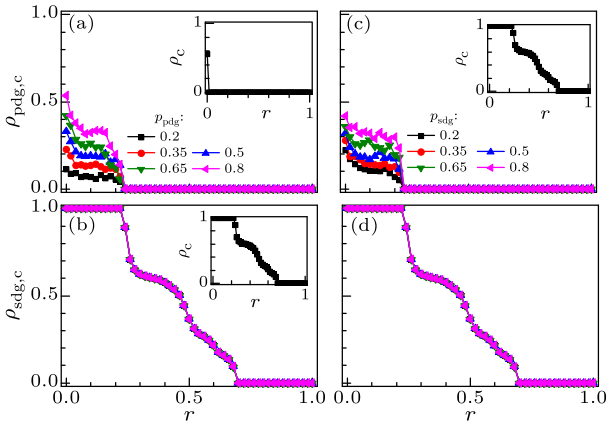

Fig. 1. The fraction of individuals holding cooperation in the PDG layer $\rho_{\rm pdg,c}$ and in the SDG layer $\rho_{\rm sdg,c}$ against the game parameter r for the situation of C-dominance. (a, b) The effects of initial fractions of cooperation $p_{\rm pdg}$ in the PDG layer. (c, d) The effects of initial fractions of cooperation $p_{\rm sdg}$ in the SDG layer. The insets in (a) and (b) show the fraction of cooperators $\rho_{\rm c}$ against r for PDG and SDG in single-layer square lattices, respectively. The inset in (c) shows the fraction of individuals taking the action of cooperation $\rho_{\rm c}$.

To understand the game dynamics in the situation of C-dominance, we consider the transitions between different strategy pairs. First, the transitions among the pairs (C, C), (C, D), and (D, C) occur at the same probability since all of them yield the same action, cooperation. The transition between (C, C) and (D, D) and the transition between (C, D) and (D, C) may be ignored since they involve the second order process (the strategies in both PDG and SDG layers must change at the same time). Secondly, the strategy pairs (C, C), (C, D) and (D, C) take the action of cooperation while only (D, D) takes the action of defection, which indicates that, in the perspective of action of cooperation/defection, the model dynamics follows the evolutionary SDG in single-layer square lattices (see the insets in Figs. 1(b) and 1(c)). Therefore, for $r < 0.22$ where the SDG in single-layer square lattices builds an all-cooperation state, the

pairs (D, D) and (C, D) cannot survive in the final strategy configuration. The winner in the competition between (C, C) and (D, C) is determined by the factions of (C, C) and (D, C) at the time when (D, D) and (C, D) become extinct. Both increasing $p_{\rm pdg}$ and decreasing r may lead to the rise of density of (C, C) and consequently the rise of $\rho_{\rm pdg,c}$. On the other hand, for $r \in (0.22, 0.7)$, the SDG in single-layer square lattices allows for the coexistence of cooperation and defection in which individuals holding defection wander around the lattices. Correspondingly, the pair (D, D) exists in the two-layer networks forever. Due to the advantage of (D, D) in payoff in the PDG layer, (C, C) and (C, D) will be replaced by (D, C) and (D, D), which results in $\rho_{\rm pdg,c} = 0$.

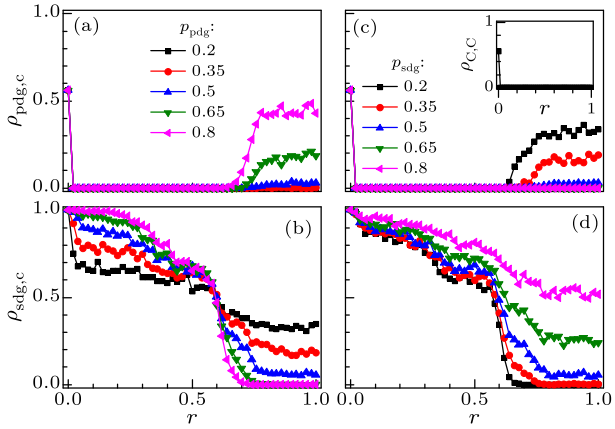

Fig. 2. The values of $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ against r for the situation of D-dominance. (a, b) The effects of initial fractions of cooperation $p_{\rm pdg}$ in the PDG layer. (c, d) The effects of initial fractions of cooperation $p_{\rm sdg}$ in the SDG layer. The inset in (c) shows the fraction of individuals taking the action of cooperation $\rho_{\rm C,C}$.

We present $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ against r in Fig. 2 for the situation of D-dominance. Generally, there exists a threshold $r_{\rm c}$ around 0.7 at which cooperation is extinct for SDG in a single-layer square lattice. Below the threshold, $\rho_{\rm pdg,c}$ stays at zero and $\rho_{\rm sdg,c}$ decreases with r. Above the threshold, both $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ may stay at nonzero values and $\rho_{\rm pdg,c}$ is inversely correlated with $\rho_{\rm sdg,c}$. The behaviors of $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ are strongly dependent on the initial conditions. As shown in Fig. 2, $p_{\rm pdg}$ and $p_{\rm sdg}$ have different effects on $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$. For example, $\rho_{\rm pdg,c}$ increases with $p_{\rm pdg}$ while $\rho_{\rm sdg,c}$ decreases (or increases) with $p_{\rm pdg}$ for r above (or below) the threshold. On the other hand, increasing $p_{\rm sdg}$ leads to the fall of $\rho_{\rm pdg,c}$ and the rise of $\rho_{\rm sdg,c}$. Similar to the situation of C-dominance, the strategy configurations settle down to absorbing states either. Considering that all strategy pairs but (C, C) act as defection, the dynamics follows the evolutionary PDG in single-layer square lattices in the perspective of action of cooperation/defection (see the insets in Figs. 2(c) and 1(a)). As a result, the pair (C, C) will be eliminated in the evolution for any $r \in (0, 1)$ except very small r and the final strategy configuration is determined by the relative densities of the pairs (C, D),(D, C) and (D, D). Similarly, the pair (C, D) is also disfavored in the evolution and it is eliminated with the pair (C, C). However, for $r > 0.7$ where $\rho_{\rm c} = 0$ for SDG in single-layer square lattices, the pair (C, C) becomes extinct very rapidly. Consequently, the fraction of (C, D) may stay close to its initial value at the time the pair (C, C) disappears, which yields to the nonzero $\rho_{\rm pdg,c}$ in Figs. 2(a) and 2(c); especially, the higher $p_{\rm pdg}$ is, and the higher $\rho_{\rm pdg,c}$ is. The effects of $p_{\rm sdg}$ can be analyzed in the same way.

It should be noted that the results in Figs. 1 and 2 are not sensitive to the population size. We take the situation of D-dominance as an example. We consider population size ranging from 50×50 to 400×400. The values of $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ in Fig. 3 show the independence on the population size. Actually, finite size effects could be prominent near the phase transition involving the extinction of cooperation. However, the detailed investigation on the phase transition is beyond this work.

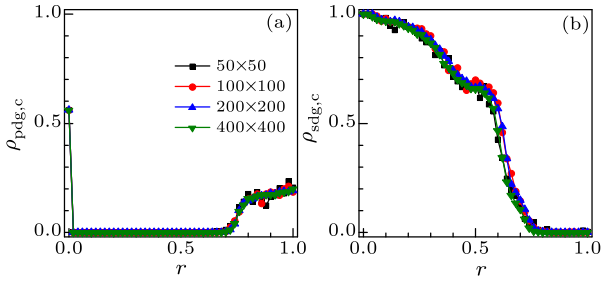

Fig. 3. The values of (a) $\rho_{\rm pdg,c}$ and (b) $\rho_{\rm sdg,c}$ against r in two-layer square lattices for different system sizes. Four system sizes from 50×50 to 400×400 have been considered. Initial fractions of cooperation are $p_{\rm pdg} = 0.65$ and $p_{\rm sdg} = 0.5$.

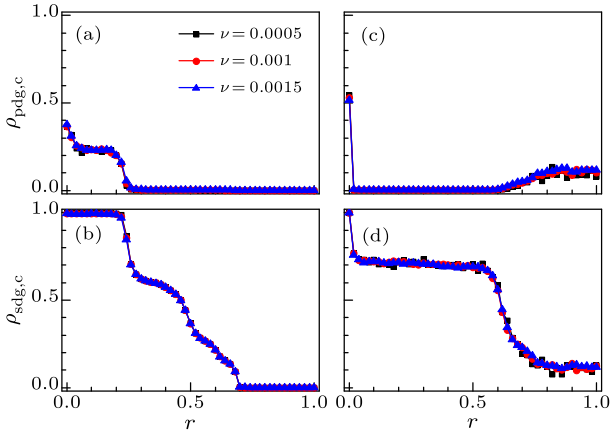

Fig. 4. The values of $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ against r in two-layer square lattices for different mutation rates ν: (a, b) the situation of C-dominance, and (c, d) the situation of D-dominance.

Then we consider the model with mutation $\nu \neq 0$. As shown in Fig. 4, $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ display the same dependence on r as those with $\nu = 0$. We also find that $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ are independent of the mutation rate. Actually, the mutation drives the model dynamics to wander through all possible states, which suggests that $\rho_{\rm pdg,c}$ and $\rho_{\rm sdg,c}$ with $\nu \neq 0$ are the

ensemble values by averaging over all realizations under different initial conditions in the absence of mutation. Different mutation rates just account for how frequently the different states are sampled, which do not change the ensemble averages of $\rho_{\mathrm{pdg,c}}$ and $\rho_{\mathrm{sdg,c}}$.

Furthermore, we study the model in two-layer ERNs to investigate the influence of population structure on the model dynamics. Here we set the size of ERN to be 10^3 and the mean degree $d = 4$. The values of $\rho_{\mathrm{pdg,c}}$ and $\rho_{\mathrm{sdg,c}}$ in Fig. 5 show the similar behaviors to those in two-layer square lattices for both the situations of C-Dominance and D-dominance. However, there exist some differences: (1) $\rho_{\mathrm{sdg,c}}$ does not follow the same relationship with r as ρ_{c} in SDG in single-layer ERNs. Especially, for $r < 0.1$ where $\rho_{\mathrm{c}} = 1$ for both the PDG and the SDG in single-layer ERNs (see the insets in Figs. 5(a) and 5(b)), $\rho_{\mathrm{pdg,c}}$ and $\rho_{\mathrm{sdg,c}}$ are lower than 1; (2) $\rho_{\mathrm{pdg,c}}$ and $\rho_{\mathrm{sdg,c}}$ may stay nonzero at a larger r than those in two-layer square lattices; and (3) $\rho_{\mathrm{pdg,c}}$ and $\rho_{\mathrm{sdg,c}}$ become dependent on the mutation rate ν when r is not sufficiently large.

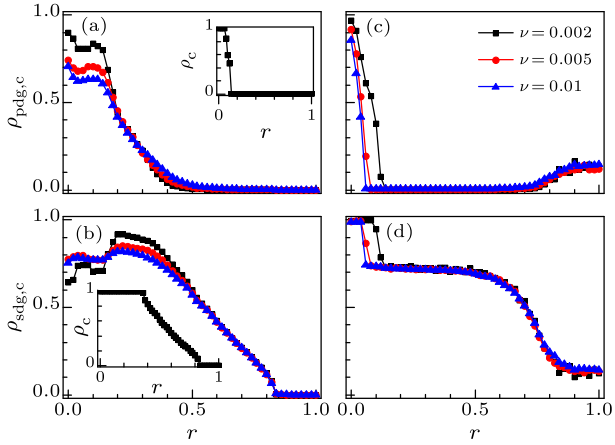

Fig. 5. The values of $\rho_{\mathrm{pdg,c}}$ and $\rho_{\mathrm{sdg,c}}$ against r for different mutation rates ν in two-layer ERNs: (a, b) the situation of C-dominance, and (c, d) the situation of D-dominance. The insets in (a) and (b) show ρ_{c} against r for PDG and SDG in single-layer ERNs, respectively.

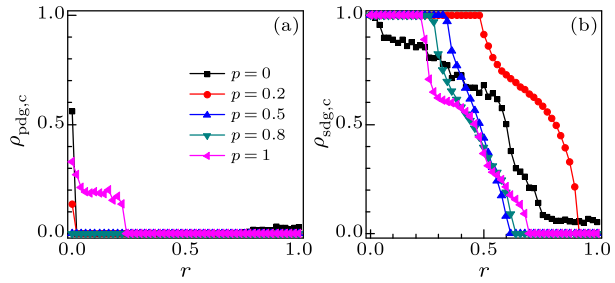

Fig. 6. The values of $\rho_{\mathrm{pdg,c}}$ in (a) and $\rho_{\mathrm{sdg,c}}$ in (b) against r for different p in two-layer square lattices, where the parameter p characterizes the proportion of individuals affected by C-dominance. Initial fractions of cooperation in both layers are set as 0.5 and the population size is 100×100.

In closing, we consider a probabilictic version of our model. We assume that, in each time step, each individual has the probability p to be in the situation of C-dominance and the probability $1-p$ in the situation of D-dominance. As shown in Fig. 6, both $\rho_{\mathrm{pdg,c}}$ and $\rho_{\mathrm{sdg,c}}$ strongly depend on p. Especially, it seems that there exists an optimal p at which $\rho_{\mathrm{sdg,c}}$ reaches a maximum.

In conclusion, we have considered multiplex networks containing two layers where individuals play different games such as PDG and SDG in different layers. We study the evolution of cooperation by introducing the correlation between two layers through the C-dominance or the D-dominance. Instead of the action of cooperation, we focus on the fractions of the strategy of cooperation in different layers, $\rho_{\mathrm{pdg,c}}$ and $\rho_{\mathrm{sdg,c}}$. We explore the dependence of these two measures on the game parameter, the initial conditions, and the network topology for the two situations of C-dominance and D-dominance, respectively. In contrast to the result in single-layer networks, two-layer networks show obvious advantages of maintaining cooperation behavior for the PDG but not for the SDG. In particular, the game parameter regime where cooperation could be maintained expands significantly in the PDG layer.

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