Abbott–Deser–Tekin Charge of Dilaton Black Holes with Squashed Horizons *

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We consider the conserved charge of static black holes with squashed horizons in the Einstein–Maxwell–dilaton theory via both the Abbott–Deser–Tekin (ADT) method and its off-shell generalization. We first make use of the original ADT method to compute the mass of the dilaton squashed black holes in terms of three different reference spacetimes, which are the asymptotic geometry, the flat background and the spacetime of the Kaluza–Klein monopole with boundary matched to the original metric, respectively. Each mass satisfies the first law of black hole thermodynamics, although the mass computed on the basis of the boundary matching the Kaluza–Klein monopole is different from that of the other two reference spacetimes. Then the mass of the black holes is evaluated through the off-shell generalized ADT method.

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In recent years, much attention has been paid to seeking higher-dimensional black hole solutions in various gravity theories motivated by string theory and brane-world scenario. Particularly in five dimensions, a nontrivial class of solutions, called the squashed Kaluza–Klein black holes, has attracted lots of interest since Ishihara *et al.* found a static charged black hole with squashed horizons in the five-dimensional Einstein–Maxwell theory.[1] Subsequently the Ishihara–Matsuno solution was generalized to the theory of five-dimensional Einstein–Maxwell–dilaton gravity.[2] By applying a so-called squashing transformation, the rotating Kaluza–Klein black holes with squashed horizons in the Einstein gravity[3] and the charged (rotating) squashed Kaluza–Klein black holes in five-dimensional supergravity[4–7] were found in succession. Compared with their un-squashed counterparts, all of the squashed Kaluza–Klein black holes have a squashed three-sphere as their horizon topology, and they possess the similar asymptotic geometry at spatial infinity, that is, a twisted S^1 bundle over the two-sphere in a four-dimensional Minkowski spacetime. On account of such an asymptotic geometry, the black holes with squashed horizons have numerous interesting properties.[9–13]

The particular asymptotic structure of the black holes with squashed horizons also makes them become a useful platform to test various approaches on conserved charges of gravity theories, such as the Komar formula, the Arnowitt–Deser–Misner (ADM) formalism, the Abbott–Deser–Tekin (ADT) method and the counter-term subtraction method.[2,3,14–20] Among them, the ADT method[21,22] is defined in terms of the Noether potential obtained through the linearized perturbation of the expression for the equation of motion in a fixed background of spacetime, which is required to satisfy the equation of motion in a vacuum. Due to this, the Noether potential in the ADT method is on-shell. Recently, in Refs. [23,24] the on-shell Noether potential in the ADT formalism was lifted to off-shell level on the basis of the linearized perturbation of the Noether current and potential in arbitrary background. In terms of the off-shell generalized potential, a new method, mentioned above as the off-shell generalized ADT method, was proposed. In comparison, the off-shell generalized method makes it more convenient to evaluate the ADT potential and it expands the range of application of the original ADT method since the contributions from matter fields are included.

As is well-known, the ADT method is background dependent. This means that one has to find proper reference backgrounds to obtain sensible conserved charges. In the previous studies,[2,3,14,15,18–20] the most popular reference background chosen to evaluate the conserved charges of black holes with squashed horizons is their asymptotic geometry. Apart from this reference spacetime, it is of great interest to find other backgrounds on which the original ADT method can produce meaningful mass. This is just the main goal of the present work. On the other hand, till now, the off-shell generalized ADT method has been extended to various spacetimes with different asymptotic structures,[25–28] while it has not been applied to spacetime with the same asymptotic topology as the black holes with squashed horizons. We expect to

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fill the gap.

In the present study, to check the background dependence of the original ADT method and the universality of its off-shell generalization, as well as to provide other novel approaches to derive the mass of static black holes with squashed horizons in the Einstein–Maxwell–dilaton theory,[2] we consider evaluating the mass of these black holes via both the methods although their mass was derived in terms of the counter-term subtraction method there. In the evaluation of the ADT mass, we adopt three different reference spacetimes, i.e., the asymptotic geometry, the flat background and the spacetime of the Kaluza–Klein monopole with boundary matched to the original metric.

Yazadjiev presented new charged solutions that describe static black holes with squashed horizons in the five-dimensional Einstein–Maxwell–dilaton gravity.[2] The line element, the dilaton field φ and the $U(1)$ gauge field A_μ for the black hole solutions take the forms

$$\begin{aligned}\frac{ds^2}{U^{\hat{\alpha}}} =& -\frac{f_+(r)}{f_+(r_\infty)}\frac{dt^2}{U^{3\hat{\alpha}}}+\frac{k^2(r)}{f_+(r)}dr^2\\ &+\frac{r^2}{4}[k(r)(d\theta^2+\sin^2\theta d\phi^2)\\ &+(d\psi+\cos\theta d\phi)^2],\\ e^{-2\alpha\varphi} =& U^{2(1-\hat{\alpha})},\\ A_\mu =& \frac{\sqrt{3}}{2}\frac{c\sqrt{\hat{\alpha}}}{s}\frac{U-1}{U}\delta^t_\mu,\end{aligned} \tag{1}$$

where $c=\cosh(q)$ and $s=\sinh(q)$. The parameter $\hat{\alpha}$ is defined in terms of the dilaton coupling parameter α through $\hat{\alpha}=4/(4+3\alpha^2)$, and both the parameters r_+ and r_∞ are assumed such that $0<r_+<r_\infty$. Moreover, in Eq. (1), the Euler angles $(\theta,\ \phi,\ \psi)$ take the values $(0<\theta<\pi,\ 0<\phi<2\pi,\ 0<\psi<4\pi)$, the radial coordinate r varies in the range of $0<r<r_\infty$, and the four functions $(f_+(r),\ f_\infty(r),\ k(r),\ U)$ are given by

$$\begin{aligned}f_+(r) =& 1-\frac{r_+^2}{r^2},\quad f_\infty(r)=1-\frac{r_\infty^2}{r^2},\\ k(r) =& \frac{r_\infty^4}{r^4}\frac{f_+(r_\infty)}{f_\infty^2(r)},\quad U=1+s^2\frac{f_\infty(r)}{f_\infty(r_+)}.\end{aligned} \tag{2}$$

The Lagrangian for the solution to Eq. (1) is

$$L=\sqrt{-g}(R-2\nabla^\mu\varphi\nabla_\mu\varphi-e^{-2\alpha\varphi}F^{\mu\nu}F_{\mu\nu}). \tag{3}$$

When $r_\infty\to\infty$, the solution (1) becomes the five-dimensional static dilaton black hole solution with $SO(4)$ symmetry. When $\alpha=0$ and $\varphi=0$, this solution reduces to the charged solution describing a static squashed Kaluza–Klein black hole found in Ref. [1] whose mass and thermodynamics have been investigated from various perspectives.[14–17] The slowly rotating generalization of this solution was presented in Ref. [8].

The spatial infinity of the black hole solution (1) lives at $r=r_\infty$. To see this clearly, two new positive parameters $(\rho_0,\ \rho_+)$ and a new radial coordinate ρ are introduced, which are

$$\begin{aligned}r_+^2 =& 4\rho_0\rho_+,\quad r_\infty^2=4\rho_0(\rho_0+\rho_+),\\ \rho =& -\rho_0/f_\infty(r).\end{aligned} \tag{4}$$

Due to the above new parameters and radial coordinate, as well as the function U_* defined by

$$U_*=1+s^2\rho_+/\rho, \tag{5}$$

the solution (1) is reformed as

$$\begin{aligned}\frac{ds^2}{U_*^{\hat{\alpha}}} =& -\frac{\rho-\rho_+}{\rho}\frac{dt^2}{U_*^{3\hat{\alpha}}}+\frac{\rho+\rho_0}{\rho-\rho_+}d\rho^2\\ &+\rho(\rho+\rho_0)(d\theta^2+\sin^2\theta d\phi^2)\\ &+\rho_0\rho\frac{\rho_0+\rho_+}{\rho_0+\rho}(d\psi+\cos\theta d\phi)^2,\\ e^{-2\alpha\varphi} =& U_*^{2(1-\hat{\alpha})},\\ A_\mu =& \frac{\sqrt{3\hat{\alpha}}}{2}\frac{cs\rho_+}{\rho+s^2\rho_+}\delta^t_\mu.\end{aligned} \tag{6}$$

Taking the limit $\rho\to\infty$, that is, $r\to r_\infty$, we obtain the asymptotic metric

$$\begin{aligned}ds^2 =& -dt^2+d\rho^2+\rho^2(d\theta^2+\sin^2\theta d\phi^2)\\ &+\frac{r_\infty^2}{4}(d\psi+\cos\theta d\phi)^2,\end{aligned} \tag{7}$$

which shows that the asymptotic geometry of the black hole solution (1) is a twisted S^1 bundle over a four-dimensional Minkowski spacetime, completely matching that of the charged squashed Kaluza–Klein black hole in Ref. [1].

After a brief review of the dilaton black holes with squashed horizons, we pay attention to calculating their mass in terms of the ADT method.[21,22] Since the ADT method is background dependent, it is of great importance to find an appropriate reference spacetime to obtain sensible conserved charges. As carried out to the black holes with squashed horizons,[2,3,14,15,18–20] a natural reference spacetime is the asymptotic geometry of the squashed black holes. Accordingly, we first adopt the asymptotic metric (7) as the background metric $\bar{g}_{\mu\nu}$ of the reference spacetime to compute the mass of the squashed black hole solution (6). According to the ADT method, the conserved charges related to the theory of five-dimensional Einstein–Maxwell–dilaton gravity are defined by

$$\mathcal{Q}=\frac{1}{96\pi}\int_{\partial\Sigma}\sqrt{-\bar{g}}Q^{\mu\nu}_{\rm ADT}\epsilon_{\mu\nu\mu_1\mu_2\mu_3}dx^{\mu_1}\wedge dx^{\mu_2}\wedge dx^{\mu_3}, \tag{8}$$

where the ADT potential $Q^{\mu\nu}_{\rm ADT}$ is given by

$$Q^{\mu\nu}_{\rm ADT} = \xi_\sigma \bar{\nabla}^{[\mu} h^{\nu]\sigma} - h^{\sigma[\mu}\bar{\nabla}_\sigma \xi^{\nu]} + \frac{1}{2} h \bar{\nabla}^{[\mu}\xi^{\nu]} - \xi^{[\mu}\bar{\nabla}_\sigma h^{\nu]\sigma} + \xi^{[\mu}\bar{\nabla}^{\nu]} h \tag{9}$$

in terms of a Killing vector ξ^μ and the divergence $h_{\mu\nu}$ between the original metric $g_{\mu\nu}$ and the background metric $\bar{g}_{\mu\nu}$, that is, $h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$. Letting $\xi^\mu = -\delta^\mu_t$ and $g_{\mu\nu}$ be given by Eq. (6), we obtain the mass of the squashed black hole

$$M_{\rm ADT} = \pi\sqrt{\rho_0(\rho_0+\rho_+)}(\rho_0 + 2\rho_+ + 3\hat{\alpha}\rho_+ s^2) = \frac{\pi r_\infty}{4(r_\infty^2 - r_+^2)^{1/2}}(r_\infty^2 + r_+^2 + 3\hat{\alpha}s^2 r_+^2), \tag{10}$$

which coincides with the mass via the counter-term method in Ref. [2] and satisfies the differential and integral first law of black hole thermodynamics, that is,

$$dM_{\rm ADT} = TdS + \Phi dQ + \mathcal{I}d\mathcal{L}, \quad 2M_{\rm ADT} = 3TS + 2\Phi Q + \mathcal{I}\mathcal{L}, \tag{11}$$

where the Hawking temperature T, the entropy S, the gauge potential evaluated on the horizon Φ and the electric charge Q are

$$T = \frac{\sqrt{\rho_0\rho_+}}{2\pi c^{3\hat{\alpha}} r_\infty \rho_+}, \quad S = \pi^2 c^{3\hat{\alpha}} r_\infty^2 \rho_+ \sqrt{\frac{\rho_+}{\rho_0}}, \quad \Phi = s\sqrt{3\hat{\alpha}}/(2c), \quad Q = \pi c s r_\infty \rho_+ \sqrt{3\hat{\alpha}}. \tag{12}$$

The size of the S^1 fiber at infinity $\mathcal{L} = 2\pi r_\infty$ and the gravitational tension of the squashed black hole solution (6) along the ψ direction is given by[15]

$$\mathcal{I} = \frac{1}{4}(2\rho_0 + \rho_+), \tag{13}$$

whose contribution to the first law of thermodynamics for the squashed black hole cannot be ignored, otherwise the integral form of the first law does not hold as in Ref. [2].

Strictly speaking, the original ADT method requires that the background metric $\bar{g}_{\mu\nu}$ satisfies the equation of motion in vacuum. Obviously, the asymptotic metric (7) is not an exact solution of the five-dimensional vacuum Einstein equations. However, the off-shell generalized ADT method[23,24] demonstrates that both the original and the off-shell generalized ADT potentials are equivalent if the contributions from the matter fields are ignored (for the Lagrangian (3) of the Einstein–Maxwell–dilaton gravity, when we ignore the contributions from the dilaton field φ and the gauge field A_μ, the off-shell generalized ADT potential $\mathcal{Q}^{\mu\nu}_{\rm ADT}$ is defined by $\nabla_\nu \mathcal{Q}^{\mu\nu}_{\rm ADT} = \mathcal{J}^\mu = \delta G^{\mu\nu}\xi_\nu + \frac{1}{2}g^{\alpha\beta}\delta g_{\alpha\beta}G^{\mu\nu}\xi_\nu - \frac{1}{2}\xi^\mu G^{\alpha\beta}\delta g_{\alpha\beta} + G^{\mu\nu}\delta g_{\nu\sigma}\xi^\sigma$ and the original ADT potential $Q^{\mu\nu}_{\rm ADT}$ is defined by $\nabla_\nu Q^{\mu\nu}_{\rm ADT} = \mathcal{J}^\mu(\delta g_{\alpha\beta} \to h_{\alpha\beta})$. The difference between them is the form for the fluctuation of metric tensor). To this point, it is unnecessary to declare whether the background metric satisfies the equation of motion in a vacuum.

Next, we compute the mass of the dilaton black hole with squashed horizons in terms of another background metric

$$ds^2 = -dt^2 + d\rho^2 + \rho^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{r_\infty^2}{4}d\psi^2, \tag{14}$$

which satisfies the five-dimensional Einstein equations in a vacuum, that is, $R_{\mu\nu} = 0$. The metric (14), first proposed as an appropriate reference spacetime to evaluate the mass of the static squashed Kaluza–Klein black hole in Ref. [15] has the topology of a product S^1 over four-dimensional Minkowski spacetime, which is different from the asymptotic metric (7). For convenience, this reference spacetime is mentioned as the flat background like in Ref. [15]. Based on the background metric (14), we obtain the ADT mass $\tilde{M}_{\rm ADT}$ that is the same as $M_{\rm ADT}$ in Eq. (10), that is,

$$\tilde{M}_{\rm ADT} = M_{\rm ADT}. \tag{15}$$

In addition to the reference spacetimes of Eqs. (7) and (14), the five-dimensional Kaluza–Klein monopole spacetime, whose boundary is assumed to match with that of the squashed black hole solution (6), can be proposed as a sensible background. The metric of the monopole takes the form

$$ds^2 = -dt^2 + \Big(1 + \frac{r_\infty}{2\rho}\Big)d\rho^2 + \rho\Big(\rho + \frac{r_\infty}{2}\Big)(d\theta^2 + \sin^2\theta d\phi^2) + \frac{r_\infty^2}{2}\frac{\rho}{2\rho + r_\infty}(d\psi + \cos\theta d\phi)^2, \tag{16}$$

which is an exact solution of the five-dimensional vacuum Einstein equations and is just the special case of the solution to Eq. (6) in the limit $\rho_+ = 0$. The metric (16) was first chosen as a meaningful reference spacetime to investigate the thermodynamics of the static squashed Kaluza–Klein black hole in Ref. [15]. Taking the asymptotic metric (7) or the flat background Eq. (14) as the reference spacetime of the Kaluza–Klein monopole, we obtain its ADT mass

$$M_{\rm mon} = \pi\frac{r_\infty^2}{4} = \pi\rho_0(\rho_0 + \rho_+). \tag{17}$$

Moreover, choosing the monopole spacetime of Eq. (16) as the reference background of the black hole solution (6), we can compute its ADT mass as

$$\mathcal{M}_{\rm ADT} = M_{\rm ADT} - M_{\rm mon} = \frac{\pi r_\infty}{4(r_\infty^2 - r_+^2)^{1/2}}(r_\infty^2 + r_+^2 + 3\hat{\alpha}s^2 r_+^2) - \pi\frac{r_\infty^2}{4}, \tag{18}$$

where the mass $\mathcal{M}_{\rm ADT}$ still satisfies the first law of black hole thermodynamics in Eq. (11) since the value of the parameter r_∞ is fixed to obtain the first law. In this sense, the mass $\mathcal{M}_{\rm ADT}$ is meaningful. It is worth noting that the monopole spacetime that takes the form as the metric (16) after setting $r_\infty \to 2\rho_0$ was chosen as the background metric to calculate the ADT mass of the five-dimensional static squashed Kaluza–Klein black hole in Ref. [14]. However, the boundary of the monopole spacetime there is not matched to that of the original metric, thus the ADT mass does not satisfy the first law.

Let us pause to make comments on the mass $M_{\rm ADT}$ and $\mathcal{M}_{\rm ADT}$. The former grows when the parameter r_∞ increases. Particularly, in the limit $r_\infty \to \infty$, it goes to infinity. However, in this limit, the latter approaches to $3\pi(c^2+s^2)r_+^2/8$, which is just the mass of the five-dimensional static dilaton black hole. Moreover, when $r_+ = 0$, the mass $M_{\rm ADT}$ becomes that of the Kaluza–Klein monopole in Eq. (17), while $\mathcal{M}_{\rm ADT}$ vanishes. The characteristic of the mass $\mathcal{M}_{\rm ADT}$ is consistent with that of the 'well-defined' mass suggested in Ref. [3], where they conjectured that the well-defined mass of the static black holes with squashed horizons is the one subtracting the monopole contribution Eq. (17). In this sense, the mass $\mathcal{M}_{\rm ADT}$ can be proposed as the well-defined mass of the static squashed black hole solution (6) and the boundary matched Kaluza–Klein monopole Eq. (16) is just the meaningful reference background one expects.

In the above, the mass of the dilaton black holes with squashed horizons is derived via the ADT method in terms of three different reference spacetimes. To compare and to check the universality of the off-shell generalized ADT method in Refs. [23,24], we evaluate the mass of the black holes through this method. Unlike the potential defined in the original ADT method, the off-shell generalized one includes the contributions from all the matter fields and does not require that the background metric must satisfy the equations of motion in vacuum. For the theory of Einstein–Maxwell–dilaton gravity with the Lagrangian (3), the off-shell ADT potential is presented as[26]

$$\begin{aligned}\tilde{\mathcal{Q}}^{\mu\nu}_{\rm ADT} &= Q^{\mu\nu}_{\rm ADT}(h_{\alpha\beta}\to\delta g_{\alpha\beta}) + Q^{\mu\nu}_{matt},\\ Q^{\mu\nu}_{matt} &= e^{-2\alpha\varphi}[\xi^\sigma A_\sigma(hF^{\mu\nu} + 4h^{\rho[\mu}F_\rho^{\nu]}\\ &\quad - 4g^{\rho[\mu}\partial^{\nu]}\delta A_\rho) + 6\xi^{[\sigma}F^{\mu\nu]}\delta A_\sigma]\\ &\quad - 4(\alpha e^{-2\alpha\varphi}\xi^\sigma A_\sigma F^{\mu\nu} + \xi^{[\mu}\partial^{\nu]}\varphi)\delta\varphi, \end{aligned} \tag{19}$$

where the Noether potential $Q^{\mu\nu}_{matt}$ is the contribution from the gauge field A_μ and the dilaton field φ. More details on the potential $\tilde{\mathcal{Q}}^{\mu\nu}_{\rm ADT}$ and its comparison with that in other methods can be found in Ref. [26].

Due to the covariance of the off-shell generalized ADT method, we can directly compute the mass of the dilaton squashed black hole solution (1) in the coordinate system $(t, r, \theta, \phi, \psi)$ without a further coordinate transformation to the radial coordinate r. The fluctuation of the metric, the gauge field and the dilaton field is determined by keeping r_∞ fixed (the parameter r_∞ cannot vary, otherwise the conserved charge is not integrable) and varying the parameters (r_+, q) as $r_+ \to r_+ + dr_+$ and $q \to q + dq$. Substituting the off-shell ADT potential $\tilde{\mathcal{Q}}^{\mu\nu}_{\rm ADT}$ into Eq. (8) and taking the limit $r \to r_\infty$, we can calculate the mass $\tilde{\mathcal{M}}_{\rm ADT}$ as

$$\begin{aligned} d\tilde{\mathcal{M}}_{\rm ADT} = \frac{\pi r_+ r_\infty}{4(r_\infty^2 - r_+^2)^{3/2}}&[(3r_\infty^2 - r_+^2 + 3\hat{\alpha}(r_+^2\\ &- 2r_\infty^2) + 3\hat{\alpha}c^2(2r_\infty^2 - r_+^2))dr_+\\ &+ 6cs\hat{\alpha}r_+(r_\infty^2 - r_+^2)dq]. \end{aligned} \tag{20}$$

The integral of the above equation yields the mass

$$\tilde{\mathcal{M}}_{\rm ADT} = \mathcal{M}_{\rm ADT}. \tag{21}$$

The equivalence between the mass $\tilde{\mathcal{M}}_{\rm ADT}$ and $\mathcal{M}_{\rm ADT}$ is reasonable. On the one hand, due to the off-shell generalized ADT method,[23,24] the manner of parameterizing the space of solutions indicates that there exists a 'hidden' reference background that coincides with the Kaluza–Klein monopole in Eq. (16). On the other hand, when $r \to r_\infty$, the component $Q^{\rm tr}_{\rm matt}$, which is the potential related to both the dilaton field and the gauge field, falls quite quickly, thus it makes no contribution to the total mass. Compared with the original ADT method, the merits of the off-shell generalized one are that it is unnecessary to perform coordinate transformation to the radial coordinate and one does not need to present a reference background in advance.

In summary, we have evaluated the mass of the static dilaton black holes with squashed horizons using the ADT method on three different reference backgrounds, which are the asymptotic geometry Eq. (7), the flat background Eq. (14) and the spacetime of the five-dimensional Kaluza–Klein monopole Eq. (16) with boundary matching to the original metric (6), respectively. The mass computed on the asymptotic geometry equals that on the flat background, while it is different from the one on the boundary matched Kaluza–Klein monopole. Each mass satisfies the first law of black hole thermodynamics. This indicates that all three reference backgrounds will be meaningful if one only considers the calculation of the mass. To obtain further understanding of the three reference backgrounds, thermodynamics of the squashed black hole solution (6) should be investigated in different methods as in Ref. [15]. Moreover, the mass of the black holes is directly evaluated through the off-shell ADT method in the coordinate system with the coordinates $(t, r, \theta, \phi, \psi)$. This coincides with the mass calculated on the boundary matched Kaluza–Klein monopole via

the original ADT method. Since the black hole solution (6) reduces to the charged squashed Kaluza–Klein black hole in Ref. [1] within a special case, our results recover all the ones of the latter.

It is worth noting that the ADT method fails to produce the meaningful mass of the (charged) rotating black holes with squashed horizons,[3–6,8] when one takes the five-dimensional Kaluza–Klein monopole Eq. (16) as their reference background. Furthermore, when the off-shell generalized ADT method is naively applied to the (charged) rotating black holes with squashed horizons, we can find that their conserved charges are not integrable. To overcome this problem, we will consider the conserved charges of these black holes in our future work.

References

[1] Ishihara H and Matsuno K 2006 *Prog. Theor. Phys.* **116** 417
[2] Yazadjiev S S 2006 *Phys. Rev.* D **74** 024022
[3] Wang T 2006 *Nucl. Phys.* B **756** 86
[4] Nakagawa T, Ishihara H, Matsuno K and Tomizawa S 2008 *Phys. Rev.* D **77** 044040
[5] Tomizawa S, Ishihara H, Matsuno K and Nakagawa T 2009 *Prog. Theor. Phys.* **121** 823
[6] Tomizawa S and Ishibashi A 2008 *Class. Quantum Grav.* **25** 245007
[7] Tomizawa S 2010 arXiv:1009.3568[hep-th]
[8] Allahverdizadeh M and Matsuno K 2010 *Phys. Rev.* D **81** 044001
[9] Ji L Y, Chen S B and Jing J L 2014 *J. High Energy Phys.* **1403** 089
Chen S B and Jing J L 2011 *Phys. Lett.* B **704** 641
Sadeghi J, Naji J and Vaez H 2014 *Phys. Lett.* B **728** 170
Matsuno K and Ishihara H 2009 *Phys. Rev.* D **80** 104037
[10] Nishikawa R and Kimura M 2010 *Class. Quantum Grav.* **27** 215020
Kimura M, Murata K, Ishihara H and Soda J 2008 *Phys. Rev.* D **77** 064015
[11] Tomizawa S 2010 *Phys. Rev.* D **82** 104047
[12] He X, Wang B and Chen S B 2009 *Phys. Rev.* D **79** 084005
He X, Chen S B, Wang B, Cai R G and Lin C Y 2008 *Phys. Lett.* B **665** 392
[13] Stetsko M M 2016 *Eur. Phys. J.* C **76** 48
Li H L 2011 *Chin. Phys.* B **20** 030402
Matsuno K and Umetsu K 2011 *Phys. Rev.* D **83** 064016
Wei S W, Li R, Liu Y X and Ren J R 2010 *Eur. Phys. J.* C **65** 281
Chen S B, Wang B and Su R K 2008 *Phys. Rev.* D **77** 024039
Ishihara H and Soda J 2007 *Phys. Rev.* D **76** 064022
[14] Cai R G, Cao L M and Ohta N 2006 *Phys. Lett.* B **639** 354
[15] Kurita Y and Ishihara H 2008 *Class. Quantum Grav.* **25** 085006
[16] Kurita Y and Ishihara H 2007 *Class. Quantum Grav.* **24** 4525
[17] Saito M, Noda H and Tashiro T 2009 *Int. J. Mod. Phys.* A **24** 2357
[18] Peng J J and Wu S Q 2010 *Nucl. Phys.* B **828** 273
[19] Stelea C, Schleich K and Witt D 2008 *Phys. Rev.* D **78** 124006
[20] Wu S Q, Wen D, Jiang Q Q and Yang S Z 2013 *Phys. Lett.* B **726** 404
[21] Abbott L F and Deser S 1982 *Nucl. Phys.* B **195** 76
Abbott L F and Deser S 1982 *Phys. Lett.* B **116** 259
[22] Deser S and Tekin B 2002 *Phys. Rev. Lett.* **89** 101101
Deser S and Tekin B 2003 *Phys. Rev.* D **67** 084009
[23] Kim W, Kulkarni S and Yi S H 2013 *Phys. Rev. Lett.* **111** 081101
[24] Hyun S, Jeong J, Park S A and Yi S H 2014 *Phys. Rev.* D **90** 104016
[25] Peng J J 2016 *Phys. Lett.* B **752** 191
Peng J J 2014 *Eur. Phys. J.* C **74** 3156
[26] Peng J J 2016 *Int. J. Mod. Phys.* A **31** 1650060
[27] Ahn B, Hyun S, Park S A and Yi S H 2016 *Phys. Rev.* D **93** 024041
[28] Hyun S, Park S A and Yi S H 2016 arXiv:1603.02530[hep-th]