Total Pairwise Quantum Correlation and Entanglement in a Mixed-Three-Spin Ising-XY Model with Added Dzyaloshinskii–Moriya Interaction under Decoherence

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We investigate the behavior of geometric global quantum discord (GGQD) and concurrence (C) between half-spins of a mixed-three-spin (1/2, 1, 1/2) system with the Ising-XY model for which spins (1, 1/2) have the Ising interaction and half-spins (1/2, 1/2) have both XY and the Dzyaloshinskii–Moriya interactions together, under the decoherence action. A single-ion anisotropy property with coefficient ζ is assumed for the spin-integer. This system which includes an analytical Hamiltonian is considered at the front of an external homogeneous magnetic field B in thermal equilibrium. Finally, we compare GGQD and C and express some interesting phase flip reactions of the total quantum correlation and pairwise entanglement between spins (1/2, 1/2). Generally, we conclude that the concurrence and GGQD have different behaviors under the phase flip channel.

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It is well known that entanglement as the most important resource in the quantum information processing makes possible tasks which are impossible in the classical domain.[1−5] Entanglement of a bipartite system with state ρ was quantified by using the concurrence (as a measure of entanglement) by Wootters *et al.*[6−8] In spite of entanglement, the most popular quantifier of non classical correlations have been introduced in literature by the quantum discord (QD),[9] as a measure of quantum correlations,[10] which goes beyond entanglement. Quantum discord is defined as the discrepancy between the total correlations between two parts $\mathcal{A}$ and $\mathcal{B}$ of a bipartite system before and after a local measurement performed on one part. According to the QD definition, separable states may have nonzero QD. Recently, this scenario has been verified for different quantum systems by many researchers.[11−19] A necessary and sufficient condition for the existence of non-zero QD for bipartite systems was proposed by Dakic *et al.* in Ref. [20], and an intelligent geometrical way to quantify QD, i.e., the so-called geometric discord (GD), as the distance of the given state from the closest classical quantum (or the zero discord) state was presented as well.

Following the work of Dakics *et al.*, a great deal of attention due to manifold interesting applications has been attracted to GD in recent years.[14,21−23] Meanwhile, the geometric quantum discord of multipartite states was characterized by Xu.[24] He proposed a geometric global quantum discord (GGQD) to geometrically measure the amount of total quantum correlation between parts of a two-qubit system. According to the expressions in Ref. [25], Yurischev illustrated the level surfaces of entanglement and quantum discord for general X-states in Ref. [26] (similar work has been carried out by Yao *et al.*[27]) and Ying *et al.* carried out work for non X-state.[28] They found explicit differences between the shape and the structure of the entanglement and the quantum discord.

Nowadays, numerous research studies have been devoted to detect and quantify entanglement and quantum discord for mixed-spin systems[18,29−32] covering composed two or more models such as the Ising–Heisenberg (XXZ, XYZ, XXX, XX) model,[33−36] the Ising–Hubbard model,[37] the Ising-XY model.[38] Some of the investigated models are considered with the added Dzyaloshinskii–Moriya (DM) interaction (introduced by Dzyaloshinskii[39] and Moriya[40]) arisen from spin–orbit coupling.

Moreover, the dynamic behavior of quantum discord under decoherence was studied in Refs. [14,41]. It was shown that a class of X-states has sudden transition between classical correlation and quantum one under the action of decoherence. In this Letter, we study the concurrence and GGQD in a half-spins bipartite (sub)system of a mixed-three-spin (1/2,1,1/2) Ising-XY model with DM interaction at the front of a homogeneous magnetic field under the decoherence action.

We introduce mixed-N-spin chain with the Ising-XY model contained $M = (N-1)/2$ mixed-three-spin cells, which is in an external homogeneous magnetic field B with analytical Hamiltonian

$$\begin{aligned} H = & \sum_{i=1}^{M}((1+\gamma)S_{i,1}^{x}S_{i,3}^{x} + (1-\gamma)S_{i,1}^{y}S_{i,3}^{y}) \\ & + \sum_{i=1}^{M}\boldsymbol{B}\cdot(\boldsymbol{S}_{i,1}+\boldsymbol{S}_{i,3}) + \sum_{i=1}^{M}\boldsymbol{D}\cdot(\boldsymbol{S}_{i,1}\times\boldsymbol{S}_{i,3}) \\ & + \sum_{i=1}^{M}J(S_{i,1}^{z}J_{i,2}^{z} + J_{i,2}^{z}S_{i,3}^{z}) + \sum_{i=1}^{M}\zeta(J_{i,2}^{z})^{2} \\ & + \sum_{i=1}^{M}\boldsymbol{B}\cdot\boldsymbol{J}_{i,2}, \end{aligned} \tag{1}$$

where i denotes the number of triangular cells in the chain, γ is the anisotropy parameter, J is the Ising

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coupling between the spins (1,1/2), D is the DM interaction between the spins (1/2, 1/2) of any cell, and ζ denotes the single-ion anisotropy considered for integer-spins. Here all these introduced parameters are considered dimensionless, and $\boldsymbol{S} = \{S^x, S^y, S^z\}$ and $\boldsymbol{J} = \{J^x, J^y, J^z\}$ are spin-half and integer-spin operators (with $\hbar = 1$), respectively.

We here consider $M = 1$ (one mixed-three-spin cell selection), also $\boldsymbol{B} = B_z$ and $\boldsymbol{D} = D_z$, which are homogeneous magnetic field and DM interaction in the z-direction respectively.

Eigenvectors of the Hamiltonian Eq. (1) are given by

$$\begin{aligned}
|\phi_1\rangle &= a|\uparrow, \bigcirc, \downarrow\rangle + |\downarrow, \bigcirc, \uparrow\rangle,\\
|\phi_2\rangle &= b|\uparrow, \bigcirc, \downarrow\rangle + |\downarrow, \bigcirc, \uparrow\rangle,\\
|\phi_3\rangle &= e|\uparrow, \Uparrow, \uparrow\rangle + |\downarrow, \Uparrow, \downarrow\rangle,\\
|\phi_4\rangle &= f|\uparrow, \Uparrow, \uparrow\rangle + |\downarrow, \Uparrow, \downarrow\rangle,\\
|\phi_5\rangle &= g|\uparrow, \bigcirc, \uparrow\rangle + |\downarrow, \bigcirc, \downarrow\rangle,\\
|\phi_6\rangle &= h|\uparrow, \bigcirc, \uparrow\rangle + |\downarrow, \bigcirc, \downarrow\rangle,\\
|\phi_7\rangle &= j|\uparrow, \Downarrow, \downarrow\rangle + |\downarrow, \Downarrow, \uparrow\rangle,\\
|\phi_8\rangle &= k|\uparrow, \Downarrow, \downarrow\rangle + |\downarrow, \Downarrow, \uparrow\rangle,\\
|\phi_9\rangle &= l|\uparrow, \Downarrow, \uparrow\rangle + |\downarrow, \Downarrow, \downarrow\rangle,\\
|\phi_{10}\rangle &= m|\uparrow, \Downarrow, \uparrow\rangle + |\downarrow, \Downarrow, \downarrow\rangle,\\
|\phi_{11}\rangle &= n|\uparrow, \Uparrow, \downarrow\rangle + |\downarrow, \Uparrow, \uparrow\rangle,\\
|\phi_{12}\rangle &= o|\uparrow, \Uparrow, \downarrow\rangle + |\downarrow, \Uparrow, \uparrow\rangle,
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
a &= -\frac{i(i-D)}{\sqrt{1+D^2}},\ b = \frac{i(i-D)}{\sqrt{1+D^2}},\\
e &= -\frac{1}{2}\gamma\Big[B + \frac{\zeta}{2}\\
&\quad - \frac{1}{2}\sqrt{(\zeta+2B)^2 + 4J(\zeta+J+2B) + \gamma^2} + J\Big]^{-1},\\
f &= -\frac{1}{2}\gamma\Big[B + \frac{\zeta}{2}\\
&\quad + \frac{1}{2}\sqrt{(\zeta+2B)^2 + 4J(\zeta+J+2B) + \gamma^2} + J\Big]^{-1},\\
g &= -\frac{1}{2}\frac{\gamma}{-\frac{1}{2}\sqrt{\gamma^2+4B^2}+B},\\
h &= -\frac{1}{2}\frac{\gamma}{\frac{1}{2}\sqrt{\gamma^2+4B^2}+B},\\
j &= -\frac{i(i-D)}{-\zeta+\sqrt{1+\zeta^2+D^2}},\\
k &= -\frac{i(i-D)}{-\zeta-\sqrt{1+\zeta^2+D^2}},\\
l &= \frac{1}{2}\gamma\Big[\frac{\zeta}{2} - B\\
&\quad + \frac{1}{2}\sqrt{(\zeta-2B)^2 + 4J(\zeta+J-2B) + \gamma^2} + J\Big]^{-1},\\
m &= \frac{1}{2}\gamma\Big[\frac{\zeta}{2} - B\\
&\quad - \frac{1}{2}\sqrt{(\zeta-2B)^2 + 4J(\zeta+J-2B) + \gamma^2} + J\Big]^{-1},
\end{aligned}$$

$$\begin{aligned}
n &= \frac{i(i-D)}{-\zeta-\sqrt{1+\zeta^2+D^2}},\\
o &= \frac{i(i-D)}{-\zeta+\sqrt{1+\zeta^2+D^2}},
\end{aligned} \tag{3}$$

and eigenvalues are

$$\begin{aligned}
E_1 &= \zeta + \frac{1}{2}\sqrt{1+D^2},\ E_2 = \zeta - \frac{1}{2}\sqrt{1+D^2}\\
E_3 &= B + \frac{\zeta}{2}\\
&\quad + \frac{1}{2}\sqrt{(\zeta+2B)^2 + 4J(\zeta+J+2B) + \gamma^2},\\
E_4 &= B + \frac{\zeta}{2}\\
&\quad - \frac{1}{2}\sqrt{(\zeta+2B)^2 + 4J(\zeta+J+2B) + \gamma^2},\\
E_5 &= \zeta + \frac{1}{2}\sqrt{\gamma^2+4B^2},\ E_6 = \zeta - \frac{1}{2}\sqrt{\gamma^2+4B^2}\\
E_7 &= \frac{\zeta}{2} - B + \frac{1}{2}\sqrt{\zeta^2+D^2+1},\\
E_8 &= \frac{\zeta}{2} - B - \frac{1}{2}\sqrt{\zeta^2+D^2+1}\\
E_9 &= \frac{\zeta}{2} - B\\
&\quad + \frac{1}{2}\sqrt{(\zeta-2B)^2 + 4J(\zeta+J-2B) + \gamma^2},\\
E_{10} &= \frac{\zeta}{2} - B\\
&\quad - \frac{1}{2}\sqrt{(\zeta-2B)^2 + 4J(\zeta+J-2B) + \gamma^2},\\
E_{11} &= \frac{\zeta}{2} + B + \frac{1}{2}\sqrt{\zeta^2+D^2+1},\\
E_{12} &= \frac{\zeta}{2} + B - \frac{1}{2}\sqrt{\zeta^2+D^2+1}.
\end{aligned} \tag{4}$$

Hence, we can derive the density matrix of the pair spins (1/2, 1/2) as

$$\boldsymbol{\rho}_{13}^{T_2} = \frac{1}{Z}\begin{pmatrix} \delta & 0 & 0 & \varsigma\\ 0 & P & \xi & 0\\ 0 & \xi^* & Q & 0\\ \varsigma^* & 0 & 0 & \chi \end{pmatrix}, \tag{5}$$

in which T_2 is the partial trace over second spin (spin-1 in the cell). $\{\delta, \varsigma, P, Q, \xi, \chi\}$ are functions of temperature T, magnetic field B, DM interaction D, anisotropy parameter γ, Ising coupling J and single-ion anisotropy ζ. The concurrence $C(\rho)$ that is defined as a measure of entanglement, is given by $C(\rho) = \max\{0, 2\lambda - \sum_{i=1}^{4}\lambda_i\}$, where $\lambda = \max\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ and λ_i are square roots of the eigenvalues of the inner product $R = \rho\tilde{\rho}$ with $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^{\dagger}(\sigma_y \otimes \sigma_y)$. In the standard basis, thermal state ρ of a quantum system with Hamiltonian H Eq. (1) in the equilibrium state is defined as $\rho_{\rm eq} = \frac{1}{\mathcal{Z}}\exp(-\beta H)$, where $\beta = 1/k_{\rm B}T$ (for simplicity we here consider $k_{\rm B} = 1$), T is the temperature, $\mathcal{Z} = {\rm Tr}[\exp(-\beta H)]$ is the partition function, $\rho^{\dagger}$ denotes the complex conjugation of the density matrix

ρ, and σ_y is Pauli matrix defined as $\sigma_y = 2S^y$. Here for considered bipartite (sub)system (1/2, 1/2) with the density matrix ρ presented in Eq. (5) the concurrence can also be given by

$$C(\rho) = 2\max\left\{\max\left(0, \left|\frac{\varsigma}{Z}\right| - P\right), \left|\frac{\xi}{Z}\right| - \left(\frac{\delta\chi}{Z^2}\right)^{1/2}\right\}. \quad (6)$$

Hereafter, we use this equation for calculating and simulating the concurrence. The QD of a bipartite quantum system $H^{\rm a} \otimes H^{\rm b}$ with state $\rho_{\mathscr{A}\mathscr{B}}$ and marginals $\rho_{\rm a}$ and $\rho_{\rm b}$ can be expressed as

$$QD(\rho_{\mathscr{A}\mathscr{B}}) = S(\rho_{\mathscr{A}}) - S(\rho_{\mathscr{A}\mathscr{B}}) + \min_{\{\mathscr{B}^k\}} S(\rho|\{\mathscr{B}^k\}), \quad (7)$$

where the minimum is taken over all possible 1D orthogonal POVMs $\{\mathscr{B}^k\}$ performed on party $\mathscr{B}$, $S(\rho|\{\mathscr{B}^k\})$ is the average conditional entropy of party $\mathscr{B}$, $p_k = {\rm Tr}_{\mathscr{A}\mathscr{B}}[(I^{\mathscr{A}} \otimes \mathscr{B}^k)\rho_{\mathscr{A}\mathscr{B}}(I^{\mathscr{A}} \otimes \mathscr{B}^k)]$ is the probability measurement outcome k, and $\rho_k = \frac{1}{p_k}[(I^{\mathscr{A}} \otimes \mathscr{B}^k)\rho_{\mathscr{A}\mathscr{B}}(I^{\mathcal{A}} \otimes \mathscr{B}^k)]$ is the corresponding conditional density matrix which describes the post-measurement state of the party $\mathscr{A}$. A good review on QD and CC as well as on minimization circumstances has been carried out in Refs. [42,43]. Dakic *et al.* introduced a geometric measure of quantum discord (GD) for a state ρ as $D(\rho) = \min_{\varrho\in\Omega_0} \| \rho - \varrho \|^2$, where Ω_0 denotes the set of zero-discord states, and $\| \rho - \varrho \|^2 := tr(\rho - \varrho)^2$ is the square of Hilbert–Schmidt norm of linear operators acting on the state space of the system. GD for a two-qubit state is given by

$$D(\rho) = \frac{1}{4}(\| \boldsymbol{x} \|^2 + \| \boldsymbol{R} \|^2 - k_{\max}), \quad (8)$$

where $\boldsymbol{x} := (x_1, x_2, x_3)^T$ is a coherent column vector, $\| \boldsymbol{x} \|^2 = \sum_\alpha x_\alpha^2$, $T = (t_{\alpha\beta})$ is the correlation matrix with $t_{\alpha\beta} = {\rm Tr}[\rho(\sigma_\alpha \otimes \sigma_\beta)]$, and $k_{\max}$ is the largest eigenvalue of matrix $\boldsymbol{x}\boldsymbol{x}^T + \boldsymbol{R}\boldsymbol{R}^T$. Straightforward calculations have been carried out for obtaining a general equation of GGQD of a two-qubits X-state system and analytical methods have been suggested in Refs. [15,44,45]. Using these suggested methods we can gain an upstanding formula of GGQD for our favorite two-qubit (sub)system (1/2, 1/2) with state Eq. (5) as

$$\begin{aligned} D(\rho) = {} & \frac{1}{\mathcal{Z}^2}(\delta^2 + P^2 + Q^2 + \chi^2) - \frac{1}{4} \\ & + 2\left(\left|\frac{\varsigma}{\mathcal{Z}}\right|^2 + \left|\frac{\xi}{\mathcal{Z}}\right|^2\right) - \max\left(\frac{1}{\mathcal{Z}^2}(\delta^2 \right. \\ & \left. + P^2 + Q^2 + \chi^2) - \frac{1}{4}, \left|\frac{\varsigma + \xi}{\mathcal{Z}}\right|^2\right). \end{aligned} \quad (9)$$

To achieve our aims, we here are interested in this formula for calculating and simulating GGQD. Recently, it was proved that the quantum discord of some of the Bell states are invariant under decoherence.[46] In this regard, we investigate such phenomena for our suggested bipartite (sub)system (1/2, 1/2). The state ρ introduced in Eq. (5) undergoes the phase flip channel by means of the Kraus operators $\Gamma^0_{\mathscr{A}} = {\rm diag}(\sqrt{1-p/2}, \sqrt{1-p/2}) \otimes I$, $\Gamma^1_{\mathscr{A}} = {\rm diag}(\sqrt{p/2}, -\sqrt{p/2}) \otimes I$, $\Gamma^0_{\mathscr{B}} = I \otimes {\rm diag}(\sqrt{1-p/2}, \sqrt{1-p/2})$ and $\Gamma^1_{\mathscr{B}} = I \otimes {\rm diag}(\sqrt{p/2}, -\sqrt{p/2})$ that describe the noise channels $\mathscr{A}$ and $\mathscr{B}$, with $p = 1 - \exp(-\kappa t)$, where κ is the phase damping rate, and $0 \le p \le 1$. The evolved state of such a bipartite (sub)system under phase flip channel is described as a completely positive trace preserving map $\mathcal{M}[\rho(\mathscr{A}\mathscr{B})] = \sum_{i,j} \Gamma^i_{\mathscr{A}} \Gamma^j_{\mathscr{B}} \rho(\mathscr{A}\mathscr{B}) \Gamma^{i\dagger}_{\mathscr{B}} \Gamma^{j\dagger}_{\mathscr{A}}$ (late, in Ref. [47] we proved that a mixed-spin chain with the two adjacent mixed-three-spin (1/2, 1, 1/2) cells can be an ideal memory-less channel for transferring quantum information).

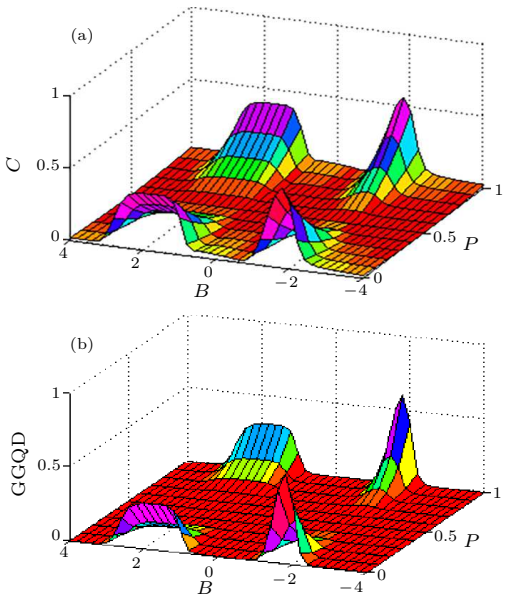

Fig. 1. The magnetic field dependence of (a) the concurrence $C \times \frac{1}{2}$ and (b) GGQD of the spins (1/2, 1/2), at finite temperature $T = 0.15J$ under the phase flip at fixed $\zeta = J$, $D = 2J$ and $\gamma = 0.2J$.

Figure 1 depicts the concurrence and GGQD behaviors versus the magnetic field under the phase flip channel. We can see that in the considered region, the behaviors of the concurrence and GGQD are generally different from each other under the decoherence action versus the magnetic field changes. By inspecting this figure, it can be seen that both functions are not symmetric with respect to the magnetic field changes. On the other hand, both functions are interestingly symmetric versus line $p = 0.5$ in the (B, p)-plane for any fixed value of the magnetic field. This phenomenon does not occur for other X-states verified in aforementioned references and there is no mention of this phenomenon above. Moreover, the GGQD surface is sharper than the concurrence surface, indeed, in some regions of the (B, p)-plane where GGQD vanishes, the concurrence does not vanish, e.g., see the region with weak magnetic field and finite p. The vanishing of GGQD denotes the sudden transition of classical and

quantum correlation, hence with regard to Fig. 1, one sees that the entanglement disappears after the sudden transition of classical and quantum correlation, that is, the concurrence keeps non-zero after the transition. Therefore, for the suggested model, pairwise entanglement is more robust against the decoherence than GGQD.

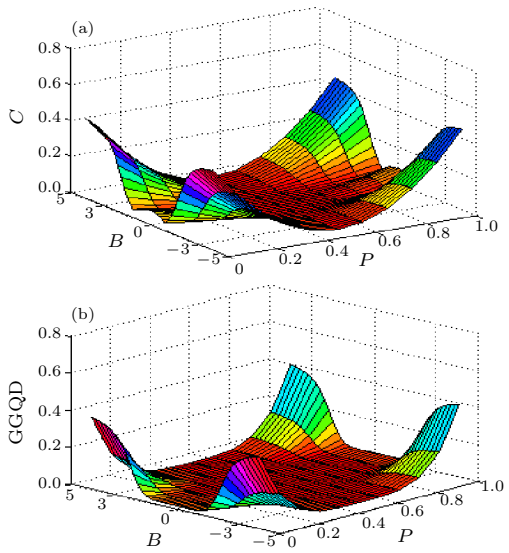

Fig. 2. The DM interaction dependence of (a) the concurrence $C \times \frac{1}{2}$ and (b) GGQD of the spins (1/2, 1/2), at finite temperature $T = 0.15J$ under the phase flip at fixed values of $\zeta = J$, $B = J$ and $\gamma = 0.2J$.

The DM interaction dependence of the concurrence and GGQD under the phase flip channel at finite temperature in the vicinity of a fixed magnetic field $B = \zeta = J$ is illustrated in Fig. 2. It is obvious that both functions are symmetric with respect to the DM interaction changes and are also symmetric versus line $p = 0.5$ in the (D, p)-plane. As shown in this figure, the behaviors of the concurrence and GGQD are also different under the decoherence action versus the DM interaction changes. Indeed, in some regions of the (D, p)-plane in which GGQD vanishes, the concurrence does not vanish, for example, see the region with weak DM interaction and finite p. For more realization, we depict the contour plots of the concurrence and GGQD under the phase flip channel versus the magnetic field and the DM interaction, respectively, in Figs. 3(a) and 3(b). These figures show all differences in behavior between the concurrence and GGQD of the favorite bipartite (sub)system containing the spins (1/2, 1/2). It is clear that the entanglement disappears after the sudden transition of classical and quantum correlation, in other words, the concurrence vanishes after GGQD and due to the symmetry, the entanglement disappearance phenomenon occurs closer to the line $p = 0.5$ than the GGQD disappearance.

Figure 3(c) shows the contours of the concurrence and GGQD under phase flip channel versus the anisotropy. Here the behaviors of the concurrence and GGQD are different near the line $p = 0.5$ in addition near line $\gamma = 0$ (due to the symmetry). In both the cases the entanglement disappears after GGQD. Finally, we conclude that the concurrence behavior is generally different from GGQD and for the suggested model pairwise entanglement is more robust against the decoherence than GGQD.

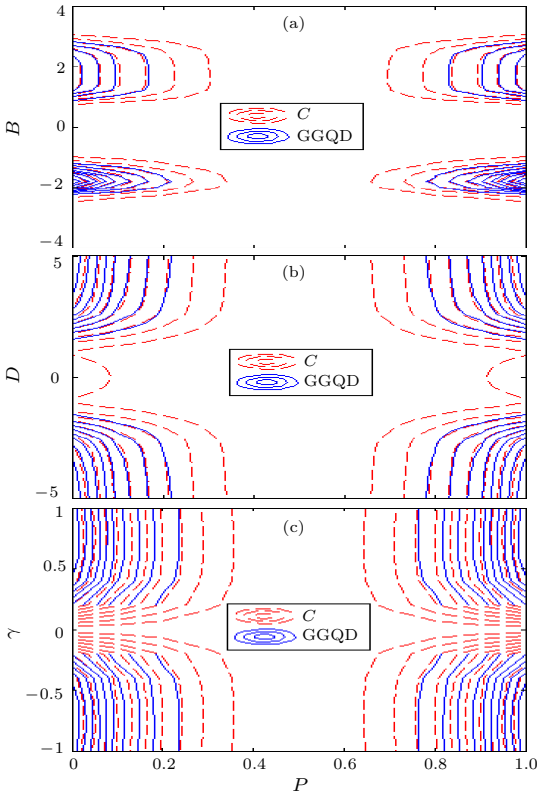

Fig. 3. Contour plots of the concurrence $C \times \frac{1}{2}$ and GGQD of the spins (1/2, 1/2) in the finite temperature $T = 0.15J$: (a) the magnetic field at fixed values of $\zeta = J$, $D = 2J$ and $\gamma = 0.2J$, (b) the DM interaction at fixed values of $\zeta = J$, $B = J$ and $\gamma = 0.2J$, and (c) the anisotropy at fixed values of $\zeta = J$, $B = J$ and $D = 2J$, under phase flip.

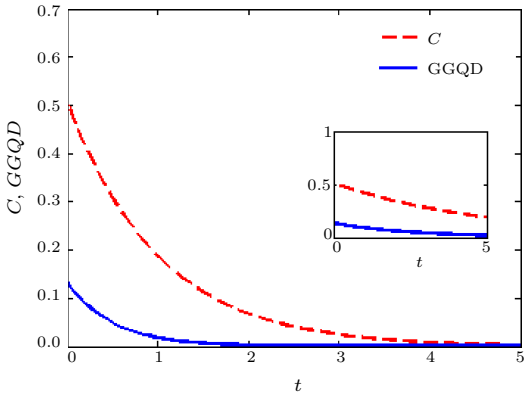

Fig. 4. Concurrence $C(\rho)$ and GGQD of the spins (1/2, 1/2), with respect to the time decoherence action at fixed values of DM interaction $D = 2J$, $B = J$, $\gamma = 0.2J$ and finite temperature $T = 0.15J$, for $\kappa = 0.1$. The inset is depicted for $\kappa = 5$.

To obtain more useful information about the behaviors of the concurrence and GGQD, we show the

comparison between both functions versus time decoherence action at fixed values of $B = J$, $D = 2J$, $\gamma = 0.2J$ and finite temperature $T = 0.15J$, for $\kappa = 0.1$ (inset denotes for $\kappa = 5$) in Fig. 4. With regard to this figure, we see that the concurrence value is much greater than GGQD at initial time $t = 0$ (almost five-fold), also it disappears at longer times. This means that when the sudden transition of classical and quantum correlation occurs (GGQD disappears) the entanglement is still established. The inset shows that with the increase of the phase damping rate κ, time disappearance of both the concurrence and GGQD becomes longer.

In summary, we have investigated the magnetic field, the DM interaction and the anisotropy dependences of the concurrence and GGQD for a bipartite half-spins (1/2, 1/2) (sub)system of a mixed-three-spin (1/2, 1, 1/2) Ising-XY model under the decoherence action at thermal equilibrium. It is concluded that the concurrence and GGQD have difference in behaviors versus all verified parameters under the phase flip channel. They are symmetric versus the line $p = 0.5$ in all of planes (B, p), (D, p) and (γ, p), also lines $D = 0$ and $\gamma = 0$, while there is no symmetry versus the magnetic field changes (line $B = 0$). Moreover, we find that the entanglement disappears after the sudden transition of classical and quantum correlation, and this disappearance occurs in longer times for the concurrence; consequently, for the considered model the pairwise entanglement is more robust against the decoherence than GGQD. These interpretations can be promoted for the longer spin chains with the same model and other new outcomes can be obtained.

References

[1] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865
[2] Cerf N J and Adami C 1997 *Fund. Theor. Phys.* **81** 77
[3] Cerf N J and Adami C 1997 *Phys. Rev. Lett.* **79** 5194
[4] Horodecki M, Oppenheim J and Winter A 2005 *Nature* **436** 673
[5] Cerf N J and Adami C 1998 *Physica* D **120** 62
[6] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[7] Hill S and Wootters W K 1997 *Phys. Rev. Lett.* **78** 5022
[8] Wootters W K 2001 *Quantum Inf. Comput.* **1** 27
[9] Ollivier H and Zurek W H 2001 *Phys. Rev. Lett.* **88** 017901
Henderson L and Vedral V 2001 *J. Phys. A: Math. Gen.* **34** 6899
[10] Liu B Q, Bin S and Jian Z 2010 *Phys. Rev.* A **82** 062119
[11] Ferraro A, Aolita L, Cavalcanti D, Cucchietti F M and Acin A 2010 *Phys. Rev.* A **81** 052318
[12] Sarandy M S 2009 *Phys. Rev.* A **80** 022108
[13] Qiong W, Qiao L J and Sheng Z H 2010 *Chin. Phys.* B **19** 100311
[14] Li B, Wang Z X and Fei S M 2011 *Phys. Rev.* A **83** 022321
[15] Ali M, Rau A R P and Alber G 2010 *Phys. Rev.* A **81** 042105
[16] Chen Q, Zhang C, Yu S, Yi X X and Oh C H 2011 *Phys. Rev.* A **84** 042313
[17] Sarandy M S, Oliveira T R D and Amico L 2013 *Int. J. Mod. Phys.* B **27** 1345030
[18] Zad H A 2015 *Acta Phys. Pol.* B **46** 1911
[19] Mamtimin T, Ahmad A, Rabigul M, Ablimit A and Pan Q P 2013 *Chin. Phys. Lett.* **30** 030303
[20] Dakić B, Vedral V and Brukner C 2010 *Phys. Rev. Lett.* **105** 190502
[21] Paula F M, Oliveira T R d and Sarandy M S 2013 *Phys. Rev.* A **87** 064101
[22] Paula F M, Montealegre J D, Saguia A, Oliveira T R d and Sarandy M S 2013 *Europhys. Lett.* **103** 50008
[23] Saif A, Hassan M and Joag P S 2012 *J. Phys. A: Math. Theor.* **45** 345301
[24] Xu J 2012 *J. Phys. A: Math. Theor.* **45** 405304
[25] Lang M D and Caves C M 2010 *Phys. Rev. Lett.* **105** 150501
[26] Yurischev M A 2015 *Quantum Inf. Process.* **14** 3399
[27] Yao Y, Li H W, Yin Z Q and Han Z F 2012 *Phys. Lett.* A **376** 358
[28] Yao J Y, Dong Yu Li and Zhu S Q 2013 *Commun. Theor. Phys.* **59** 693
[29] Ning W Q, Gu S J, Chen Y G, Wu C Q and Lin H Q 2008 *J. Phys.: Condens. Matter* **20** 235236
[30] Han S D and Aydiner E 2014 *Chin. Phys.* B **23** 050305
[31] Tonegawa T, Hikihara T, Kaburagi M, Nishino T, Miyashita S and Mikeska H J 1998 *J. Phys. Soc. Jpn.* **67** 1000
[32] Ivanov N, Richter B J and Schulenburg J 2009 *Phys. Rev.* B **79** 104412
[33] Ananikian N S, Ananikyan L N, Chakhmakhchyan L A and Rojas O 2012 *J. Phys.: Condens. Matter* **24** 256001
[34] Faizi E and Eftekhari H 2015 *Chin. Phys. Lett.* **32** 100303
[35] Čanová L, Strečka J and Lučivjanský T 2009 *Condens. Matter Phys.* **12** 353
[36] Rojas V, Souza S M D, Ohanyan V and Khurshudyan M 2011 *Phys. Rev.* B **83** 094430
[37] Nalbandyan M, Lazaryan H, Rojas O, Souza S M D and Ananikian N 2014 *J. Phys. Soc. Jpn.* **83** 074001
[38] Zad H A 2016 *Chin. Phys.* B **25** 030303
[39] Dzyaloshinskii I 1958 *J. Phys. Chem. Solids* **4** 241
[40] Moriya T 1960 *Phys. Rev. Lett.* **4** 228
[41] Maziero J, Celeri L C, Serra R M and Vedral V 2009 *Phys. Rev.* A **80** 044102
[42] Modi K, Brodutch A, Cable H, Paterek T and Vedral V 2012 *Rev. Mod. Phys.* **84** 1655
[43] Ciliberti L, Rossignoli R and Canosa N 2010 *Phys. Rev.* A **82** 042316
[44] Girolami D and Adesso G 2011 *Phys. Rev.* A **83** 052108
[45] Qiang W C, Zhanga H P and Zhang L 2014 *Int. J. Theor. Phys.* **55** 1833
[46] Mazzola L, Piilo J and Maniscalco S 2010 *Phys. Rev. Lett.* **104** 200401
[47] Zad H A and Movahhedian H 2016 *Chin. Phys.* B **25** 080307