Inhibition of Atomic Decay in Strongly Coupled Photonic Crystal Cavities *

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We discuss the evolution dynamics of a quantum system consisting of two two-level atoms separately embedded within two strongly coupled photonic crystal cavities. Although the quantum system is subjected to dissipation and decoherence from the cavity leakage and the atomic decay, it does allow for eigenstates that are not influenced by one of the two dissipation channels and results in dissipation-inhibition quantum states. These dissipation-free quantum states can help to achieve an extremely long photon and atom storage lifetime and provide a new perspective to realize efficient quantum information storage via reducing the negative influence of the dissipation from the environment.

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Recently cavity quantum electrodynamics (QED) has been actively pursued for its potential insight into the fundamental problems of light–matter interaction.[1–4] Many QED schemes describe cavities that contain trapped qubits to realize quantum information storage,[5–9] and various approaches have been proposed to realize quantum memories, where quantum information can be stored and retrieved. As a traditional optical cavity, the Fabry–Perot etalon is widely used in optical components and photonic devices,[10] while whispering gallery modes of microspheres offer an alternative avenue of high quality with small modal volume.[11,12] The recently developed photonic crystal (PC) micro-cavity can provide easy fabrication and flexible design and has attracted much attention.[13–15] The photon local density of states[16,17] is directly connected to the coupling strength with the quantum emitter, for example, a quantum dot, which is often considered to be a two-level atom-like emitter.[18,19] However, the interaction between a quantum system and its surroundings, which is traditionally seen as a negative feature since it is the root cause of dissipation and decoherence, is unavoidable.[20,21] There is currently a huge theoretical and experimental effort aimed at controlling dissipative effects in cavity quantum optics.[22,23] In this work we show that the dissipation of a photon–atom quantum system coupled to its environment can be effectively inhibited when they are embedded into two strongly coupled PC micro-cavities.

The system under consideration is depicted in Fig. 1, which is created in the PC platform. It consists of two identical atoms trapped in two separate cavities, where each atom is resonant with the cavity. The dissipation induced by photon loss due to cavity leakage and atom coupling with the environment is taken into account. To obtain a significantly simplified and clarified insight, the two atoms are placed at the equivalent position of the two cavities. As a result, the position dependence of the coupling strength g, the cavity leaky rate $\kappa_{\rm c}$, and the atomic dissipative decay rate $\gamma_{\rm a}$ are the same in the two subsystems.

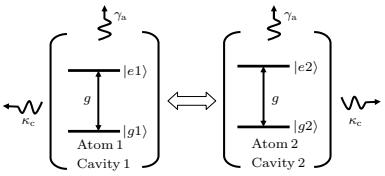

Fig. 1. Schematic diagram of a quantum system formed by two coupled cavities, each of which contains a two-level atom. The atoms are coupled to their cavity modes with strength g. Photons can hop between the cavities at rate λ, and $\gamma_{\rm a}$ and $\kappa_{\rm c}$ are the cavity leaky rate and the atomic dissipative decay rate, respectively.

In the interaction picture, the Hamiltonian governing the system is in the form

$$H_{\rm I} = \hbar \sum_{j=1,2} [g(a_j^+ \sigma_{j-} + \sigma_{j+} a_j)] + \lambda\hbar(a_1^+ a_2 + a_2^+ a_1), \quad (1)$$

where a_j^+ and a_j are creation and annihilation operators for cavity photon mode j, σ_{j+} and σ_{j-} ($j = 1$ and 2) are rising and lowering operators of the atom j, and λ is the cavity–cavity hopping strength. Considering the decay of the atom and the dissipation of the cavity field, the dissipative term can be given by

$$H_{\rm d} = -(i\hbar\gamma_{\rm a}\sigma_{11} + i\hbar\gamma_{\rm a}\sigma_{22}) - i\hbar\kappa_{\rm c}(a_1^+ a_1 + a_2^+ a_2), \quad (2)$$

where $\sigma_{ii} = |e_i\rangle\langle e_i|$ ($i = 1$ and 2). The Hamiltonian for the system is

$$H = H_{\rm I} + H_{\rm d}. \quad (3)$$

For simplicity, the solution and discussion of the Hamiltonian are restricted to the subspace only containing zero and one excited state. Thus the state

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vector of the system at time t is

$$\begin{aligned}|\psi(t)\rangle =& |\psi_0(t)\rangle + |\psi_1(t)\rangle,\\ |\psi_0(t)\rangle =& |g1\rangle|g2\rangle|0\rangle_1|0\rangle_2,\\ |\psi_1(t)\rangle =& a(t)|g1\rangle|g2\rangle|1\rangle_1|0\rangle_2\\ &+ b(t)|g1\rangle|g2\rangle|0\rangle_1|1\rangle_2\\ &+ c(t)|e1\rangle|g2\rangle|0\rangle_1|0\rangle_2\\ &+ d(t)|g1\rangle|e2\rangle|0\rangle_1|0\rangle_2, \end{aligned} \tag{4}$$

where $|gj\rangle$ and $|ej\rangle$ are the ground and excited states of atom j, $|0\rangle_j$ and $|1\rangle_j$ are vacuum and one-photon state of cavity field j. The zero-excitation component $|\psi_0(t)\rangle$ is invariant under the action of the Hamiltonian, thus we only consider the dynamics of the single-excitation component $|\psi_1(t)\rangle$. The motion for the probability amplitudes $a(t)$, $b(t)$, $c(t)$ and $d(t)$ can be derived from the Schrödinger equation $i\hbar\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle$ as follows:

$$\begin{aligned} i\dot{a}(t) =& gc(t) + \lambda b(t) - i\kappa_{\rm c}a(t),\\ i\dot{b}(t) =& gd(t) + \lambda a(t) - i\kappa_{\rm c}b(t),\\ i\dot{c}(t) =& ga(t) - i\gamma_{\rm a}c(t),\\ i\dot{d}(t) =& gb(t) - i\gamma_{\rm a}d(t). \end{aligned} \tag{5}$$

To solve the above equations, four trial solutions can be given as follows:

$$\begin{aligned} a(t) =& Ae^{xt}, \quad b(t) = Be^{xt},\\ c(t) =& Ce^{xt}, \quad d(t) = De^{xt}. \end{aligned} \tag{6}$$

Inserting the trial solutions into Eq. (5), we can obtain the following equations

$$\begin{aligned} i(x+\kappa_{\rm c})A - \lambda B - gC =& 0,\\ -\lambda A + i(x+\kappa_{\rm c})B - gD =& 0,\\ gA - i(x+\gamma_{\rm a})C =& 0,\\ gB - i(x+\gamma_{\rm a})D =& 0. \end{aligned} \tag{7}$$

The above equations can be transformed into a set of linear equations of a simple 4×4 eigensystem

$$\begin{pmatrix} -\kappa_{\rm c} & -i\lambda & -ig & 0\\ -i\lambda & -\kappa_{\rm c} & 0 & -ig\\ -ig & 0 & -\gamma_{\rm a} & 0\\ 0 & -ig & 0 & -\gamma_{\rm a} \end{pmatrix} \begin{pmatrix} A\\ B\\ C\\ D \end{pmatrix} = x \begin{pmatrix} A\\ B\\ C\\ D \end{pmatrix}, \tag{8}$$

where x is the eigenvalue. By solving the linear equations, we can obtain four roots as follows:

$$\begin{aligned} x_{1,2} =& -\frac{1}{2}(\kappa_{\rm c}+\gamma_{\rm a}-i\lambda)\\ &\pm\frac{1}{2}\sqrt{(\kappa_{\rm c}-\gamma_{\rm a}-i\lambda)^2-4g^2},\\ x_{3,4} =& -\frac{1}{2}(\kappa_{\rm c}+\gamma_{\rm a}+i\lambda)\\ &\pm\frac{1}{2}\sqrt{(\kappa_{\rm c}-\gamma_{\rm a}+i\lambda)^2-4g^2}. \end{aligned} \tag{9}$$

For the current atom–cavity coupled system, in the situation of $\lambda \gg g \gg \gamma_{\rm a}, \kappa_{\rm c}$, the four eigenvalues can be simplified by adopting some algebraic approximations to

$$\begin{aligned} x_{1,4} =& -\kappa_{\rm c} \pm i\Big(\lambda + \frac{g^2}{\lambda}\Big),\\ x_{2,3} =& -\gamma_{\rm a} \mp i\frac{g^2}{\lambda}. \end{aligned} \tag{10}$$

Here we have used a phenomenological interaction Hamiltonian considering the dissipation of atom and cavity, which is easier to understand and work on. Our phenomenological theoretical formalism is the same as those used in our previous work.[24] Moreover, our calculation results are equivalent to those which are achieved by using the solution of master equations.

The quantities of $x_n (n = 1, 2, 3, 4)$ basically determine the dynamics of the dual-cavity photon–atom quantum system. As shown in Eq. (9), the four eigenvalues generally are complex numbers, with the imaginary part governing the oscillatory behavior and the real part describing the decay process.

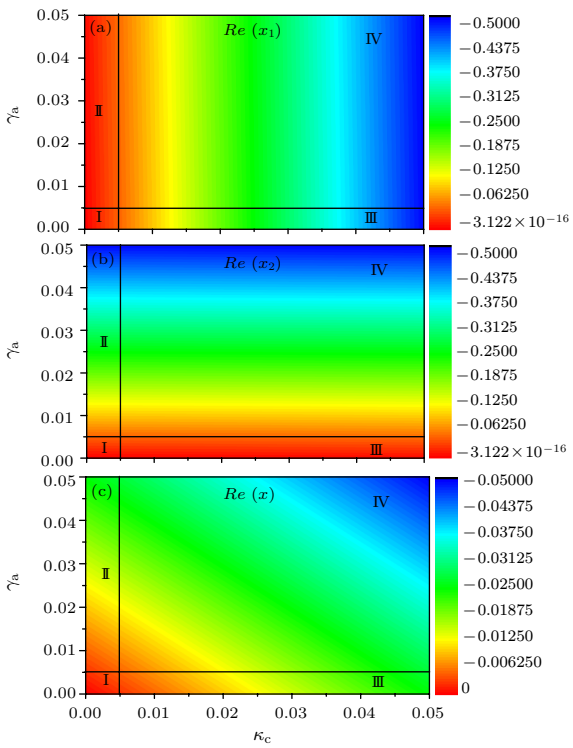

Fig. 2. (Color online) Calculated quantum dynamics eigenvalue (a) Re(x_1), (b) Re(x_2) of the dual atom–cavity system, and (c) Re(x) of a single atom–cavity system as functions of the dissipation parameter $\gamma_{\rm a}$ and $\kappa_{\rm c}$ in the case of $\lambda \gg g$. The other parameters are $g = 1$, $\lambda = 10$. Here $\gamma_{\rm a}$, $\kappa_{\rm c}$, g and λ are dimensionless.

The simplified form of x_n as addressed in Eq. (10) shows that $\mathrm{Re}(x_1) = \mathrm{Re}(x_4)$ and $\mathrm{Re}(x_2) = \mathrm{Re}(x_3)$. To have some intuitive ideas about the dynamics of the quantum system, we calculate $\mathrm{Re}(x_1)$ and $\mathrm{Re}(x_2)$, the decay rate of the whole system, as functions of the two dissipative parameters $\kappa_{\rm c}$ and $\gamma_{\rm a}$ directly from

Eq. (9), where no approximation is made. The results are displayed in Fig. 2. Generally speaking, the dynamics of the quantum system decay modes in all situations, including the information of dissipation-inhibition eigenstates can be visualized from these pictures. To discuss the inhibited decay property clearly, the decay mode can be divided into four regimes as labeled by I, Ⅱ, Ⅲ, and Ⅳ in Fig. 2. In regime I, the decay rate of both dissipation channels, κ_c and γ_a are small. The overall decay rate of the quantum state is also low and eventually goes to 0 when the two dissipation channels are completely closed, that is, κ_c and γ_a are close to zero, as it should be. Photons can slowly decay through the cavity cladding wall to the reservoirs with oscillation between atom and cavity. In regime Ⅱ of Fig. 2(a), it can be found that when κ_c is kept in a small-number range, the decay mode corresponding to $\mathrm{Re}(x_1)$ has an overall dissipation rate that is completely insensitive to the atomic decay rate γ_a, and it remains in a small-number range even if γ_a is large. In this case, the dissipation of the whole quantum system is heavily suppressed. A similar property of dissipation inhibition is found in regime Ⅲ of Fig. 2(b) for the decay mode corresponding to $\mathrm{Re}(x_2)$. In this regime, γ_a is kept in a small value while κ_c can be large. From Figs. 2(a) and 2(b) we find that the quantum system embedded in strongly coupled cavities can support dissipation-inhibition eigenstates.

To have clarified vision of the virtual of the new scheme of dual atom–cavity system in suppressing the dissipation and decoherence of the atomic quantum state, we make a comparison with the dynamics of a single atom–cavity quantum system. The corresponding eigenvalues of the evolution dynamics are

$$x_{1,2} = \frac{1}{2}[-(\kappa_c + \gamma_a) \pm \sqrt{(\kappa_c - \gamma_a)^2 - 4g^2}]. \quad (11)$$

In the condition of $\lambda \gg g \gg \gamma_a, \kappa_c$, the approximate form is

$$x_{1,2} = -(\kappa_c + \gamma_a)/2 \pm ig. \quad (12)$$

We calculate $\mathrm{Re}(x)$, the decay rate of the quantum system, as functions of the two parameters κ_c and γ_a from Eq. (11), and the result is illustrated in Fig. 2(c). Comparing the three panels, the story for the single atom–cavity system is very different for the dual atom–cavity system. The inhibition behavior only occurs in regime I, where both κ_c and γ_a are small. The decay modes, which are symmetrical to the diagonal line, change together with both κ_c and γ_a. When either of them grows, the decay rate also increases.

The above dissipation behavior of the quantum states is more clarified when we look at Eqs. (9)–(12). For the dual-cavity structure (Eq. (10)), the eigenstate corresponding to x_1 or x_4 has a decay rate that is not influenced by γ_a, while the eigenstate corresponding to x_2 or x_3 has a decay rate independent of κ_c. In contrast, for the single-cavity structure (Eq. (12)), the eigenstate corresponding to x_1 or x_2 has a decay rate that is dependent on both γ_a and κ_c together. This means that the strongly coupled dual-cavity structure has opened a regime where one of the two dissipation channels of atoms has no influence upon the atomic dynamics. This will bring a promising aspect of engineering and suppressing dissipation and decoherence in quantum systems. If one particular dissipation channel is completely closed, for instance, the cavity leakage of light is in an extremely low level, i.e., $\kappa_c \approx 0$, as can be realized in a high-Q PC micro-cavity, then the dual-cavity structure still supports some dissipation-inhibition eigenstate even if the atomic decay channel originating from either radiative or nonradiative decay processes remains open and γ_a has a relatively large magnitude. The clear physical image of this dissipation-inhibition phenomenon can be described as follows. For the single-cavity structure, energy can decay through the cavity cladding wall to the reservoirs rapidly. However, for the dual-cavity structure, owing to the strong coupling interaction between the two closely-neighboring cavities, a photon can hop periodically from one cavity to another cavity for a number of times before it leaves the cavity finally.

To explore the dissipation-inhibition characteristic in more depth and to have a more clarified picture of the underlying physics, we investigate the time evolution of the atomic states and the cavity photon states in the strongly coupled dual cavity system. After the eigenvalues x_n are solved from Eq. (9), we substitute the four eigenvalues into Eq. (7) and obtain the exact solution of the corresponding eigenstate coefficients[24] as follows:

$$\begin{aligned} B_n &= i\frac{(x_n + \kappa_c)(x_n + \gamma_a) + g^2}{\lambda(x_n + \gamma_a)} A_n, \\ C_n &= \frac{g}{i(x_n + \gamma_a)} A_n = i\frac{-g}{(x_n + \gamma_a)} A_n, \\ D_n &= i\frac{-g}{(x_n + \gamma_a)} B_n = g\frac{(x_n + \kappa_c)(x_n + \gamma_a) + g^2}{\lambda(x_n + \gamma_a)^2} A_n. \end{aligned} \quad (13)$$

The probability amplitude of four one-excitation states can be obtained by substituting the above eigenstate coefficients into the trial solution, and the results are

$$\begin{aligned} a(t) &= \sum_{n=1}^{4} A_n e^{x_n t}, \\ b(t) &= \sum_{n=1}^{4} B_n e^{x_n t} \\ &= \sum_{n=1}^{4} i\frac{(x_n + \kappa_c)(x_n + \gamma_a) + g^2}{\lambda(x_n + \gamma_a)} A_n e^{x_n t}, \end{aligned}$$

$$c(t) = \sum_{n=1}^{4} C_n e^{x_n t} = \sum_{n=1}^{4} i\frac{-g}{(x_n+\gamma_a)} A_n e^{x_n t},$$
$$d(t) = \sum_{n=1}^{4} D_n e^{x_n t}$$
$$= \sum_{n=1}^{4} g\frac{(x_n+\kappa_c)(x_n+\gamma_a)+g^2}{\lambda(x_n+\gamma_a)^2} A_n e^{x_n t}. \quad (14)$$

It can be found from Eq. (4) that $a(t)$ and $b(t)$ are the probability amplitude of two cavity photon states, and $c(t)$ and $d(t)$ are the probability amplitude of the atomic states, respectively. Any quantum state can be expressed as the superposition of these four eigenstates. In principle, we can choose a special initial state described by the coefficient vector (A_1, A_2, A_3, A_4). If the coefficient vector contains only one nonzero value, then only the corresponding eigenstate occurs in the whole system based on Eqs. (13) and (14).

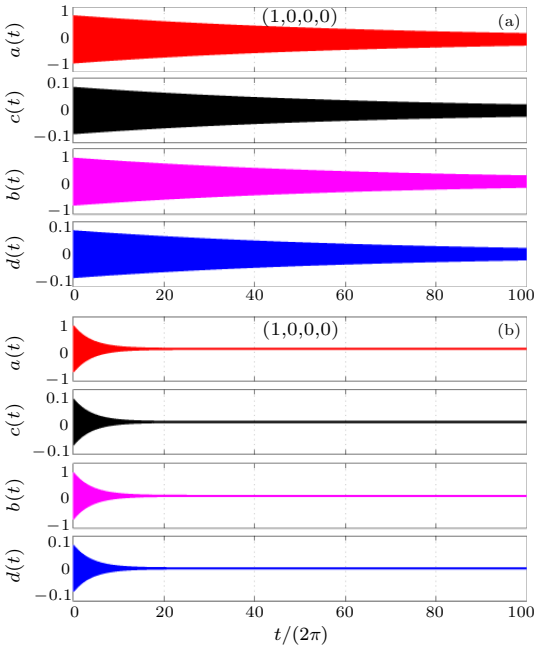

Fig. 3. (Color online) Dynamics of atomic and field excitation amplitude in the case of $\lambda \gg g$ under the dissipation parameters of (a) $\kappa_c = 0.002$ and $\gamma_a = 0.04$, and (b) $\kappa_c = 0.04$ and $\gamma_a = 0.002$. The initial eigenstate coefficient vector is set as (1, 0, 0, 0) and only the eigenstate x_1 takes effect. The other parameters are the same as used in Fig. 2. The time parameter t is dimensionless.

We assume that initially the coefficient vector is (1, 0, 0, 0), and thus the eigenstate x_1 dominates the quantum dynamics of the dual atom–cavity system. The time evolution of the amplitudes obtained from Eq. (14) is displayed in Fig. 3. In Fig. 3(a), the parameters are set as $\kappa_c = 0.002$, $\gamma_a = 0.04$, which correspond to regime Ⅱ of Fig. 2(a). All the photon and atomic states remain in a sufficiently large population even in a long time of $100 \times 2\pi$. In this situation, κ_c is small and x_1 corresponds to a dissipation-inhibition state. In contrast, the dynamics of the field mode and atomic mode presents a fast decay property when the decay rate is changed to $\kappa_c = 0.04$, $\gamma_a = 0.002$, as shown in Fig. 3(b). The population can be nearly negligible at a short time of $t = 10 \times 2\pi$.

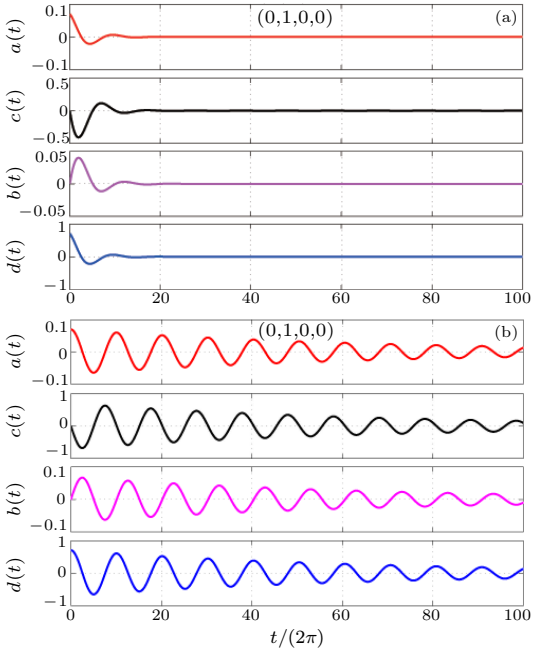

Fig. 4. (Color online) Dynamics of atomic and field excitation amplitude in the case of $\lambda \gg g$ under the dissipation parameters of (a) $\kappa_c = 0.002$ and $\gamma_a = 0.04$, and (b) $\kappa_c = 0.04$ and $\gamma_a = 0.002$. The initial eigenstate coefficient vector is set as (0, 1, 0, 0) and only the eigenstate x_2 takes effect. The other parameters are the same as those used in Fig. 2. The time parameter t is dimensionless.

Similarly, if the coefficient vector is initially prepared as (0, 1, 0, 0), the system dynamics is governed by the eigenvalue x_2 and it displays different properties as shown in Fig. 4. Each quantum state now decays fast when $\kappa_c = 0.002$, $\gamma_a = 0.04$ (see Fig. 4(a)), while is heavily inhibited in the case of $\kappa_c = 0.04$, $\gamma_a = 0.002$ (see Fig. 4(b)). It can be seen from Eq. (10) that the eigenvalue x_2 is only subjected to the influence from the parameter γ_a. Therefore, the system dynamics is not affected by the decay parameter κ_c at all.

The above discussions on the evolution dynamics of the quantum system clearly indicate that the key factor to the efficient suppression of photonic and atomic decay in the dual-cavity structure is to prepare the quantum system into a desirable dissipation-inhibition eigenstate, which will remain decay-free with time evolving. Even if the initial quantum state is not in the dissipation-inhibition eigenstate, with a certain superposition of all the four eigenstates, the system can still evolve into a decay-free state when times goes on, due to the fact that other strongly-dissipative eigenstates will eventually decay to zero and contribute nothing to the quantum system dynamics. In this regard, the dissipation-free state in

the dual-cavity structure is robust.

In summary, we have theoretically shown that the strong coupling between two closely-neighboring cavities together with the resonant coupling between atom and photon within each cavity can strongly suppress the overall dissipation and decoherence induced by the cavity leakage and atomic dissipative decay. The key is that the quantum states are completely insensitive to one of the two dissipation channels. If one dissipation channel is completely closed, the system can support quantum states that are decay free. The existence of dissipation-inhibition quantum state can help to achieve an extremely long photon and atom storage lifetime and to provide a new perspective to realize efficient quantum information storage through reducing the negative influence of the dissipation from environment.

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