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Time-Modulated Hamiltonian for Interpreting Delayed-Choice Experiments via Mach–Zehnder Interferometers *

Zhi-Yuan Li(李志远)[1,2]**

[1]Laboratory of Optical Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190
[2]College of Physics and Optoelectronics, South China University of Technology, Guangzhou 510641

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Many delayed-choice experiments based on Mach–Zehnder interferometers (MZI) have been considered and made to address the fundamental problem of wave-particle duality. Conventional wisdom long holds that by inserting or removing the second beam splitter (BS2) in a controllable way, microscopic particles (photons, electrons, etc.) transporting within the MZI can lie in the quantum superposition of the wave and particle state as $\psi = a_{\rm w}\psi_{\rm wave} + a_{\rm p}\psi_{\rm particle}$. *Here we present an alternative interpretation to these delayed-choice experiments. We notice that as the BS2 is purely classical, the inserting and removing operation of the BS2 imposes a time-modulated Hamiltonian* $H_{\rm mod}(t) = a(t)H_{\rm in} + b(t)H_{\rm out}$, *instead of a quantum superposition of* $H_{\rm in}$ *and* $H_{\rm out}$ *as* $H = a_{\rm w}H_{\rm in} + a_{\rm p}H_{\rm out}$, *to act upon the incident wave function. Solution of this quantum scattering problem, rather than the long held quantum eigen-problem yields a synchronically time-modulated output wave function as* $\psi_{\rm mod}(t) = a(t)\psi_{\rm wave} + b(t)\psi_{\rm particle}$, *instead of the stationary quantum superposition state* $\psi = a_{\rm w}\psi_{\rm wave} + a_{\rm p}\psi_{\rm particle}$. *As a result, the probability of particle output from the MZI behaves as if they are in the superposition of the wave and particle state when many events over time accumulation are counted and averaged. We expect that these elementary but insightful analyses will shed a new light on exploring basic physics beyond the long-held wisdom of wave-particle duality and the principle of complementarity.*

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The interpretation of quantum theory, a conceptual foundation problem in physics, has been an issue ever since its founding nearly 100 years ago,[1–11] although the operational power of quantum mechanics to solve practical problems in the microscopic world is of no dispute. This conceptual foundation has raised extensive hot controversies in history, e.g., between Einstein and Bohr,[1–3,5] and is still attracting much attention in current days and raising intensive disputes and discussions. The wave-particle duality of quantum objects, and more generally, the principle of complementarity, stands on the central conceptual core of quantum theory.

In history, numerous studies, either theoretical or experimental, have been made to test the wave-particle duality aiming at gaining deeper understanding of the conceptual foundation of quantum mechanics.[12] Most experiments favored the orthodox Copenhagen interpretation that one can never observe simultaneously the wave and particle behavior of quantum objects using a single set of experimental apparatus, even in theory and in principle. In this study we revisit this historical problem by analyzing quantitatively delayed-choice experiments in the framework of orthodox quantum mechanics operation formulation.

A popular instrument to demonstrate and analyze the puzzling feature of wave–particle duality is the Mach–Zehnder interferometer (MZI). A standard form of MZI is schematically illustrated in Fig. 1. It involves several key components, including the first 50:50 beam splitter (BS1), two mirrors (M), a phase shift plate introducing phase difference ϕ between the two arms (paths X and Y), and the second 50:50 beam splitter (BS2). In the presence of both BS1 and BS2, this MZI can perfectly demonstrate the wave nature of quantum particles. When a high-coherence particle beam is sent into this MZI, the signal intensity measured by the detectors x and y in this MZI exhibits a perfect cosine function of the phase difference ϕ with a well-defined, in principle infinitely fine signal peak-to-valley contrast, and the signal is given by $P_{y(x)}(\phi) = |\psi_0|^2[1 \pm \cos\phi]/2$. The presence of the perfect interference fringe curve is an ideal representation of the wave nature of the quantum particle.

On the other hand, if one removes BS2 from the setup, the path of the particle can be unambiguously determined by looking at the signal recorded by detectors x and y in this new MZI, as schematically illustrated in Fig. 2. Yet, the price is that now one completely loses the capability to observe the wave nature of the particle due to the fact that the interference fringe with respect to the phase difference ϕ is 100% smeared out and leaves a constant line in both detectors x and y. Mathematically the signal is $P_{y(x)}(\phi) = |\psi_0|^2/2$. Obviously both MZIs cannot simultaneously give the wave and particle nature of the quantum object, instead they only give either one or the other nature (i.e., wave or particle).

The desire of physicists to fully uncover the secret behind the motion of the quantum particle ignites flashes of wisdom from time to time. The delayed-choice thought experiments as first proposed by Wheeler in the late 1970s are among such kinds of prominent examples.[13–18] A classic example of delayed-choice experiment is illustrated in Fig. 3 based on the MZI. The BS2 is controlled by external operation to either go into the interferometer, called the

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**Corresponding author. Email: lizy@aphy.iphy.ac.cn
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IN state, or come out of the interferometer, called the OUT state. However, the classical wave-particle duality firmly stands in these smart thought or practical experiments. The wave and particle nature of quantum particles are still exclusively repulsive to each other and nobody is able to observe simultaneously the wave and particle nature of quantum particles. More recently, several works have moved one further step and gone into the regime of so-called quantum delayed-choice experiments by replacing the classical BS2 with a quantum BS2 device that can lie in the quantum superposition state of the IN and OUT state.[19–25] Notice that even in these MZI experiments, the composite optical devices, including the beam splitter, the half-wave plate, and others, are still classical, due to the fact that their size and mass are far larger than a quantum particle. Yet, their interaction and modulation to the wave function of photons can be quantum mechanical. Theoretical and experimental results show that the output of the MZI exhibits the coexistence (or mixture) of partial wave nature and partial particle nature of quantum particles.

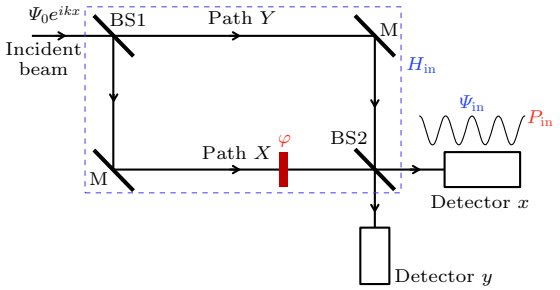


Fig. 1. Schematic setup based on the Mach–Zehnder two-arm interferometer used to probe the wave behavior of quantum particles. The blue dotted rectangular box consists of two beam splitters (BS1 and BS2), two mirrors (M), and a phase delay (ϕ). The quantum scattering system is described by the Hamiltonian $H_{\rm in}$, which works upon the incident wave function $\psi_0 e^{ikx}$ and gives rise to the output wave function $\psi_{\rm in}$. The signal recorded by the detector x varies periodically with respect to the phase delay, remarking the wave nature of the quantum particle.

It is worth noting that although the problems in Figs. 1–3 are far from complicated, a thorough and systematic quantum-mechanical analysis of them has been very rare in history, perhaps due to the fact that they seem to be too simple and thus a simple conceptual argument is sufficient to find the correct answer. Here we follow an alternative path to make an elementary but insightful analysis of these MZI experimental schemes used to test the wave–particle duality, hoping to shed a new light on this old problem. Our analysis is strictly based on the formalism of classical quantum mechanics. We find that quantum mechanics of the current MZI problem is a quantum scattering problem, rather than the usual quantum mechanical eigen-problem that most of the literature have exclusively adopted until now. This means that the output physical quantity measured and analyzed by detectors is the quantum scattering amplitude or intensity of a particle beam passing through the MZI, a quantum device. This output should closely rely on the quantum state of the input beam, and can be calculated precisely if strict quantum mechanical analysis and solution are made. Notice that the principle of the MZ in Figs. 1–3 not only works for photons, as most theoretical and experimental studies have involved, but also works for massive particles such as atoms, although the involved composite devices and technologies are quite different. Therefore, the principle of our following quantum mechanical analyses applies to both massless photons and massive particles. Nonetheless, we focus on photons as in most studies.

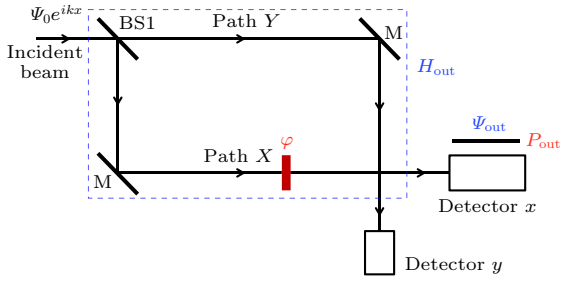


Fig. 2. Schematic setup based on the Mach–Zehnder two-arm interferometer used to probe the particle behavior of quantum particles. The blue dotted rectangular box consists of one beam splitter (BS1) but without the second splitter BS2, two mirrors (M), and a phase delay (ϕ). The quantum scattering system is described by the Hamiltonian $H_{\rm out}$, which works upon the incident wave function $\psi_0 e^{ikx}$ and gives rise to the output wave function $\psi_{\rm out}$. The signal recorded by the detector x determines which-path information of the quantum particle, and is a constant with respect to the phase delay.

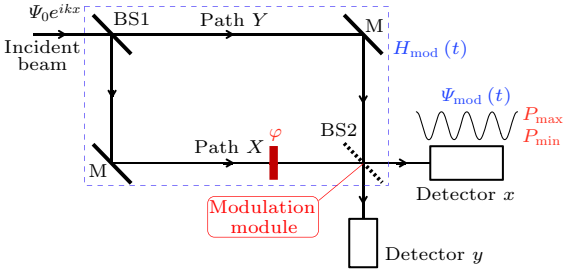


Fig. 3. Schematic setup of delayed-choice experiment based on the Mach–Zehnder two-arm interferometer. The blue dotted rectangular box consists of the first beam splitter BS1, two mirrors (M), a phase delay (ϕ), and the second beam splitter BS2 connected with modulation module to control its insertion into and removal from the path. The quantum scattering system is described by the time-dependent Hamiltonian $H_{\rm mod}(t)$, which works upon the incident wave function $\psi_0 e^{ikx}$ and gives rise to the output time-dependent wave function $\psi_{\rm mod}(t)$. The signal recorded by the detector x varies in time with a complicated behavior that can be described by the superposition of the wave and particle behavior of the quantum particle examined in Figs. 1 and 2, respectively.

The quantum mechanics for the MZI satisfies the following Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(R,t) = H(R,t)\,\Psi(R,t), \tag{1}$$

where $H(R,t)$ is the Hamiltonian of the MZI, which should rely on the specific geometric and physical configuration (denoted as R) of all the composite devices of the MZI as enclosed within the dashed rectangular box in Figs. 1–3, including the two mirrors, two beam splitters, and the phase plate. In most MZI experiments one can straightforwardly write down the

explicit form of this Hamiltonian, and usually it is an elastic scattering Hamiltonian where the energy E of the quantum particle is conserved. Here $\Psi(R,t)$ is the wave function of the quantum particle that transports within the MZI. Since this is a quantum scattering problem, the determination of $\Psi(R,t)$ should closely rely on the boundary condition (at R_∞) as well as the initial condition (at t_0), which is written as the input wave function $\Psi_{\rm input}$ at the entrance port of the MZI. At the exit port of the MZI, the wave function is $\Psi_{\rm output}$, which is the quantity that the detectors x and y probe and analyze for testing the wave-particle duality.

Given the explicit form of both $H(R,t)$ and $\Psi_{\rm input}$, the unknown wave function $\Psi_{\rm output}$ can be readily calculated. For photons, the analysis is about electromagnetic wave (representing photons). The methodology of analysis has long been well established and can be found in classical optics textbooks. It eventually traces back to the solution of Maxwell's equations for photon transporting through all the optical devices of the MZI, which are purely classical in nature. For massive particles such as electrons one needs to solve the Schrödinger equation for these particles transporting through all the composite devices of MZI. In essence, one can readily follow the path of the quantum particle and deal with the transmission and reflection of wave function at each device one-by-one.

Now that the general framework of analyzing the quantum mechanical problem of MZI has been clarified, we proceed to see what happens in Figs. 1–3. The MZI setup used to illustrate the wave nature of a quantum particle, with the BS2 present, has already been depicted in Fig. 1. The Hamiltonian of the setup is denoted as $H_{\rm in}(R)$, where the subscript IN means the IN state of BS2. Due to the fact that this Hamiltonian is time independent (or stationary), the scattering problem has a stationary solution, $\Psi_{\rm in}(R,t) = \Psi_{\rm in}(R)e^{-iEt/\hbar}$, where E is the energy (more precisely the kinetic energy) of the particle. The signal intensity is $P_{\rm in} = |\Psi_{\rm in}(R,t)|^2 = |\Psi_{\rm in}(R)|^2$ and also time independent.

Assume that the incident particle beam is described by a wave function $\Psi_{\rm input} = \psi(\boldsymbol{r},x) = \psi_0(\boldsymbol{r})e^{ikx}$, where $\boldsymbol{r}$ and x denote the transverse and longitudinal space coordinates of the particle beam, respectively. Thus $\psi_0(\boldsymbol{r})$ and e^{ikx} represent the transverse and longitudinal freedoms of the particle beam wave function. In many experiments, $\psi_0(\boldsymbol{r}) = \psi_0$ is a constant, thus $\Psi_{\rm input}$ is a plane wave transporting along the x-axis, i.e., the horizontal direction. A simple classical-optics-like calculation shows that the solution of this MZI quantum mechanical scattering problem at the x output port is

$$\Psi_{{\rm output},x} = \Psi_{{\rm in},x} = \psi_0(1-e^{i\phi})e^{ikx}/2. \tag{2}$$

The corresponding signal intensity is

$$P_{\rm in} = |\Psi_{{\rm in},x}|^2 = |\psi_0|^2[1-\cos\phi]/2 = P_0[1-\cos\phi], \tag{3}$$

where $P_0 = |\psi_0|^2/2$ is the signal intensity at each path. As illustrated in Fig. 1, the signal shows periodic oscillation with respect to the change of phase shift ϕ, which is a perfect interference pattern with the fringe visibility $V^2 = 1$.[26] One can judge from this feature that the quantum particle behaves completely as a wave, and thus the corresponding quantum state can be designated as $\Psi_{\rm in} = \psi_{\rm wave}$. Yet, the path information of the particle is completely lost now with the path distinguishability being $D^2 = 0$.[26]

Now let us look at Fig. 2, where BS2 is removed from the MZI. The corresponding time-independent Hamiltonian and wave function are denoted as $H_{\rm out}(R)$ and $\Psi_{\rm out}(R,t) = \Psi_{\rm out}(R)e^{-iEt/\hbar}$ respectively, where the subscript OUT means the OUT state of BS2. The solution of the output wave function under a given input wave function $\Psi_{\rm input} = \psi(\boldsymbol{r},x) = \psi_0(\boldsymbol{r})e^{ikx} = \psi_0 e^{ikx}$ can be calculated, which is

$$\Psi_{{\rm output},x} = \Psi_{{\rm out},x} = \psi_0 e^{i\phi}e^{ikx}/\sqrt{2}. \tag{4}$$

The signal intensity is

$$P_{\rm out} = |\Psi_{{\rm out},x}|^2 = |\psi_0|^2/2 = P_0. \tag{5}$$

As illustrated in Fig. 2, the signal is just a constant irrespective of the phase shift ϕ, and with a fringe visibility $V^2 = 0$, and it does not show any wave nature. Yet, the path information of the quantum particle can be unanimously identified now, and the corresponding path distinguishability is $D^2 = 1$. The corresponding quantum state can be designated as $\Psi_{\rm out} = \psi_{\rm particle}$. The above two simple quantum mechanical calculations confirm the well-known results in history about the MZI used to test the wave-particle duality.

We proceed to look at the more complicated situation as shown in Fig. 3. The status of BS2 is externally controlled to move into or out of the MZI, which can be random or other forms. The Hamiltonian of this new system is thus time dependent, denoted as $H_{\rm mod}(R,t)$. The corresponding wave function should be much more complicated than the stationary solution $\Psi(R,t) = \Psi(R)e^{-iEt/\hbar}$. Yet, in most practical situations, some approximations can be adopted, rendering easy solution of the quantum mechanical problem.

One prominent point in Fig. 3 is that two sets of instrumental setup, which are still all purely classical devices, are used, that is, the setup with BS2 inserted or removed, respectively. These two setups cannot be in place simultaneously in time, due to the fact that they obviously repel each other. In the methodology of quantum theory, these two sets of instrument arrangement correspond to two mutually repulsive quantum mechanical problems corresponding to the two different setups of MZI as illustrated in Figs. 1 and 2, respectively. Their quantum mechanical solution, as has been discussed in Eqs. (2)–(5), only allows for either wave or particle behavior observation of quantum particles. The current scheme of Fig. 3 uses a single set of measurement setup involving two instruments placed in different space-time domains, in the form of some deliberate temporal operation and modulation to BS2. Keeping this in mind, the Hamiltonian in Eq. (1) is written into a time-modulated form as

$$H_{\rm mod}(R,t) = a(t)H_{\rm in}(R) + b(t)H_{\rm out}(R), \tag{6}$$

where $a(t)$ and $b(t)$ are temporal functions used to collectively describe how BS2 goes into (operation IN) and moves out of (operation OUT) the MZI. Usually they are step functions given by $a(t) = 1$, $b(t) = 0$, $t \subset [t_1, t_2]$ or $a(t) = 0$, $b(t) = 1$, $t \subset [t_1, t_2]$. This simple expression means that the two operations cannot happen simultaneously at any time interval $[t_1, t_2]$. Moreover, the time series $[t_1, t_2]$ and duration $\Delta t{=}t_2 - t_1$ can be of arbitrary form, depending on how the state of BS2 is controlled and modulated.

Suppose that in practical experiments the modulation time Δt is far longer than the transport time of the particle through BS2, then the transient behavior of the wave function at this transition window can be ignored when the overall evolution dynamics of the particle is concerned. Keeping this in mind, we find the output wave function from the MZI as detected and analyzed by the detectors x and y can be expressed as

$$\begin{aligned}\Psi_{\rm output}(R,t) &= \Psi_{\rm mod}(R,t)\\ &= a(t)\,\Psi_{\rm in}(R,t) + b(t)\,\Psi_{\rm out}(R,t). \quad (7)\end{aligned}$$

Thus the output wave function is simply a superposition of the wave behavior state and particle behavior state in the temporal domain, called temporal superposition, in contrast to the usual superposition of stationary states in quantum mechanics. When one follows the regular practice in carrying out this kind of quantum mechanical experiment, and accumulate in time and average over the signal recorded and analyzed by the detectors, the expected outcome of signal intensity is given by

$$\begin{aligned}P_{\rm output} = P_{\rm mod} &= A|\,\Psi_{\rm in}|^2 + B|\,\Psi_{\rm out}|^2\\ &= AP_{\rm in} + BP_{\rm out}. \quad (8)\end{aligned}$$

The coefficients A and B are given by

$$A = \frac{1}{T}\int_0^T |a(t)|^2 dt, \quad B = \frac{1}{T}\int_0^T |b(t)|^2 dt, \quad (9)$$

where T is the total accumulation time of the signal. Obviously A and B are just the ratio of the total time interval of IN and OUT operation states of BS2, respectively, over the total accumulation time T. In all the cases of temporal modulation, $A + B = 1$. Without loss of generality, we can define $A = \sin^2\theta$ and $B = \cos^2\theta$, then the signal in the MZI scheme of Fig. 3 is calculated from Eq. (8) as

$$P_{\rm output} = P_{\rm mod} = P_0[1 - \sin^2\theta\cos\phi]. \quad (10)$$

This equation can be written into another equivalent form as

$$P_{\rm output} = P_{\rm mod} = 2P_0[\cos^2\frac{\phi}{2}\sin^2\theta + \frac{1}{2}\cos^2\theta]. \quad (11)$$

It is interesting to note that Eq. (11) is exactly the same as Eq. (6) in Ref. [21]. As illustrated in Fig. 3, the detected signal is just the superposition (or more precisely, the mixture) of the particle and wave state of quantum particle, that is, a perfect cosine function superimposed with a non-zero offset signal basis. It involves a partial wave feature exhibiting an interference fringe with visibility of $V^2 = \sin^2\theta$ and partial particle feature exhibiting the which-path information with path distinguishability $D^2 = \cos^2\theta$. Obviously the outcome of the signal can continuously transform from the purely wave state ($\theta = 0$) to the purely particle state ($\theta = \pi/2$), and locate in any intermediate superposition (mixture) state between them. In any case, we find $V^2 + D^2 = 1$, thus this delayed-choice experimental scheme is impossible for simultaneous observation of perfect wave and particle nature of a quantum particle, which requests $V^2 = D^2 = 1$ and $V^2 + D^2 = 2 >> 1$ under ideal conditions.[19–27]

Our analysis made in the above sections involves several key points. First, the analysis completely and strictly follows the operation formulation of standard quantum mechanics (the Schrödinger equation and wave function, together with boundary and initial conditions), which are well-established, have good consensus, and are free from controversies. Secondly, the analysis uses the probability interpretation of wave function against practical experimental observations, that is, any expected result of measurement over a physical quantity against a quantum state is an average over the outcome of many identical measurement operations against that specific quantum state, and this is again free from controversy. In the current situation of MZI, the outcome of wave nature (the interference pattern) and particle nature (which-way information) measurement is determined via accumulation and average in time over the detector outcome when the MZI experiment is performed over single particle one-by-one in time. Thus we justify that our analysis is made on a solid basis of quantum mechanics rather than pure conceptual arguments.

As the only way that the traditional delayed-choice experiment designers can carry out is to change the status of BS2 to test the wave-particle duality via the MZI in Fig. 3, there is no way that this MZI instrument can tell simultaneously the wave and particle nature of quantum particles according to our above analysis. The measurement outcome of the detectors has three options. It either gives unambiguously the wave nature of the quantum particle by permanently inserting BS2 in the MZI, or gives unambiguously the wave nature of the quantum particle by permanently removing BS2 from the MZI, or yields a result that seemingly matches with an intermediate superposition state between the wave and particle as intensively discussed in recent literature of quantum delayed-choice experiments.[19–25] Surely there are new physics, while the fundamental cause is far from being completed.

At the first glance, our observation based on quantum mechanical analysis over a quantum scattering problem under a time-modulated Hamiltonian seems to be exactly the same as the prediction made by previous delayed-choice experiment analyses.[19–25] In particular, let us recall again the exact identification of Eq. (11) derived from our time-modulated Hamiltonian analysis to Eq. (6) in Ref. [21] derived by the classical quantum-delayed choice experimental analysis. Nonetheless, the physical picture is very differ-

ent between our analysis and previous analyses. Although the outcome of measurement over the which-path information and interference fringe visibility in this MZI as illustrated in Fig. 3 is the same in both analyses, the detailed physical processes in the analyses are very different. In our analysis, the wave–particle duality test is a quantum mechanical scattering problem. The motion of BS2 in the MZI induces a time-modulated Hamiltonian acting over the incident wave function of the quantum particle, leading to the output of a synchronously time-modulated quantum state and the seeming coexistence and mixture of partial wave and partial particle behavior. In previous analyses, the wave–particle duality test is a quantum mechanical eigen problem. The motion of BS2 under external modulation action induces a new time-independent stationary quantum state, that is, the superposition state of wave state and particle state, $\psi = a_{\rm w}\psi_{\rm wave} + a_{\rm p}\psi_{\rm particle}$, where $a_{\rm w}$ and $a_{\rm p}$ are coefficients. Obviously this new quantum state carries both the information of wave and particle. The detectors directly interact and deal with this quantum state, and record the same outcome as our current analysis, that is, Eq. (10) (or Eq. (11)). Frankly speaking, apart from conceptual differences, where only the technical issue is concerned, our analysis is much simpler and more straightforward than previous delayed-choice experiment analysis.

A simple argument shows that the new quantum state should be the solution of a stationary quantum superposition Hamiltonian as $H = a_{\rm w}H_{\rm in} + a_{\rm p}H_{\rm out}$. It is not evident from previous analyses how the classical device BS2 can stay in such a superposition state and how its Hamiltonian can be in such a superposition (or merger) of two completely exclusive Hamiltonians as $H_{\rm in}$ and $H_{\rm out}$. Although seemingly similar in form, the time-modulated Hamiltonian $H_{\rm mod}(t) = a(t)H_{\rm in} + b(t)H_{\rm out}$ is completely different in essence from the stationary superposition Hamiltonian $H = a_{\rm w}H_{\rm in} + a_{\rm p}H_{\rm out}$. Here we leave it an open problem for researchers to judge which way is closer to the true physics of these historical and modern MZI problems.

In our above quantum mechanical analysis of delayed-choice MZ interferometers, we only consider single-particle experiment, which is the case for classical delayed-choice experiments. These experiments only involve single-particle wave function and its evolution within the MZ interferometer, and the solution is a single-particle quantum scattering problem. Yet, in recent quantum delayed-choice experiments,[20–23] photon pairs are involved. Thus one needs to solve more complicated two-particle quantum mechanical problem. Still, the conceptual and theoretical framework of current analysis is applicable, while the detailed procedure of solution needs further studies, which will be reported in our future work.

In summary, we have used the standard quantum mechanical formalism to revisit and evaluate several classical MZI experiments used to test wave–particle duality of microscopic particles. It is found that they are a standard quantum mechanical scattering problem and the classical delayed-choice experiments can be described by a time-modulated Hamiltonian $H_{\rm mod}(t) = a(t)H_{\rm in} + b(t)H_{\rm out}$. As a result of this modulation, the final output wave function is also synchronically time-modulated as $\psi_{\rm mod}(t) = a(t)\psi_{\rm wave} + b(t)\psi_{\rm particle}$, thus the outcome of measurement behaves as if they are in the superposition of the wave and particle quantum state when many events over time accumulation are counted and averaged. The picture is completely different from the conventional wisdom that this is a quantum eigen-problem described by the stationary Hamiltonian $H = a_{\rm w}H_{\rm in} + a_{\rm p}H_{\rm out}$ and a quantum superposition state as $\psi = a_{\rm w}\psi_{\rm wave} + a_{\rm p}\psi_{\rm particle}$. We expect that our new analysis over the delayed-choice MZI schemes may stimulate more studies over the conceptual basis of quantum physics, in particular, the wave-particle duality and orthodox Copenhagen principle of complementarity.[27–30]

References

[1] Bohr N 1928 *Naturwissenschaften* **16** 245
[2] Bohr N 1928 *Nature* **121** 580
[3] Bohr N 1949 *Albert Einstein: Philosopher Scientist* (Evanston: Library of Living Philosophers) p 200
Bohr N 1983 *Quantum Theory and Measurement* (Princeton: Princeton University Press) p 9
[4] Heisenberg W 1930 *The Physical Principles of the Quantum Theory* (Chicago: University of Chicago Press)
[5] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[6] Schrödinger E 1935 *Proc. Camb. Philos. Soc.* **31** 555
[7] Bohm D A 1952 *Phys. Rev.* **85** 166
Bohm D A 1952 *Phys. Rev.* **85** 180
[8] Feynman R, Leighton R and Sands M, *The Feynman Lectures on Physics* (Cambridge: Addison-Wesley) vol Ⅲ chap I
[9] Jammer M 1974 *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective* (New York: Wiley)
[10] Bell J S 1988 *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press)
[11] Wootters W K and Zurek W H 1979 *Phys. Rev.* D **19** 473
[12] Cronin A D et al 2009 *Rev. Mod. Phys.* **81** 1051
[13] Wheeler J A 1978 *Mathematical Foundations of Quantum Theory* (New York: Academic Press) p 948
Wheeler J A 1979 *Problems in the Formulations of Physics* (Amsterdam: North-Holland)
[14] Scully M O and Druhl K 1982 *Phys. Rev.* A **25** 2208
[15] Hellmuth T et al 1987 *Phys. Rev.* A **35** 2532
[16] Lawson-Daku B J, Asimov R, Gorceix O, Miniatura Ch, Robert J and Baudon J 1996 *Phys. Rev.* A **54** 5042
[17] Jacques V, Wu E, Grosshans F, Treussart F, Grangier P, Aspect A and Roch J F 2007 *Science* **315** 966
[18] Jacques V, Wu E, Grosshans F, Treussart F, Grangier P, Aspect A and Roch J F 2008 *Phys. Rev. Lett.* **100** 220402
[19] Ionicioiu R and Terno D R 2011 *Phys. Rev. Lett.* **107** 230406
[20] Tang J S, Li Y L, Xu X Y, Xiang G Y, Li C F and Guo G C 2012 *Nat. Photon.* **6** 600
[21] Kaiser F, Coudreau T, Milman P, Ostrowsky D B and Tanzilli S 2012 *Science* **338** 637
[22] Peruzzo A, Shadbolt P, Brunner N et al 2012 *Science* **338** 634
[23] Tang J S, Li Y L, Li C F and Guo G C 2013 *Phys. Rev.* A **88** 014103
[24] Yan H, Liao K, Deng Z, He J, Xue Z Y, Zhang Z M and Zhu S L 2015 *Phys. Rev.* A **91** 042132
[25] Xin T, Li H, Wang B X and Long G L 2015 *Phys. Rev.* A **92** 022126
[26] Englert B G 1996 *Phys. Rev. Lett.* **77** 2154
[27] Li Z Y 2014 *Chin. Phys.* B **23** 110309
[28] Jia A A, Yang J, Yan S H, Hu Q Q, Luo Y K and Zhu S Y 2015 *Chin. Phys.* B **24** 080302
[29] Sun J, Sun Y N, Li C F and Guo G C 2015 *Chin. Phys. Lett.* **32** 090302
[30] Zhu X W, Fang X M, Peng X H, Feng X H, Gao K L and Du F 2001 *J. Phys.* B **34** 4349