Optimization Design of Electromagnetic Nihility Nanoparticles *

Ya-Ming Xie(解亚明)[1,2], Chang-Yu Liu(刘昌宇)[3], Ze-Jun Ding(丁泽军)[4,1], Zhi-Guo Wang(王治国)[3,2]**

[1]*Department of Physics, University of Science and Technology of China, Hefei 230026*
[2]*Beijing Computational Science Research Center, Beijing 100193*
[3]*School of Physics Science and Engineering, Tongji University, Shanghai 200092*
[4]*Hefei National Laboratory for Physical Sciences at Microscale, University of Science and Technology of China, Hefei 230026*

(Received 15 May 2016)

Nihility material is a medium whose relative permittivity and permeability tend to zero simultaneously. In this work, comparing with the scattering properties of perfect nihility nanoparticles (made from nihility material), we provide an optimization design of electromagnetic nihility nanoparticles, which is a coated hybrid nanosphere constituted by commutative ε-negative (ENG) and μ-negative (MNG) media. Compared to a single ENG or MNG nanosphere, it is found that the total and back scattering spectra of coated hybrid nanospheres are much closer to those of perfect nihility nanospheres. Moreover, it is observed that the scattered electromagnetic field distribution of coated hybrid nanospheres is identical to that of perfect nihility nanospheres. These results indicate that the combination of commutative ENG and MNG media can constitute a composite structure which gives the closest approximation of electromagnetic scattering of perfect nihility nanospheres in a wide frequency range.

PACS: 42.25.Fx, 78.67.Pt, 78.20.Ci DOI: 10.1088/0256-307X/33/9/094208

The study and realization of metamaterials have undergone considerable development[1] in recent years, which can surpass the diffraction limit that is considered insurmountable in many conventional situations.[2–4] One of the primary purposes for metamaterials is to realize prescribed anomalous electromagnetic responses by designing complex electromagnetic structures, whose optical properties are decided by their relative permittivities ε and permeabilities μ.

Classified by ε and μ, the commonly used metamaterials include double-negative (DNG) media ($\varepsilon < 0$, $\mu < 0$),[5] ε-negative (ENG) media ($\varepsilon < 0$, $\mu > 0$),[6] μ-negative (MNG) media ($\varepsilon > 0$, $\mu < 0$),[7] and zero-index metamaterials (ZIMs).[8–20] Among these materials, ZIMs have attracted much interest due to their unique electromagnetic properties, which include cloaking,[8,9] directional radiation patterns,[10,11] infinite phase velocity and zero phase accumulation.[12] To achieve zero refractive index, it is required that at least one of the relative permittivities and permeabilities should be equal to zero. When relative permittivity and permeability go to zero simultaneously (known as nihility[13] material), the ZIM is impedance matched to vacuum, which implies an effective coupling of light at normal incidence. Lakhtakia has indicated that a perfect lens is equivalent to a nihility slab,[14] and Ziolkowski has offered a detailed discussion about one-, two-, and three-dimensional sources propagating in and scattering from a nihility slab.[15] Moreover, by inserting defects into a nihility slab, it is shown that total transmission or reflection can be achieved when the size and permittivities of defects are varied.[16] In addition to a nihility slab, Lakhtakia *et al.* have also studied plane waves scattered by single nihility spheres[17] and cylinders[18] utilizing the traditional Mie theory.[21]

Due to the lack of nihility material in nature, it is realized that artificial composite structures need to be introduced, and several possible designs have been proposed to do this. Silveirinha *et al.* have offered a feasible structure by embedding resonant nonmagnetic inclusions in a ε-near-zero host medium and it is proven that the transmission through a waveguide bend can be improved when such a nihility material is applied.[19] To avoid ohmic loss caused by metallic components, Huang *et al.* have presented an ingenious design of all-dielectric nihility medium,[9] which has recently been realized experimentally by Moitra *et al.* at infrared frequencies.[20] In this study, comparing with the scattering properties of perfect nihility nanospheres (constituted by nihility material), we provide a detailed discussion of the use of non-zero refractive index media to construct electromagnetic nihility nanospheres, with electromagnetic response approaching that of perfect nihility nanospheres. For nanospheres made from a double-positive (DPS) or DNG material (keeping $\varepsilon = \mu$), it is observed that the total scattering spectrum will move towards larger or smaller size parameter directions, respectively, when compared with perfect nihility nanospheres, while zero back scattering is reserved. In contrast, if an ENG or MNG material (keeping $\varepsilon = -\mu$) is applied, then it is

*Supported by the National Natural Science Foundation of China under Grant Nos 11174222 and 91230203.
**Corresponding author. Email: zgwang@tongji.edu.cn
© 2016 Chinese Physical Society and IOP Publishing Ltd

shown that the positions of total scattering peaks will remain the same, while a non-zero back scattering is obtained. More interestingly, building coated hybrid spherical nanoparticles with commutative ENG and MNG media, it is noticed that compared with single ENG or MNG nanospheres, the total and back scattering spectra of coated nanospheres are much closer to those of perfect nihility nanospheres, and the scattered electromagnetic field distributions of coated and perfect nihility nanoparticles are identical.

Considering an isotropic homogenous spherical particle with radius a is placed in vacuum, whose relative permittivity and permeability are ε and μ. The fields scattered by the particle can be expanded by using the Mie scattering coefficients a_n and b_n,[22] where coefficients a_n represent the amplitudes of transverse magnetic (TM) scattered fields and coefficients b_n give the amplitudes of transverse electric (TE) scattered fields. Following the form of Bohern *et al.*,[22] these Mie coefficients can be written as

$$a_n = \frac{\sqrt{\varepsilon}\psi_n(mx)\psi_n'(x) - \sqrt{\mu}\psi_n(x)\psi_n'(mx)}{\sqrt{\varepsilon}\psi_n(mx)\xi_n'(x) - \sqrt{\mu}\xi_n(x)\psi_n'(mx)}, \quad (1)$$

$$b_n = \frac{\sqrt{\mu}\psi_n(mx)\psi_n'(x) - \sqrt{\varepsilon}\psi_n(x)\psi_n'(mx)}{\sqrt{\mu}\psi_n(mx)\xi_n'(x) - \sqrt{\varepsilon}\xi_n(x)\psi_n'(mx)}, \quad (2)$$

where the radial functions ψ_n and ξ_n are the Ricatti Bessel and Hankel functions of size parameter $x = 2\pi a/\lambda$ and refractive index $m = \sqrt{\varepsilon}\sqrt{\mu}$, and λ is the incident wavelength in vacuum. When nihility material is used, these Mie coefficients can be simplified to

$$a_n = b_n = \frac{\psi_n(x)}{\xi_n(x)}. \quad (3)$$

The total scattering efficiency can be expressed in terms of the Mie coefficients as

$$Q_{\rm sca} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)(|a_n|^2 + |b_n|^2), \quad (4)$$

while the forward and back scattering efficiencies can be obtained as

$$Q_{\rm f} = \frac{1}{x^2}\Big|\sum_{n=1}^{\infty}(2n+1)(a_n + b_n)\Big|^2, \quad (5)$$

$$Q_{\rm b} = \frac{1}{x^2}\Big|\sum_{n=1}^{\infty}(-1)^n(2n+1)(a_n - b_n)\Big|^2. \quad (6)$$

It is noticed from Eqs. (1) and (2) that $a_n = b_n$ can be satisfied when $\varepsilon = \mu$, thus back scattering will always be zero[23] due to Eq. (6). The impedance of a particle will be 1 if $\varepsilon = \mu$ and it happens to match the impedance of vacuum, which will cause zero reflection under normal incidence. However, the total and forward scatterings of a particle with $\varepsilon = \mu$ are still affected by its parameters, and are contributed by TM (a_n) and TE (b_n) polarization modes with equal weight.

Figure 1(a) shows the total scattering efficiency $Q_{\rm sca}$ as a function of size parameter x for a perfect nihility nanosphere. Moreover, in Fig. 1(a) the contributions in total scattering from the first six Mie coefficients representing TM (a_1, a_2 and a_3) and TE (b_1, b_2 and b_3) polarization modes are also shown. It is necessary to mention that the contribution of the Mie coefficient a_n is exactly the same as that of the Mie coefficient b_n with the same order n, for the condition $a_n = b_n$ is not affected by the variation of x in perfect nihility nanospheres.

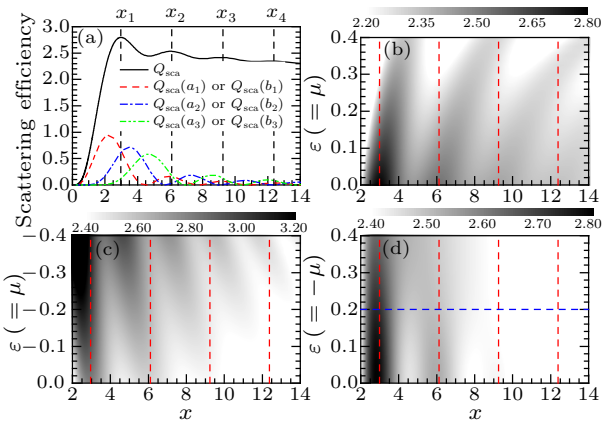

Fig. 1. (Color online) (a) Total scattering efficiency $Q_{\rm sca}$ of a perfect nihility nanosphere as a function of size parameter x. Contributions from the first six Mie coefficients representing TM (a_1, a_2 and a_3) and TE (b_1, b_2 and b_3) polarization modes in total scattering are also provided. (b)–(d) The total scattering efficiency $Q_{\rm sca}$ as functions of size parameter x and relative permittivity ε for spherical nanoparticles composed of the DPS ($0 \le \varepsilon = \mu \le 0.4$), DNG ($-0.4 \le \varepsilon = \mu \le 0$), and MNG ($0 \le \varepsilon = -\mu \le 0.4$) materials, respectively.

For large x ($x \gg n^2$), the Ricatti Bessel function $\psi_n(x)$ will have the asymptotic relation[22]

$$\psi_n(x) \to \sin\Big(x - \frac{n\pi}{2}\Big), \quad (7)$$

which indicates that in large parameter domain, the contribution in total scattering from each Mie coefficient (proportional to $|\psi_n(x)|^2$) has a period of π. In fact, as can be noticed from Fig. 1(a), the π period is satisfied approximately after the first peak of each mode where x is not large enough. Thus the total scattering obtained from the summation of all Mie coefficients' contributions will also experience a π period. Since the position of the first total scattering peak is at $x_1 = 2.98$, the lth total scattering peak position x_l can be approximately determined by

$$x_l = 2.98 + (l-1)\pi, \quad l = 1, 2, 3, \ldots, \quad (8)$$

and the positions of the first four total scattering peaks are denoted by the vertical dashed lines shown

in Fig. 1(a). It is noticed that the positions of total scattering peaks for perfect nihility nanospheres are constant when using x as the parameter, which implies that the corresponding wavelengths at total scattering peaks are proportional to the radius of particle.

From Eq. (7) it is realized that the phase difference between contributions in total scattering from two arbitrary Mie coefficients can be divided into two types: integral multiple of π or odd multiple of $\pi/2$. The superposition of contributions with phase difference of integral multiple of π or odd multiple of $\pi/2$ will make a constructive or destructive contribution to the oscillation behavior of total scattering spectrum. With the increase of the size parameter, a growing number of coefficients will take effect on total scattering, thus the oscillation feature of total scattering spectrum disappears gradually, which can be seen plainly in Fig. 1(a).

Because of the lack of nihility material in nature, several composite structures have been proposed to realize effective nihility medium in practice.[9,19,20] In the following, comparing with the scattering properties of perfect nihility nanospheres, we demonstrate the feasibility of using non-zero refractive index materials to construct electromagnetic nihility nanoparticles, which can imitate the electromagnetic response of perfect nihility nanospheres.

Firstly, the DPS, DNG, ENG and MNG materials with small while non-zero refractive index are utilized for the attempt to construct electromagnetic nihility nanospheres. When the magnitude of refractive index is small, Eqs. (1) and (2) can be approximately reduced to

$$a_n = \frac{(n+1)\psi_n(x) - \varepsilon x\psi_n'(x)}{(n+1)\xi_n(x) - \varepsilon x\xi_n'(x)}, \tag{9}$$

$$b_n = \frac{(n+1)\psi_n(x) - \mu x\psi_n'(x)}{(n+1)\xi_n(x) - \mu x\xi_n'(x)}. \tag{10}$$

In general, for traditional materials, Eqs. (9) and (10) are valid only when the limit $|mx| \ll 1$ is satisfied. However, it has been confirmed by our numerical calculations that Eqs. (9) and (10) are still good approximations for Eqs. (1) and (2) for the nanospheres made from the DPS, DNG, ENG or MNG material as long as the magnitude of refractive index is relatively small, even if $|mx|$ is larger than 1.

Figure 1(b) gives the total scattering efficiency $Q_{\rm sca}$ as functions of size parameter x and relative permittivity ε for a spherical nanoparticle composed of the DPS material, while in Fig. 1(c) the nanoparticle is constructed with the DNG material. To ensure impedance matched to vacuum, here we assume that the relative permittivities and permeabilities of the DPS and DNG materials are equal to each other. The vertical dashed lines in Figs. 1(b) and 1(c) represent the first four total scattering peaks of perfect nihility nanospheres, which are used as reference.

From Eqs. (4) and (9) it is known that the zeros of contribution in total scattering from the Mie coefficient a_n are determined by $(n+1)\psi_n(x) = \varepsilon x\psi_n'(x)$, which can be reduced to $\psi_n(x) = 0$ if the nihility material is applied. From a simple analysis, the following conclusions can be obtained: the solution of $(n+1)\psi_n(x) = \varepsilon x\psi_n'(x)$ will be larger than that of the equation $\psi_n(x) = 0$ if $\varepsilon > 0$, and vice versa. Thus compared with perfect nihility nanospheres, the total scattering spectrum of a single DPS nanosphere will have a blue shift (i.e., towards larger x), as illustrated in Fig. 1(b). Moreover, it is observed from Fig. 1(b) that the intensities of total scattering peaks are gradually decreased with the increase of ε. In contrast, the total scattering spectrum of a single DNG nanoparticle will experience a red shift (i.e., towards smaller x) when compared with perfect nihility nanoparticles, and the scattering intensities are amplified with the diminishing of ε, which are illustrated plainly in Fig. 1(c). Like perfect nihility nanospheres, the back scatterings of single DPS and DNG nanospheres are always zero due to the assumption $\varepsilon = \mu$.

Figure 1(d) gives the total scattering efficiency $Q_{\rm sca}$ for a spherical nanoparticle made of the MNG material as functions of size parameter x and relative permittivity ε. As shown above, the relative permeability and permittivity of the material are assumed to be equal in magnitude while opposite in sign, and the vertical dashed lines show the corresponding total scattering peaks of perfect nihility nanospheres.

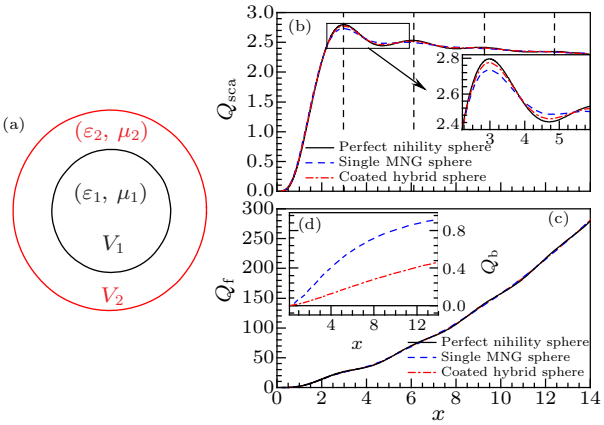

Fig. 2. (Color online) (a) Schematic diagram of a coated spherical particle. Comparisons of total (b), forward (c) and back (d) scattering spectra among three different nanoparticles. The solid, dashed and dash-dotted lines represent perfect nihility, single MNG and coated hybrid nanospheres, respectively. Inset in (b) gives the enlarged area for highlighting the difference in total scattering among three different nanoparticles.

Based on the above analysis, for a single MNG nanoparticle, the contributions in total scattering related to ε have a blue shift, while the parts related to μ experience a red shift. The partial compensation between blue and red shifts will cause nearly

the same positions while reduced intensities in total scattering peaks when compared with perfect nihility nanospheres, as can be observed in Fig. 1(d). These phenomena can be seen more clearly in comparison of the solid and dashed lines in Fig. 2(b), which represent the total scattering spectra of perfect nihility and single MNG ($\varepsilon = -\mu = 0.2$, denoted by the horizontal dashed line in Fig. 1(d)) nanospheres, respectively. Figures 2(c) and 2(d) provide the comparisons of forward and back scattering spectra between perfect nihility and single MNG nanospheres. It is observed that the forward scattering spectra of these two particles are hardly distinguishable, while their back scattering spectra exhibit completely different behaviors. The MNG nanosphere has a non-zero back scattering while the back scattering of perfect nihility nanospheres is always zero.

According to Eqs. (1) and (2), it is realized that relative permittivity and permeability are commutative, which means that the exchange of values between permittivity and permeability does not affect the scattering properties of particles. Thus the substitution of the ENG material for the MNG material will obtain exactly the same results.

Compared the plots in Fig. 1, it is found that under condition of the same magnitude of refractive index, the electromagnetic scattering of single MNG or ENG nanospheres is much closer to that of perfect nihility nanospheres than DPS and DNG nanospheres. This result provides a basis for further optimal design of electromagnetic nihility nanoparticles. To obtain a better simulation for the electromagnetic scattering of perfect nihility nanospheres than single MNG or ENG nanospheres, a coated spherical model is proposed, whose schematic diagram is given in Fig. 2(a). The volumes of the inner core and the outer shell are V_1 and V_2, and the optical parameters of inner and outer materials are (ε_1, μ_1) and (ε_2, μ_2). To imitate the nihility material, it is required that the parameters of the coated sphere fulfil these two constraints,

$$\varepsilon_1 V_1 + \varepsilon_2 V_2 = 0, \tag{11}$$

$$\frac{\mu_1}{V_1} + \frac{\mu_2}{V_2} = 0. \tag{12}$$

From Eqs. (11) and (12), one can notice that there are various combinations of inner and outer materials under different values of V_1/V_2, to guarantee impendence matching between inner and outer parts here we choose $V_1 = V_2$. To make calculation comparable, the material in the inner core is chosen to be the same ($\varepsilon_1 = -\mu_1 = 0.2$) as that of the single MNG nanosphere shown in Fig. 2(b), and in consequence the material in the outer shell will be the commutative ENG ($-\varepsilon_2 = \mu_2 = 0.2$) medium. The total, forward and back scattering spectra of selected coated hybrid nanoparticles are denoted by the dash-dotted lines in Figs. 2(b)–2(d). It is observed distinctly from the inset of Fig. 2(b) that the total scattering spectrum of coated hybrid nanospheres is much closer to that of perfect nihility nanospheres, as compared with single MNG nanospheres. As for back scattering, although the back scattering of coated hybrid nanoparticles is nonzero, its magnitude is almost reduced by half when compared with that of a single MNG nanoparticle, as illustrated clearly in Fig. 2(d).

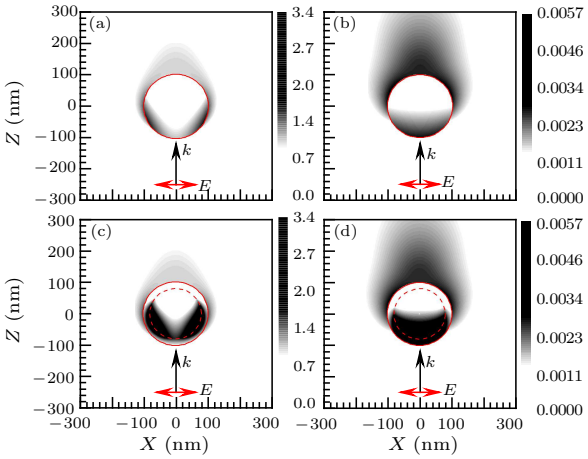

Fig. 3. (Color online) Scattered (a) electric and (b) magnetic field intensity distributions around a perfect nihility nanosphere with radius $a = 100$ nm at incident wavelength $\lambda = 210.8$ nm (i.e., at the first total scattering peak x_1). Scattered (c) electric and (d) magnetic field intensity distributions around the coated hybrid nanosphere shown in Fig. 2(b) with outer radius $a = 100$ nm and inner radius $b = 79.4$ nm at the same incident wavelength.

Figures 3(a) and 3(b) present the scattered electric and magnetic field intensity distributions of a perfect nihility nanosphere with radius $a = 100$ nm at the first total scattering peak x_1 (converting to incident wavelength $\lambda = 210.8$ nm), while Figs. 3(c) and 3(d) show the scattered electric and magnetic field intensity distributions of the coated hybrid nanosphere shown in Fig. 2(b) with outer radius $a = 100$ nm and inner radius $b = 79.4$ nm at the same incident wavelength. The propagation direction of incident wave is the $+z$-direction and the incident electric vector polarizes along the x-axis. Ignoring the transmitted fields inside the particles, it is noticed that the scattered electric and magnetic field distributions of perfect nihility nanospheres are just the same as those of coated hybrid nanospheres. From Figs. 3(a) and 3(c) it is noticed that the back scatterings of perfect nihility and coated hybrid nanoparticles are rather small, while their forward scatterings are relatively large, which are consistent with the results obtained from Figs. 2(d) and 2(c). However, the behaviors of electromagnetic field inside these two particles are quite different: the electric and magnetic field intensities of perfect nihility nanospheres reach the maxima near the surface of the particle, while for the coated hybrid nanosphere

the maxima of fields appear near the interface between the inner and outer parts.

In summary, compared with the scattering properties of perfect nihility nanospheres, we have discussed the feasibility of using non-zero refractive index materials to constitute electromagnetic nihility nanoparticles, which can approach the electromagnetic response of perfect nihility nanospheres. Compared with perfect nihility nanoparticles, the total scattering spectrum of nanoparticles will have a blue shift if their relative permittivity and permeability both are positive, and vice versa. For a single ENG or MNG nanoparticle, the blue shift in the total scattering caused by one positive optical parameter will partially offset the red shift resulting from the other negative optical parameter, which leads to nearly unchanged positions but reduced intensities in total scattering peaks, as compared with perfect nihility nanoparticles. Moreover, an optimization design of electromagnetic nihility nanoparticles is proposed, which is a coated hybrid spherical nanoparticle constituted by commutative ENG and MNG materials. It is noticed that the differences of total and back scattering spectra between perfect nihility and coated hybrid nanospheres are much smaller than those between perfect nihility and single ENG or MNG nanospheres. By further comparing the electromagnetic field distributions of perfect nihility and coated hybrid nanoparticles, it is observed that although the transmitted fields inside these two particles have obvious differentiation, their scattered electromagnetic field distributions are identical. The results in this work reveal that composite structures made from commutative ENG and MNG materials have the opportunity to provide the closest approximation of electromagnetic scattering of perfect nihility nanoparticles in a wide frequency range.

References

[1] Sarychev A K and Shalaev V M 2007 *Electrodynamics of Metamaterials* (Singapore: World Scientific)
[2] Pendry J B 2000 *Phys. Rev. Lett.* **85** 3966
[3] Monticone F and Alù A 2014 *Chin. Phys.* B **23** 047809
[4] Alù A and Engheta N 2006 *J. Opt. Soc. Am.* B **23** 571
[5] Shelby R A, Smith D R and Schultz S 2001 *Science* **292** 77
[6] Pendry J B, Holden A J, Stewart W J and Youngs I 1996 *Phys. Rev. Lett.* **76** 4773
[7] Yuan H K, Chettiar U K, Cai W, Kildishev A V, Boltasseva A, Drachev V P and Shalaev V M 2007 *Opt. Express* **15** 1076
[8] Hao J, Yan W and Qiu M 2010 *Appl. Phys. Lett.* **96** 101109
[9] Huang X, Lai Y, Hang Z H, Zheng H and Chan C T 2011 *Nat. Mater.* **10** 582
[10] Enoch S, Tayeb G, Sabouroux P, Guérin N and Vincent P 2002 *Phys. Rev. Lett.* **89** 213902
[11] Alù A, Silveirinha M G, Salandrino A and Engheta N 2007 *Phys. Rev.* B **75** 155410
[12] Li Y, Kita S, Muñoz P, Reshef O, Vulis D I, Yin M, Lončar M and Mazur E 2015 *Nat. Photon.* **9** 738
[13] Lakhtakia A 2001 *Int. J. Infrared Millimeter Waves* **22** 1731
[14] Lakhtakia A 2002 *Int. J. Infrared Millimeter Waves* **23** 339
[15] Ziolkowski R W 2004 *Phys. Rev.* E **70** 046608
[16] Nguyen V C, Chen L and Halterman K 2010 *Phys. Rev. Lett.* **105** 233908
[17] Lakhtakia A 2006 *Microwave Opt. Technol. Lett.* **48** 895
[18] Lakhtakia A and Geddes Ⅲ J B 2007 *AEU-Int. J. Electron. Commun.* **61** 62
[19] Silveirinha M and Engheta N 2007 *Phys. Rev.* B **75** 075119
[20] Moitra P, Yang Y, Anderson Z, Kravchenko I I, Briggs D P and Valentine J 2013 *Nat. Photon.* **7** 791
[21] Mie G 1908 *Ann. Phys.* (Berlin Germany) **330** 377
[22] Bohren C F and Huffman D R 1983 *Absorption and Scattering of Light by Small Particles* (New York: John Wiley & Sons)
[23] Kerker M, Wang D S and Giles C L 1983 *J. Opt. Soc. Am.* **73** 765