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The Three-Arm Michelson–Fabry–Pérot Detector for Gravitational Waves *

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A three-arm Michelson–Fabry–Pérot detector for gravitational waves is designed. It consists of three Michelson–Fabry–Pérot interferometers, one for each pair of arms. The new detector can be used to confirm whether the gravitational waves are in general relativity polarization states and to set the strong constraints on non-GR gravitational wave polarization states. By the new detectors, the angular resolution of sources can be improved significantly. With the new detector, it is easier to search for and confirm a gravitational wave signal in the observation data.

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LIGO Scientific Collaboration and Virgo Collaboration (LVC) declared that they had observed a transient gravitational wave signal GW150914 from the coalescence of a binary black hole ($36M_\odot$ and $29M_\odot$) to a $62M_\odot$ black hole.[1,2] The event can be explained by general relativity (GR) very well. In GR, gravitational radiation is mainly quadruple radiation. There are two independent polarizations (h_+ and $h_\times$). During the inspiral stage of a binary, the leading terms of two polarization states are[3]

$$h_+(t) = A_{\rm GW}(t; D_{\rm L})(1+\cos^2\iota)\cos\Phi_{\rm GW}(t; D_{\rm L}), \quad (1)$$

$$h_\times(t) = -2A_{\rm GW}(t; D_{\rm L})\cos\iota\sin\Phi_{\rm GW}(t; D_{\rm L}), \quad (2)$$

where ι is the inclination of the orbital plane of source, $A_{\rm GW}$ and $\Phi_{\rm GW}$ are the amplitude and phase, respectively, of the gravitational wave, and $D_{\rm L}$ is the luminosity distance of the binary black hole. In GR, a gravitational radiation can always be expressed in terms of the linear combination of two polarizations. Each detector will measure strains

$$h_k = F_k^{(+)}(\alpha,\delta,\psi;\vartheta,\varphi)h_+ + F_k^{(\times)}(\alpha,\delta,\psi;\vartheta,\varphi)h_\times, \quad (3)$$

where k=LIGO at Hanford, LIGO at Livingston, Virgo, GEO, KaGra, LIGO at India, etc., $F_k^{(+,\times)}(\alpha,\delta,\psi;\theta,\varphi)$ depend on the source location in the sky, the polarization of the waves,[4,5] and the orientation of the detectors.[6] Here α is the right ascension of the source, δ is the declination, ψ is the polarization, and ϑ and φ denote the orientation of the detectors. To maximize the response to a signal of gravitational wave, detectors are designed to be Michelson–Fabry–Pérot (MFP) interferometers with a right angle between two arms (referred to as, hereafter, L-shaped MFP interferometer). The two detectors for LIGO have 4 km arms and are 10 ms optical distance apart. Their orientation is chosen to be as closely aligned (modulo 90°) as possible,[7] to maximize the common response to a signal. With the detectors, the gravitational waves are first directly detected and by the observation data the basic properties of the source are provided.[3]

It is well known that the polarization is a vitally important property of gravitational radiation. In different theories of gravitation, gravitational waves have different polarizations. For example, in the scalar tensor theory of gravitation, there are six different polarization states.[8] For $n(>4)$-dimensional spacetime, the number of independent polarization states is $(n-3)n/2$. Hence, the determination of the polarization states of gravitational radiation will confirm whether the gravitation is described by GR, check the universality of the theory of gravitation, determine the dimension of the universe, understand the equation of states of matter fields in a neutron star, calibrate the luminosity distance of a source, etc. Unfortunately, the LIGO detectors almost ignore another independent combination of polarization states. This can be seen easily when a gravitational wave is incident perpendicularly to the detector. For the normal incidence, an L-shaped MFP interferometer only responds to one polarization, h_+, but does not respond to the other polarization, $h_\times$ correspondingly. Therefore, the presence of non-GR polarization states in GW150914 has not yet been excluded.[9]

On the other hand, gravitational wave astronomy or multi-messenger astronomy requires a high angular resolution for sources. However, a ground-based detector for gravitational waves similar to LIGO is an all sky monitor. It has no intrinsic angular resolution capability for transient signals. The incident direction of gravitational wave is obtained by the reconstruction of the observation data via time of arrival, and amplitude and phase consistency across the two detectors. Thus the angular resolution of advanced LIGO (aLIGO) is still very poor. They determine the position of the source in about 600 $\deg^2$ in 90% confidential region.[3]

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To improve the angular resolution for the sources, it is widely accepted that a world array of the detectors is needed (see Ref. [10]).

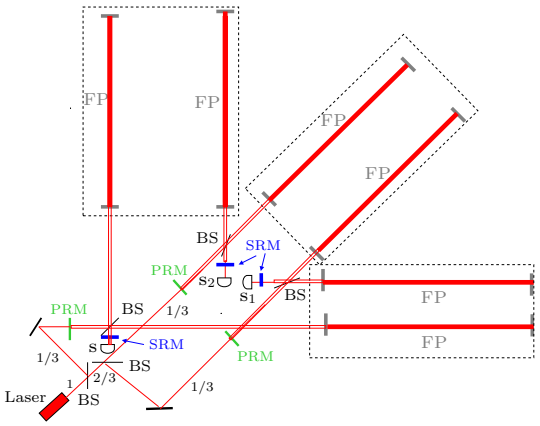


Fig. 1. The setup of the three-arm Michelson–Fabry–Pérot detector for gravitational waves. FPs are Fabry–Pérot cavities. PRM is the power recycling mirror. SRMs are signal recycling mirrors.

In fact, the detectors may be modified so that they can respond to all GR polarization states of gravitational waves. The key idea is that one arm is added along the internal bisector between the two arms as shown in Fig. 1. Thus it is called a three-arm MFP detector for gravitational waves. It actually contains three interferometers, one is the usual L-shaped MFP interferometer, and two are 45° MFP interferometers (referred to as V-shaped interferometers). It provides 3 sets of signals: s the signal between x- and y-arms as usual, s_1 and s_2 the signals between x-arm and bisector arm and between bisector arm and y-arm, respectively.

On the beam splitter near the laser, one third of light is reflected and two thirds are transmitted. The other four beam splitters are 50/50 ones. Each interferometer shares the one third of light intensity. The two Fabry–Pérot (FP) cavities in the same direction (in a dashed rectangle) are in one vacuum tube. Each interferometer has an independent power recycling mirror (PRM) and an independent signal recycling mirror (SRM). The laser is first locked on the arm in the bisector. The x and y arms are then locked on the laser.

The settings are the same as aLIGOs except that the power of the laser is raised from 20 W to 60 W. That is, the length of arms, the size, the weight, and the quality of mirrors, the finesse of FP cavities, etc. are the same as aLIGOs. The signal-to-noise ratio (SNR) will be a factor $\sqrt{2} = 1.414$ higher than that for a single detector of aLIGO. This is due to the fact that two V-shaped interferometers with 45° opening angle will contribute a factor $\sqrt{2}\sin\frac{\pi}{4} = 1$, and adding an L-shaped interferometer an overall factor becomes $\sqrt{2}$. If the detector is built underground, the SNR for low frequency will be enhanced even more. More importantly, the data for gravitational waves will be folded.

To analyze how the new detector responds to an incident gravitational wave, we may assume that the length of each arm is L, denote the signals in x-, y-, bisector-arms by s_x, s_y, and $s_/$, respectively, and use s, s_1 and s_2 for the signals in the L-shaped, the first V-shaped (x-arm and bisector-arm), and the second V-shaped (bisector-arm and y-arm) interferometers. For simplicity, the signal for a round trip in each arm in a Michelson interferometer is studied. The discussion is easily generalized to the multiple round trip in each arm in a Michelson–Fabry–Pérot interferometer.

When a monochromatic, linearized plane gravitational wave enters the detector perpendicularly, and if the angle between h_+ polarization and x-arms is ψ, the metric can be written as

$$\begin{aligned} ds^2 = & -dt^2 + [1 + h_+\cos(2\psi) + h_\times \sin(2\psi)]dx^2 \\ & + [1 - h_+\cos(2\psi) - h_\times \sin(2\psi)]dy^2 \\ & + 2[h_\times \cos(2\psi) - h_+ \sin(2\psi)]dxdy + dz^2. \end{aligned} \quad (4)$$

The signals for a round trip in the single arm in x-, y- and bisector arms will be approximately

$$s_x \approx L[h_+\cos(2\psi) + h_\times \sin(2\psi)], \quad (5)$$

$$s_y \approx -L[h_+\cos(2\psi) + h_\times \sin(2\psi)], \quad (6)$$

$$s_/ \approx L[h_\times \cos(2\psi) - h_+ \sin(2\psi)], \quad (7)$$

when the frequency of a gravitational wave is much less than L^{-1}. The signal detected by an L-shaped detector is

$$s = s_x - s_y \approx 2L[h_+\cos(2\psi) + h_\times \sin(2\psi)]. \quad (8)$$

The signals detected by two V-shaped interferometers are, respectively,

$$s_1 \approx \sqrt{2}L\Big[h_+\cos(2\psi - \frac{\pi}{4}) + h_\times \sin(2\psi - \frac{\pi}{4})\Big], \quad (9)$$

$$s_2 \approx \sqrt{2}L\Big[-h_+\sin(2\psi - \frac{\pi}{4}) + h_\times \cos(2\psi - \frac{\pi}{4})\Big]. \quad (10)$$

The three signals are not independent of each other and they satisfy

$$s = s_1 + s_2, \quad (11)$$

where s_1, s_2, and s provide a cross check for the observation data. This is the first advantage over the usual L-shaped detector. From the signals s_1 and s_2, h_+ and $h_\times$ can be solved as

$$h_+ = \frac{\sqrt{2}}{2L}\Big(s_1\cos(2\psi - \frac{\pi}{4}) - s_2\sin(2\psi - \frac{\pi}{4})\Big), \quad (12)$$

$$h_\times = \frac{\sqrt{2}}{2L}\Big(s_1\sin(2\psi - \frac{\pi}{4}) + s_2\cos(2\psi - \frac{\pi}{4})\Big). \quad (13)$$

Compared with Eqs. (1) and (2), whether the gravitational waves are in GR polarization states or not can be easily checked.

It is easy to see that the L-shaped detector only responds to a particular combination of two polarizations. In contrast, the three-arm detector not only can give the value of the L-shaped detector, but also can

determine the values of h_+ and $h_\times$, respectively. The observation data can be used to check whether the gravitational wave is in GR polarization states or not, and to figure out the angle ψ and the inclination ι. This is the second advantage over an LIGO detector.

If the monochromatic plane gravitational wave propagates with a wave vector $(k_x, k_y, k_z) = k(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\phi)$ in the detector-reference system, where k is the magnitude of the wave vector, satisfying $k = k_0$,

$$h_{\mu\nu} = e_{\mu\nu} e^{ik_\lambda x^\lambda}, \tag{14}$$

where $e_{\mu\nu} = e_{\nu\mu}$ is a polarization tensor. In GR, a gravitational wave is a traceless and transverse (TT) wave. The TT conditions are

$$-e_{00} + e_{11} + e_{22} + e_{33} = 0, \tag{15}$$

$$e_\mu{}^\lambda k_\lambda = \frac{1}{2} k_\mu e_\lambda{}^\lambda. \tag{16}$$

For TT waves, one may always choose

$$e_{00} = e_{01} = e_{10} = e_{02} = e_{20} = e_{03} = e_{30} = 0, \tag{17}$$

without loss of generality. Then, Eq. (16) becomes

$$e_{11}k_x + e_{12}k_y = 0, \tag{18}$$

$$e_{12}k_x + e_{22}k_y = 0, \tag{19}$$

$$e_{13}k_x + e_{23}k_y = 0 \tag{20}$$

for $k_z = 0$, and

$$e_{13} = -(e_{11}k_x + e_{12}k_y)/k_z, \tag{21}$$

$$e_{23} = -(e_{12}k_x + e_{22}k_y)/k_z, \tag{22}$$

$$e_{33} = \left(2e_{12}k_xk_y + e_{11}(k_x)^2 + e_{22}(k_y)^2\right)/(k_z)^2, \tag{23}$$

for $k_z \neq 0$.

For $k_z = 0$, there are two subcases: one of k_x and k_y also vanishes and neither k_x nor k_y are equal to zero. In the first subcase, let $k_y = 0$ with loss of generality. Then, $e_{11} = e_{12} = e_{13} = 0$, $e_{33} = -e_{22}$. The metric becomes

$$ds^2 = -dt^2 + dx^2 + (1+h_{22})dy^2 + (1-h_{22})dz^2 + 2h_{23}dydz. \tag{24}$$

On the detector plane, it reduces to

$$ds^2 = -dt^2 + dx^2 + (1+h_{22})dy^2. \tag{25}$$

It can be detected by the second V-shaped and the L-shaped interferometers with the same signal

$$\left.\begin{array}{l} s_2 = s_/ - s_y \\ s = s_x - s_y \end{array}\right\} \approx -Lh_{22} \neq 0, \tag{26}$$

if the frequency of the gravitational wave is low enough. The first V-shaped interferometer has no response. The above results are independent of the sign in $k_1 = \pm k_0$.

In the second subcase,

$$e_{12} = -e_{11}\frac{k_x}{k_y},\ e_{22} = e_{11}\frac{(k_x)^2}{(k_y)^2},\ e_{23} = -e_{13}\frac{k_x}{k_y}.$$

The metric is

$$\begin{aligned} ds^2 =& -dt^2 + (1+h_{11})dx^2 + (1+h_{11}\frac{(k_x)^2}{(k_y)^2})dy^2 \\ &+ (1-h_{11}-h_{11}\frac{(k_x)^2}{(k_y)^2})dz^2 - 2h_{11}\frac{k_x}{k_y}dxdy \\ &+ 2h_{13}dxdz - 2h_{13}\frac{k_x}{k_y}dydz, \end{aligned} \tag{27}$$

satisfying the traceless condition. On the detector plane, it reduces to

$$\begin{aligned} ds^2 =& -dt^2 + (1+h_{11})dx^2 + (1+h_{11}\frac{(k_x)^2}{(k_y)^2})dy^2 \\ &- 2h_{11}\frac{k_x}{k_y}dxdy. \end{aligned} \tag{28}$$

It shows that the detector only responds one polarization state. The signals recorded by the detector are

$$\begin{aligned} s_1 &\approx Lh_{11}(1+\frac{k_x}{k_y}), \\ s_2 &\approx -Lh_{11}\frac{k_x}{k_y}(1+\frac{k_x}{k_y}), \\ s &\approx Lh_{11}(1-\frac{(k_x)^2}{(k_y)^2}), \end{aligned} \tag{29}$$

under the low-frequency approximation. Using $(k_x)^2 + (k_y)^2 = (k_0)^2$, we have

$$k_x = \pm\frac{s_2k_0}{\sqrt{(s_1)^2+(s_2)^2}},\ k_y = \mp\frac{s_1k_0}{\sqrt{(s_1)^2+(s_2)^2}}, \tag{30}$$

or equivalently

$$\cos\phi = \frac{\pm s_2}{\sqrt{(s_1)^2+(s_2)^2}},\ \sin\phi = \frac{\mp s_1}{\sqrt{(s_1)^2+(s_2)^2}}, \tag{31}$$

and

$$h_{11} \approx \frac{c}{L}\frac{s_1}{1+k_x/k_y} = \frac{c}{L}\frac{s_1^2}{s_1-s_2}. \tag{32}$$

In comparison, a usual L-shaped interferometer only records the last signal Eq. (28). It just sets up the relation between h_{11} and incident direction.

For $k_z \neq 0$, the TT condition gives rise to

$$e_{22} = -\frac{(k_x)^2+(k_z)^2}{(k_y)^2+(k_z)^2}e_{11} - \frac{2k_xk_y}{(k_y)^2+(k_z)^2}e_{12}. \tag{33}$$

Then, in the reference system of the detector, the met-

ric is

$$\begin{aligned} ds^2 =& -dt^2 + (1+h_{11})dx^2 \\ &+ \Big(1 - \frac{(k_x)^2+(k_z)^2}{(k_y)^2+(k_z)^2}h_{11} - \frac{2k_xk_y}{(k_y)^2+(k_z)^2}h_{12}\Big)dy^2 \\ &+ \Big(1 + \frac{(k_x)^2-(k_y)^2}{(k_y)^2+(k_z)^2}h_{11} + \frac{2k_xk_y}{(k_y)^2+(k_z)^2}h_{12}\Big)dz^2 \\ &+ 2h_{12}dxdy - \frac{2}{k_z}(k_xh_{11}+k_yh_{12})dxdz \\ &+ \frac{2}{k_z}\Big(\frac{(k_x)^2+(k_z)^2}{(k_y)^2+(k_z)^2}k_yh_{11} \\ &+ \frac{(k_y)^2-(k_z)^2}{(k_y)^2+(k_z)^2}k_xh_{12}\Big)dydz. \end{aligned} \tag{34}$$

Again, on the detector plane, it reduces to

$$\begin{aligned} ds^2 =& -dt^2 + (1+h_{11})dx^2 + 2h_{12}dxdy \\ &+ \Big(1 - \frac{(k_x)^2+(k_z)^2}{(k_y)^2+(k_z)^2}h_{11} \\ &- \frac{2k_xk_y}{(k_y)^2+(k_z)^2}h_{12}\Big)dy^2. \end{aligned} \tag{35}$$

Under the low-frequency approximation, the signals detected by the detector are

$$s_1 \approx L(h_{11}-h_{12}), \tag{36}$$

$$\begin{aligned} s_2 \approx& L\Big(\frac{(k_x)^2+(k_z)^2}{(k_y)^2+(k_z)^2}h_{11} \\ &+ \frac{2k_xk_y+(k_y)^2+(k_z)^2}{(k_y)^2+(k_z)^2}h_{12}\Big), \end{aligned} \tag{37}$$

$$\begin{aligned} s \approx& L\Big(\frac{(k_0)^2+(k_z)^2}{(k_y)^2+(k_z)^2}h_{11} + \frac{2k_xk_y}{(k_y)^2+(k_z)^2}h_{12}\Big) \\ =& L\frac{(2(k_0)^2-(k_x)^2-(k_y)^2)h_{11}+2k_xk_yh_{12}}{(k_0)^2-(k_x)^2}. \end{aligned} \tag{38}$$

The two independent polarization states of a gravitational wave can be determined as follows:

$$\begin{aligned} h_{11} \approx& \frac{1}{L}\frac{1}{2(k_0)^2-(k_x-k_y)^2} \\ &\cdot[((k_0)^2-(k_x)^2+2k_xk_y)s_1 + ((k_0)^2-(k_x)^2)s_2] \\ =& \frac{1}{L}\frac{2k_xk_ys_1+((k_0)^2-(k_x)^2)s}{2(k_0)^2-(k_x-k_y)^2} \\ =& \frac{1}{2L}\frac{s_1\sin^2\theta\sin(2\phi)+s(1-\sin^2\theta\cos^2\phi)}{1-\sin^2\theta\sin^2(\phi-\frac{\pi}{4})}, \end{aligned} \tag{39}$$

$$\begin{aligned} h_{12} \approx& \frac{1}{L}\frac{((k_y)^2-(k_0)^2)s_1+((k_0)^2-(k_x)^2)s_2}{2(k_0)^2-(k_x-k_y)^2} \\ =& \frac{1}{L}\frac{((k_x)^2+(k_y)^2-2(k_0)^2)s_1+((k_0)^2-(k_x)^2)s}{2(k_0)^2-(k_x-k_y)^2} \\ =& \frac{1}{2L}\frac{s_1(\sin^2\theta\sin^2\phi-1)+s_2(1-\sin^2\theta\cos^2\phi)}{1-\sin^2\theta\sin^2(\phi-\frac{\pi}{4})}. \end{aligned} \tag{40}$$

For a gravitational radiation of a binary, both h_{11} and h_{12} can be expressed in terms of h_+ and $h_\times$[6], where h_+ and $h_\times$ are two polarization states in the source-reference system, and h_+ and $h_\times$ are closely related by Eqs. (1) and (2). Hence, whether there are non-GR polarization states in a gravitational wave can be determined.

From Eqs. (38) and (39), one can obtain

$$\sin^2\theta = \frac{2Lh_{11}-s}{s_1\sin(2\phi)-s\cos^2\phi+2Lh_{11}\sin^2(\phi-\frac{\pi}{4})}, \tag{41}$$

$$\sin^2\theta = \frac{2Lh_{12}+2s_1-s}{s_1-s\cos^2\phi+2Lh_{12}\sin^2(\phi-\frac{\pi}{4})}. \tag{42}$$

Equating them gives the relation of h_{11}, h_{12} and ϕ. In contrast, a usual L-shaped interferometer only records the signal Eq. (37). It gives the relation of h_{11}, h_{12}, θ and ϕ. In this sense, the three-arm detector can improve the angular resolution significantly. In principle, two three-arm detectors at different sites can determine the position of sources. This is the third advantage of the three-arm detector over the usual L-shaped detector.

The three-arm MFP detector is also beneficial for searching for gravitational wave signals from raw data due to the fact that one may search coincidence signals on a single detector, also there is a mathematical constraint Eq. (11) among the three sets of data, and there is one less free parameter in the direction of sources. On the other hand, the multi-signals in one detector and Eq. (11) provide an opportunity to check the statistical methods used in data analysis.

In conclusion, a three-arm MFP detector is designed and suggested to replace the current L-shaped MFP interferometer for observing gravitational waves. In comparison with the L-shaped MFP interferometer, the modification is minimal. One arm, a bisector arm, is added. With the added arm, two new independent V-shaped interferometers with 45° opening angle are formed. There is no technical obstacle to building the V-shaped interferometers.

A new detector consists of a pair of V-shaped and L-shaped MFP interferometers. The redundant setting may provide a useful cross-check. It may give more information on gravitational waves than the same-sized same-designed L-shaped detector, including the polarization states of the gravitational waves and the direction of sources. It can set strong constraints on or even exclude the non-GR polarization states in gravitational radiation in GW150914-like events. Since only one free parameter for the position of the source in the sky is left, the position of the source can be determined if two new-type detectors are built at different sites. The new detector is also helpful for searching for gravitational wave signals more efficiently.

Finally, Eqs. (39) and (40) show that polarization states of a gravitational wave can be determined completely in GR when the direction of the source is known. In the realistic observations, however, the direction of the source is usually unknown. The two

equations seem insufficient to determine the two polarizations and two parameters of the position of the source in the sky. In fact, the parameters of the position of the source are constants for transient sources. The time-varying law of the two polarization states $h_+(t)$ and $h_\times(t)$ for the gravitational radiation of a given binary in its inspiral stage are known. The two polarization states, h_{11} and h_{12}, in the system of detector reference can always be expressed in terms of h_+ and $h_\times$. The coefficients only depend on the directions of the source. Therefore, the remaining unknown quantities are the initial values of $h_+(t_0)$ and $h_\times(t_0)$ and two directional parameters. On the other hand, Eqs. (39) and (40) are valid at every instance. If we mark the period of the observation by n ticks, then we have $2n$ equations to determine 4 unknown quantities. Therefore, the polarization states and the position of the source can be determined at the same time by the three-arm MFP detector with the help of filters provided by numerical relativity.

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