Painlevé Integrability, Consistent Riccati Expansion Solvability and Interaction Solution for the Coupled mKdV-BLMP System *

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The integrability of the coupled, modified KdV equation and the potential Boiti–Leon–Manna–Pempinelli (mKdV-BLMP) system is investigated using the Painlevé analysis approach. It is shown that this coupled system possesses the Painlevé property in both the principal and secondary branches. Then, the consistent Riccati expansion (CRE) method is applied to the coupled mKdV-BLMP system. As a result, it is CRE solvable for the principal branch while non-CRE solvable for the secondary branch. Finally, starting from the last consistent differential equation in the CRE solvable case, soliton, multiple resonant soliton solutions and soliton-cnoidal wave interaction solutions are constructed explicitly.

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The investigation of integrability and exact solution of nonlinear evolution equations plays an important role in mathematical physics. Many powerful methods to solve integrable nonlinear models such as inverse scattering transformation, the Darboux transformation, Hirota bilinear methods, the Painlevé analysis, symmetry reduction, the variable separation approach and the function expansion method have been established. Among these methods, the Painlevé analysis is a systematic method to identify the integrability of nonlinear evolution equations, in which one may use many kinds of approaches, that is, the Ablowitz–Ramani–Segur (ARS) algorithm,[1] the Weiss–Tabor–Carnevale (WTC) approach,[2] Kruskal's simplification[3] and Conte's invariant method.[4] In particular, it is worth pointing out that all these approaches are sufficient but unnecessary conditions in determining the Painlevé integrability.

Recently, inspired by the novel results obtained through the nonlocal symmetry reduction,[5–12] Lou[13] proposed the consistent Riccati expansion (CRE)/consistent tanh expansion (CTE) method, which is a more direct but much simpler method to study solvability and interaction solution. Based on this expansion method, a new solvability involving whether or not an original nonlinear system has a CRE is defined. Furthermore, it is shown that numerous CRE solvable models possess quite similar interaction solutions between a soliton and other nonlinear waves such as cnoidal wave, Painlevé wave, Airy wave, Bessel wave.[14–21] Moreover, the CRE method can be applied to classify the possible CRE solvabilities of nonlinear evolution equation, and the result coincides with that of the Painlevé analysis and the higher order generalized symmetry consideration.[13]

In this study, we consider the integrable coupled system of the modified KdV equation and the potential Boiti–Leon–Manna–Pempinelli (mKdV-BLMP) equation[22]

$$u_t + 3u^2u_x + u_{xxx} + \frac{3}{2}(uv_x)_x + \frac{3}{2}\partial_y^{-1}(uv_y)_{xx} = 0, \tag{1}$$

$$v_{yt} + \frac{3}{2}(v_xv_y)_x + v_{xxxy} + 3(u^2v_y)_x + \frac{3}{2}(uu_y)_{xx} + \frac{3}{2}(uu_{xy})_x = 0. \tag{2}$$

For $v = 0$, Eqs. (1) and (2) can be reduced to the modified KdV equation, while this coupled system becomes the potential BLMP equation[23] for $u = 0$. To construct a new nonlinear integrable system, Wang *et al.* constructed infinitely many generalized symmetries of a coupled (2+1)-dimensional Burgers system by means of the formal series symmetry approach.[22] Due to the close connection between infinitely many symmetries and integrable hierarchies, a series of new integrable models have been attained. Here the first one of the positive flow of such symmetries corresponds to the coupled system (1) and (2). By using the simplified Hirota's method, multiple kink solutions for the coupled mKdV-BLMP system (1) and (2) were derived in Ref. [24].

In the following the detailed Painlevé analysis for the coupled mKdV-BLMP system is presented. This coupled system is proved to be Painlevé integrable for both the principal and secondary branches. Then, we apply the CRE method to the coupled mKdV-BLMP system and the result shows that it is CRE solvable for the principal branch while non-CRE solvable for the secondary branch. Finally, we construct exact interaction solutions from the last consistent differential equation in the CRE solvable case. As a result, soliton, multiple resonant soliton solutions and soliton-

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cnoidal wave interaction solutions are provided explicitly.

Let $w_y = uv_y$, Eqs. (1) and (2) are transformed into the following system

$$u_t + 3u^2u_x + u_{xxx} + \frac{3}{2}(uv_x)_x + \frac{3}{2}w_{xx} = 0, \tag{3}$$

$$v_{yt} + \frac{3}{2}(v_xv_y)_x + v_{xxxy} + 3(u^2v_y)_x + \frac{3}{2}(uu_y)_{xx} + \frac{3}{2}(uu_{xy})_x = 0, \tag{4}$$

$$w_y - uv_y = 0. \tag{5}$$

To carry out a complete Painlevé analysis for the mKdV-BLMP system (3)–(5), we take the following Laurent expansion of the functions u, v and w in the neighborhood of a noncharacteristic singular manifold $\phi \equiv \phi(x, y, t)$,

$$u = \sum_{j=0}^{\infty} u_j\phi^{j+\alpha},\ v = \sum_{j=0}^{\infty} v_j\phi^{j+\beta},\ w = \sum_{j=0}^{\infty} w_j\phi^{j+\gamma}. \tag{6}$$

Substituting the leading terms of Eq. (1) ($j = 0$) into Eqs. (3)–(5), and balancing the nonlinear and the dispersion terms, we can obtain two possible branches

$$\alpha = \beta = -1,\ \gamma = -2, \tag{7}$$

under the conditions

$$\text{(i) } u_0 = \sigma\phi_x,\ v_0 = \phi_x,\ w_0 = \frac{1}{2}\sigma\phi_x^2, \tag{8}$$

$$\text{(ii) } u_0 = 2\sigma\phi_x,\ v_0 = 2\phi_x,\ w_0 = 2\sigma\phi_x^2,\ \sigma^2 = 1. \tag{9}$$

Now substituting the full expansion (1) into Eqs. (3)–(5) yields the recursion relation

$$A\begin{pmatrix} u_j \\ v_j \\ w_j \end{pmatrix} = \begin{pmatrix} F_{1,j-1} \\ F_{2,j-1} \\ F_{3,j-1} \end{pmatrix}, \tag{10}$$

where the functions $F_{i,j-1}, (i = 1, 2, 3)$ are complicated functions and only dependent on $u_k, v_k, w_k, (k = 0, 1, \ldots, j-1)$ and the derivatives of ϕ. For the first branch (8), the coefficient matrix reads

$$\text{(i) } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \tag{11}$$

where $a_{11} = \frac{1}{2}\phi_x^3(j-3)(2j^2-6j+7)$, $a_{12} = \frac{3}{2}\sigma\phi_x^3(j-1)(j-3)$, $a_{13} = \frac{3}{2}\phi_x^2(j-2)(j-3)$, $a_{21} = 3\sigma\phi_y\phi_x^3(j-1)(j-3)(j-4)$, $a_{22} = \phi_y\phi_x^3(j-1)(j-2)(j-3)(j-4)$, $a_{23} = 0$, $a_{31} = \phi_x\phi_y$, $a_{32} = -\sigma\phi_x\phi_y(j-1)$, and $a_{33} = \phi_y(j-2)$. For the second branch (9), the coefficient matrix reads

$$\text{(ii) } A = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}, \tag{12}$$

where $b_{11} = \phi_x^3(j-3)(j^2-3j+11)$, $b_{12} = 3\sigma\phi_x^3(j-1)(j-3)$, $b_{13} = \frac{3}{2}\phi_x^2(j-2)(j-3)$, $b_{21} = 6\sigma\phi_y\phi_x^3(j-4)(j^2-4j+1)$, $b_{22} = \phi_y\phi_x^3(j-1)(j-4)(j^2-5j+12)$, $b_{23} = 0$, $b_{31} = 2\phi_x\phi_y$, $b_{32} = -2\sigma\phi_x\phi_y(j-1)$, and $b_{33} = \phi_y(j-2)$.

All the functions $\{u_j, v_j, w_j\}$ can be determined by the recursion relation (10) except for the special resonance j which cause the determinant of the coefficient matrix A to vanish. Thus the resonances of the above two branches are listed as follows:

$$\text{(i) } j = -1, 1, 1, 2, 3, 3, 4, 5, \tag{13}$$

$$\text{(ii) } j = -2, -1, 1, 2, 3, 4, 5, 6. \tag{14}$$

Lastly, all the resonances located at Eqs. (13) and (14) should be satisfied identically. This means that for the principal branch (i), six arbitrary functions (ϕ, two of $\{u_i, v_j, w_j\}$ for $j = 1$ and 3 and one of $\{u_i, v_j, w_j\}$ for $j = 2$, 4 and 5) should be included in the expansion (1). By combining Kruskal's simplification, the careful analysis shows that the resonance conditions at 1, 1, 2, 3, 3, 4 and 5 are satisfied identically. For the secondary branch (ii), the same detailed calculations show that Eqs. (3)–(5) pass the Painlevé test. Therefore, it is obtained that the mKdV-BLMP system of Eqs. (3)–(5) has the Painlevé property and then it is Painlevé integrable.

Through the leading order analysis, we take the form of solution as

$$u = u_0 + u_1R(\phi), \tag{15}$$

$$v = v_0 + v_1R(\phi), \tag{16}$$

$$w = w_0 + w_1R(\phi) + w_2R(\phi)^2, \tag{17}$$

where u_0, v_0, w_0, u_1, v_1, w_1, w_2 and ϕ are functions of x, y and t, and the function $R(\phi)$ needs to satisfy the Riccati equation

$$R_\phi = a_0 + a_1R + a_2R^2,\ R \equiv R(\phi). \tag{18}$$

Substituting (15)–(17) with (18) into the mKdV-BLMP system (3)–(5), we have

$$\frac{3}{2}a_2\phi_x[4a_2^2u_1\phi_x^2 + 3a_2(u_1v_1 + 2w_2)\phi_x + 3u_1^3]R^4 + \sum_{i=0}^{3} F_{1,i}R^i = 0, \tag{19}$$

$$6a_2^2\phi_x\phi_y[4a_2^2v_1\phi_x^2 + a_2(5u_1^2 + v_1^2)\phi_x + 2v_1u_1^2]R^5 + \sum_{j=0}^{4} F_{2,j}R^j = 0, \tag{20}$$

$$a_2\phi_y(2w_2 - u_1v_1)R^3 + \sum_{k=0}^{2} F_{3,k}R^k = 0, \tag{21}$$

where $F_{1,i}$, $F_{2,j}$ and $F_{3,k}$ ($i = 0, \ldots, 3$, $j = 0, \ldots, 4$, $k = 0, \ldots, 2$) are complicated ϕ-dependent while R-independent functions.

Vanishing the coefficients of (R^4, R^5, R^3) in Eqs. (15)–(17), we know that two main cases should be included,

$$\text{(i) } u_1 = \sigma a_2 \phi_x,\ v_1 = -a_2\phi_x,\ w_2 = -\frac{1}{2}\sigma a_2^2\phi_x^2, \quad (22)$$

$$\text{(ii) } u_1 = 2\sigma a_2 \phi_x,\ v_1 = -2a_2\phi_x,\ w_2 = -2\sigma a_2^2\phi_x^2, \quad (23)$$

with $\sigma^2 = 1$.

For the first case (22), we continue to solve five functions $\{u_0, v_0, w_0, u_1, \phi\}$ by vanishing coefficients of (R^i, R^j, R^k) for $i = 0, \ldots, 3$, $j = 0, \ldots, 4$ and $k = 0, \ldots, 2$. As a result, we obtain

$$u_0 = \frac{1}{2}\sigma(a_1 - \sqrt{\delta})\phi_x, \quad (24)$$

$$v_0 = -\frac{1}{2}(a_1 - \sqrt{\delta})\phi_x + \sigma\lambda_1, \quad (25)$$

$$w_1 = -\frac{1}{2}\sigma(a_1 - \sqrt{\delta})\phi_x^2, \quad (26)$$

$$w_0 = \frac{1}{4}\sigma(\sqrt{\delta} - a_1^2 + 2a_0a_2)\phi_x^2 + \frac{1}{2}\sigma\lambda_1^2 + \lambda_2, \quad (27)$$

and the associated ϕ equation

$$\phi_t + \phi_{xxx} + \delta\phi_x^3 + 3\sqrt{\delta}\phi_{xx}\phi_x = 0, \quad (28)$$

where $\delta = a_1^2 - 4a_1a_2$, and λ_1 and λ_2 are integral constants.

We can obtain the following transformation theorem: (theorem 2.1) if ϕ is a solution of Eq. (28), then

$$u = \frac{1}{2}\sigma(a_1 - \sqrt{\delta})\phi_x + \sigma a_2\phi_x R(\phi), \quad (29)$$

$$v = -\frac{1}{2}(a_1 - \sqrt{\delta})\phi_x + \sigma\lambda_1 - a_2\phi_x R(\phi), \quad (30)$$

$$w = \frac{1}{4}\sigma(\sqrt{\delta} - a_1^2 + 2a_0a_2)\phi_x^2 + \frac{1}{2}\sigma\lambda_1^2 + \lambda_2 - \frac{1}{2}\sigma(a_1 - \sqrt{\delta})\phi_x^2 R(\phi) - \frac{1}{2}\sigma a_2^2\phi_x^2 R(\phi)^2, \quad (31)$$

i.e., a solution of the mKdV-BLMP system (3)–(5) with $R(\phi)$ being a solution of the Riccati Eq. (18).

According to the definition of the CRE solvability, we deduce that the mKdV-BLMP system (3)–(5) is CRE solvable for the first case (22).

For the second case (23), we proceed to eliminate the coefficients of (R^3, R^4, R^2) in Eqs. (19)–(21), then one can obtain

$$u_0 = \sigma a_1\phi_x + \sigma\frac{\phi_{xx}}{\phi_x},\ w_1 = -2\sigma a_2(\phi_{xx} + a_1\phi_x^2). \quad (32)$$

Vanishing the coefficients of (R^2, R^3, R^1) in Eqs. (18)–(21), we have

$$v_0 = -a_1\phi_x - \frac{\phi_{xx}}{\phi_x} + \lambda_3, \quad (33)$$

with λ_3 being a constant of integration and

$$\phi_{xxx} = \frac{1}{2}\delta\phi_x^3 + \frac{1}{2}\phi_t + \frac{3}{2}\frac{\phi_{xx}^2}{\phi_x}. \quad (34)$$

From the coefficients of (R^1, R^2, R^0) in Eqs. (18)–(21), one can derive

$$w_{0y} = -\sigma a_1\phi_{xxy} - \sigma a_1^2\phi_x\phi_{xy} + \sigma\frac{\phi_{xx}^2\phi_{xy}}{\phi_x^3} - \sigma\frac{\phi_{xx}\phi_{xxy}}{\phi_x^2}. \quad (35)$$

Then, eliminating the coefficients of (R^0, R^1) in Eqs. (18)–(20) yields

$$\begin{aligned} w_{0xx} = &-\sigma\frac{3}{4}(\phi_{xx} + 4a_1\phi_x^2)\frac{\phi_{xx}^3}{\phi_x^4} + \Big[\frac{1}{2}\sigma(16a_0a_2 \\ &- 9a_1^2) + \sigma\frac{C}{\phi_x^2}\Big]\phi_{xx}^2 + \Big[2\sigma a_1 C - 3\sigma a_1\delta\phi_x^2 \\ &+ \frac{1}{2}\sigma\frac{C_x}{\phi_x}\Big]\phi_{xx} + \frac{1}{4}\sigma\delta(4a_0a_2 - 3a_1^2)\phi_x^4 \\ &+ \frac{1}{2}\sigma\phi_x(C_x + a_1^2C\phi_x) \\ &+ \frac{1}{4}\sigma(SC + C^2 + 4C_{xx}). \end{aligned} \quad (36)$$

Lastly, considering the R^0 term in Eq. (20) leads to

$$\begin{aligned} \phi_{xxyt} = &3KC\frac{\phi_{xx}^3}{\phi_x^2} + \Big(3KC_x + \frac{9}{2}CK_x - \frac{3}{2}C_y\Big)\frac{\phi_{xx}^2}{\phi_x} \\ &+ [\delta KC\phi_x^2 + 4SKC + KC_{xx} + 4K_xC_x \\ &+ 3CK_{xx} - 2C_{xy}]\phi_{xx} + \frac{1}{2}\delta(2CK_x \\ &- C_y)\phi_x^3 + (2SKC_x + SCK_x \\ &+ K_xC_{xx} + 2C_xK_{xx} + CC_y)\phi_x, \end{aligned} \quad (37)$$

where the notations C, S and K are defined as $C \equiv -\frac{\phi_t}{\phi_x}$, $K \equiv -\frac{\phi_y}{\phi_x}$ and $S \equiv \frac{\phi_{xxx}}{\phi_x} - \frac{3}{2}\frac{\phi_{xx}^2}{\phi_x^2}$.

With the help of the relations (34) and (37), it can be verified that $w_{0yxx} = w_{0xxy}$ is consistent. However, the direct calculation shows that

$$\phi_{xxxyt} - \phi_{xxytx} = -\frac{1}{2}\phi_x(2CC_{xy} + 3C_xC_y + C_{yt}). \quad (38)$$

Obviously, the right-hand side of Eq. (38) is nonzero, thus the consistent condition of Eqs. (34) and (37) is not satisfied. Therefore, according to the definition of CRE solvability, we know that the mKdV-BLMP system (3)–(5) is non-CRE solvable for the second case (23).

From the theorem 2.1, we know that the solution of the mKdV-BLMP system (3)–(5) can be constructed via the associated ϕ Eq. (28). In the following, some special exact solutions are listed. To this end, the solution of the Riccati Eq. (18) is chosen as

$$R(\phi) = -\frac{1}{2a_2}\Big[a_1 + \sqrt{\delta}\tanh\Big(\frac{1}{2}\sqrt{\delta}\phi\Big)\Big], \quad (39)$$

then the solution (29)–(31) can be written as

$$u = -\frac{1}{2}\sigma\sqrt{\delta}\phi_x\Big[\tanh\Big(\frac{1}{2}\sqrt{\delta}\phi\Big)+1\Big], \tag{40}$$

$$v = \frac{1}{2}\sqrt{\delta}\phi_x\Big[\tanh\Big(\frac{1}{2}\sqrt{\delta}\phi\Big)+1\Big]+\sigma\lambda_1, \tag{41}$$

$$w = \frac{1}{8}\sigma\delta\phi_x^2\mathrm{sech}^2\Big(\frac{1}{2}\sqrt{\delta}\phi\Big)-\frac{1}{4}\sigma\delta\phi_x^2\Big[\tanh\Big(\frac{1}{2}\sqrt{\delta}\phi\Big)+1\Big]+\frac{1}{2}\sigma\lambda_1^2+\lambda_2. \tag{42}$$

To construct the interaction solution, we take the form of function ϕ as

$$\begin{aligned}\phi &= k_1x+l_1y+h_1t+\Phi(x,y,t)\\ &\equiv k_1x+l_1y+h_1t+\Phi,\end{aligned} \tag{43}$$

then Eq. (28) is converted to the differential equation as follows:

$$\Phi_t+\Phi_{xxx}+\delta\Phi_x^3+3(k_1+\Phi_x)(\sqrt{\delta}\Phi_{xx}+k_1\delta\Phi_x)+\delta k_1^3+h_1=0. \tag{44}$$

Firstly, in the case of single soliton solution, Eq. (44) has the trivial solution $\Phi=0$ and $h_1=-\delta k_1^3$, which lead to the single soliton solution

$$u = -\frac{1}{2}\sigma\sqrt{\delta}k_1\Big\{\tanh\Big[\frac{1}{2}\sqrt{\delta}(k_1x+l_1y-\delta k_1^3t)\Big]+1\Big\}, \tag{45}$$

$$v = \frac{1}{2}\sqrt{\delta}k_1\{\tanh\Big[\frac{1}{2}\sqrt{\delta}(k_1x+l_1y-\delta k_1^3t)\Big]+1\}+\sigma\lambda_1, \tag{46}$$

$$w = \frac{1}{8}\sigma\delta\phi_x^2\mathrm{sech}^2\Big[\frac{1}{2}\sqrt{\delta}(k_1x+l_1y-\delta k_1^3t)\Big]-\frac{1}{4}\sigma\delta k_1^2\Big\{\tanh\Big[\frac{1}{2}\sqrt{\delta}(k_1x+l_1y-\delta k_1^3t)\Big]+1\Big\}+\frac{1}{2}\sigma\lambda_1^2+\lambda_2. \tag{47}$$

In the case of multiple resonant soliton solutions, for Eq. (44) it is easy to verify that it has the following multiple wave solution

$$\Phi = c\ln[1+\sum_{i=1}^{n}\exp(\hat{k}_ix+\hat{l}_iy+\hat{h}_it)], \tag{48}$$

where the coefficient c and the dispersion relation are determined by

$$\begin{aligned}c &= \frac{\sqrt{\delta}+\sum_{i=1}^{n}\hat{k}_i}{\delta k_1+\sqrt{\delta}\sum_{i=1}^{n}\hat{k}_i},\\ \hat{h}_i &= -\hat{k}_i(3\sqrt{\delta}k_1\hat{k}_i+\hat{k}_i^2+3\delta k_1^2).\end{aligned} \tag{49}$$

Considering the solution (40)–(42) with the solution (48) and (49), the $(n+1)$ resonant soliton solutions of Eqs. (3)–(5) can be directly obtained

$$u = -\frac{1}{2}\sigma\sqrt{\delta}K_1\Big[\tanh\Big(\frac{1}{2}\sqrt{\delta}\xi\Big)+1\Big], \tag{50}$$

$$v = \frac{1}{2}\sqrt{\delta}K_1\Big[\tanh\Big(\frac{1}{2}\sqrt{\delta}\xi\Big)+1\Big]+\sigma\lambda_1, \tag{51}$$

$$w = \frac{1}{8}\sigma\delta K_1^2\mathrm{sech}^2\Big(\frac{1}{2}\sqrt{\delta}\xi\Big)-\frac{1}{4}\sigma\delta K_1^2\Big[\tanh\Big(\frac{1}{2}\sqrt{\delta}\xi\Big)+1\Big]+\frac{1}{2}\sigma\lambda_1^2+\lambda_2, \tag{52}$$

with

$$\begin{aligned}\xi &= k_1x+l_1y+h_1t+c\ln[1+\sum_{i=1}^{n}\exp(\hat{k}_ix+\hat{l}_iy+\hat{h}_it)],\\ K_1 &= k_1+\frac{c\sum_{i=1}^{n}\hat{k}_i\exp(\hat{k}_ix+\hat{l}_iy+\hat{h}_it)}{1+\sum_{i=1}^{n}\hat{k}_i\exp(\hat{k}_ix+\hat{l}_iy+\hat{h}_it)}.\end{aligned}$$

In the case of soliton interaction with periodic wave solutions, we take the solution of Eq. (44) as the form

$$\Phi = \frac{1}{\sqrt{\delta}}\ln[\sum_{i=1}^{n}\cos(\hat{k}_ix+\hat{l}_iy+\hat{h}_it)\cdot\exp(\tilde{k}_ix+\tilde{l}_iy+\tilde{h}_it)], \tag{53}$$

where the parameters need to satisfy the following conditions

$$\hat{h}_i = -\hat{k}_i(6\sqrt{\delta}k_1\tilde{k}_i-\hat{k}_i^2+3\tilde{k}_i^2+3\delta k_1^2), \tag{54}$$

$$\tilde{h}_i = \tilde{k}_i(3\hat{k}_i^2-\tilde{k}_i^2)+3\sqrt{\delta}k_1(\hat{k}_i^2-\tilde{k}_i^2)-3\delta k_1\tilde{k}_i-\sqrt{\delta}(h_1+\delta k_1^3). \tag{55}$$

Then, we can obtain an interaction solution between soliton and periodic wave

$$u = -\frac{1}{2}\sigma\sqrt{\delta}K_2\Big[\tanh\Big(\frac{1}{2}\sqrt{\delta}\xi\Big)+1\Big], \tag{56}$$

$$v = \frac{1}{2}\sqrt{\delta}K_2\Big[\tanh\Big(\frac{1}{2}\sqrt{\delta}\xi\Big)+1\Big]+\sigma\lambda_1, \tag{57}$$

$$w = \frac{1}{8}\sigma\delta K_2^2\mathrm{sech}^2\Big(\frac{1}{2}\sqrt{\delta}\xi\Big)-\frac{1}{4}\sigma\delta K_2^2\Big[\tanh\Big(\frac{1}{2}\sqrt{\delta}\xi\Big)+1\Big]+\frac{1}{2}\sigma\lambda_1^2+\lambda_2, \tag{58}$$

with

$$\xi = k_1x+l_1y+h_1t+\frac{1}{\sqrt{\delta}}\ln[\sum_{i=1}^{n}\cos(\hat{k}_ix+\hat{l}_iy+\hat{h}_it)\exp(\tilde{k}_ix+\tilde{l}_iy+\tilde{h}_it)],$$

$$K_2 = k_1 + [\sum_{i=1}^{n} \tilde{k}_i \cos(\hat{k}_i x + \hat{l}_i y + \hat{h}_i t) \cdot \exp(\tilde{k}_i x + \tilde{l}_i y + \tilde{h}_i t) - \sum_{i=1}^{n} \hat{k}_i \sin(\hat{k}_i x + \hat{l}_i y + \hat{h}_i t) \exp(\tilde{k}_i x + \tilde{l}_i y + \tilde{h}_i t)] / [\sqrt{\delta} \sum_{i=1}^{n} \cos(\hat{k}_i x + \hat{l}_i y + \hat{h}_i t) \cdot \exp(\tilde{k}_i x + \tilde{l}_i y + \tilde{h}_i t)].$$

In summary, we have studied the integrability of the coupled mKdV-BLMP system using two different methods: the Painlevé analysis and the CRE method. The detailed Painlevé analysis is presented and then this coupled system is proved to be Painlevé integrable for both the principal and secondary branches. The application of the CRE method to the coupled mKdV-BLMP system shows that it is CRE solvable for the principal branch while non-CRE solvable for the secondary branch. Soliton, multiple resonant soliton and soliton-cnoidal wave interaction solutions are constructed explicitly from the last consistent differential equation in the CRE solvable case.

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