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Self-Organized Optimization of Transport on Complex Networks *

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We propose a self-organized optimization mechanism to improve the transport capacity of complex gradient networks. We find that, regardless of network topology, the congestion pressure can be strongly reduced by the self-organized optimization mechanism. Furthermore, the random scale-free topology is more efficient to reduce congestion compared with the random Poisson topology under the optimization mechanism. The reason is that the optimization mechanism introduces the correlations between the gradient field and the local topology of the substrate network. Due to the correlations, the cutoff degree of the gradient network is strongly reduced and the number of the nodes exerting their maximal transport capacity consumedly increases. Our work presents evidence supporting the idea that scale-free networks can efficiently improve their transport capacity by self-organized mechanism under gradient-driven transport mode.

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Network transport has been an important research topic due to the fact that transport is encountered in many natural and human-made networks.[1–10] The quantities to be transported include power, goods, informational packets and so on. The transport capacity of a network usually depends on the structure of the network, as well as on the routing rules. Recent studies have shown that many networks are strongly heterogeneous and often display scale-free topology, that is, they possess a power-law degree distribution.[11–14] In addition, the universal interactions of nodes and the internal traffic demands of the system will determine the topology evolution of technological networks,[15] and the moderate strength of community structure is more convenient for the information transfer of real complex systems.[16] On the other hand, gradient-induced transport has been shown to play an important role in physics systems, social systems, and certainly in biology.[17] A question naturally arises: is the scale-free topology propitious to improve the transport capacity of network under the mechanism of gradient-induced transport?

To answer this question, Toroczkai *et al.*[18] introduced a transport network model called the gradient network to model the gradient-induced transport mode. They found that the gradient network has a scale-free topology structure whether the substrate network is random Poisson or random scale-free, while the growing binomial random networks are more susceptible to jamming than the growing scale-free networks for the network growth processes. Later, Park *et al.*[19] made a comparison for the jamming in different complex networks with the same average degree $\langle k\rangle$ and the network size N and found that scale-free network can lead to efficient transport compared with random Poisson network only when $\langle k\rangle$ is larger than about 10. It seems that the scale-free topology is not effective to reduce the jamming since the average degree of many real networks is usually less than 10.[1,11,20]

It is noted that the gradient networks considered in Refs. [18,19] are random gradient networks due to the fact that the scale field is random there. Recently, to improve the transport efficiency of the networks, researchers find that introducing the correlations between the scale field and the local topology of networks can reduce congestion.[21,22] Piontti *et al.*[21] used a relaxation dynamics to obtain the structural correlations, and found that this procedure can decrease congestion if the substrate network is scale-free, while the same mechanism has hardly any effect on congestion in the case of random Poisson network. Gulbahce[22] found that the structural correlations can be obtained by a Monte-Carlo optimization scheme. This scheme can decrease the jamming in both random Poisson and scale-free networks, and the scale-free topology is more effective to reduce congestion under this scheme.

Although the above two different strategies can improve the transport capacity of complex gradient networks, the optimization mechanism is not self-organized behavior of systems. In this Letter, we propose a self-organized optimization mechanism to improve the transport capacity of complex gradient networks. It is found that, regardless of network topology, the congestion pressure can be strongly reduced by the optimization mechanism, and the random scale-free topology is more effective to reduce congestion compared with the random Poisson topology under the optimization mechanism. Furthermore, the self-organized optimization mechanism is more efficient to reduce congestion compared with the Monte-Carlo optimization scheme introduced by Gulbahce.[22] Finally, we study the effects of our optimization mechanism on the properties of scale field, and on the structure of the gradient networks, and discuss the relations between the transport capacity and the topology structure of the gradient networks.

The random gradient network can be obtained as follows:[18] consider a scalar field defined on a substrate network, the value $h(i)$ of the scalar field at

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node i is a random potential from the uniform distribution [0,1]. Then the gradient network can be shaped as the collection of directed edges that point from i to the neighbor with the minimum potential among all the neighbors. If the node i is isolated, a self-loop is set up. It is noted that the self-loop exists only for the isolated nodes as Ref. [22]. The measure of congestion of the network is defined by a jamming factor.[18] Let $N_{\rm send}$ be the number of nodes sending information and $N_{\rm receive}$ be the number of nodes that receive at least one gradient in-link. We assume that every node sends out one out-link, and then $N_{\rm send} = N$. The jamming factor J of the whole network can thus be defined as

$$J = 1 - \langle\langle N_{\rm receive}/N_{\rm send}\rangle_{\rm V}\rangle_{\rm network}, \tag{1}$$

where $\langle\cdots\rangle_{\rm V}$ and $\langle\cdots\rangle_{\rm network}$ denote the statistical average over local potential and the network configuration, respectively. Of course, $J = 1$ corresponds to the maximal jamming and $J = 0$ occurs when every link receives a gradient in-link, and the queue formation occurs when $N_{\rm receive} < N_{\rm send}$.

The jamming level $J(i)$ of each node i can be defined as[23]

$$J(i) = [L_{\rm in}(i) - L_{\rm out}(i)], \tag{2}$$

where $L_{\rm in}(i)$ denotes the number of gradient in-links received by node i, and $L_{\rm out}(i)$ denotes the number of gradient out-links sent by node i, and $L_{\rm out}(i) = 1$. The symbol $[x]$ means $[x] = x$ if $x > 0$; otherwise $[x] = 0$. If i is isolated, and then $J(i) = 1$. In fact, the jamming factor J of the whole network is also defined as the average over the jamming degree $J(i)$ of every node i,

$$J = \left\langle\left\langle \frac{\sum\limits_i J(i)}{N}\right\rangle_{\rm h}\right\rangle_{\rm network}. \tag{3}$$

In real networks, the scale field cannot be random and should be a dynamic quantity evolving in time. Here we propose a dynamic self-organized evolving mechanism of the scale field. The evolving mechanism is based on the idea that all the nodes i with congestion ($J(i) > 0$) have a desire to reduce their local congestion by increasing their potential, while these nodes cannot synchronously achieve their desire. The detailed strategy is described as follows: (1) according to the actual scale field, the gradient network is set up and the jamming level $J(i)$ of each node i is calculated. (2) A node i with local congestion is randomly selected, and then its potential is modified as $h(i) = h(i) + r$, where r is a random value from the uniform distribution in [0,1]. (3) Go to step (1), and repeat.

In the following we find that the congestion pressure can be strongly reduced by the self-organized optimization mechanism.

We study the effects of self-organized optimization mechanism on jamming. The substrate networks we considered are random Poisson and scale-free networks. To avoid the effects of the isolated nodes on jamming and to aim at studying the scale-free topology's effects on the transportation efficiency, for scale-free network, we choose the standard Barabási–Albert model,[24] where the growth and preferential attachment are two basic mechanisms, according to which a newly created node is connected to a pre-existing one with a probability exactly proportional to the number of links (degree) of the target node. For a random Poisson network, we choose the random growth network model,[25] where a newly created node is connected to a randomly chosen pre-existing one. The network size is chosen to be $N = 10000$, and we use the same $\langle k\rangle$ when compared with the two types of networks.

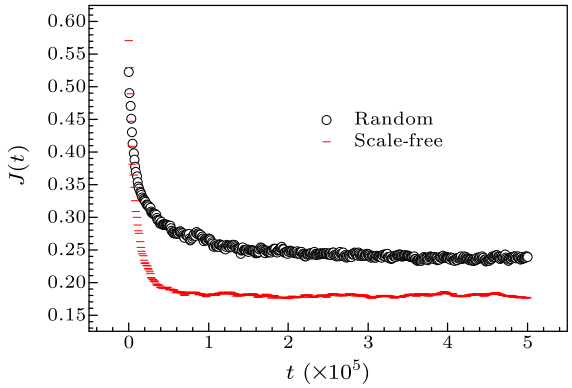

Fig. 1. The relation between jamming coefficient J and the evolutionary time t for the gradient networks based on scale-free network and random network. The average degree of the substrate network $\langle k\rangle = 4$.

In Fig. 1, the evolution of jamming coefficient J during optimization is shown as a function of evolutionary time t for average degree $\langle k\rangle = 4$. We can easily find that, regardless of network topology, the jamming coefficient J decreases with increasing the evolving time. The system reaches its optimal steady state and J reaches its minimum when the evolutionary time is enough longer. Furthermore, the jamming coefficient of scale-free network decreases rapidly and strongly compared with the random Poisson network. Thus the optimal scale-free network has a lower jamming coefficient after the optimization is completed, although it has a higher jamming when the gradient field is a random scale field. It is noted that the optimal jamming coefficient $J_{\rm R} \approx 0.24$, $J_{\rm SF} \approx 0.18$, which are less than the corresponding optimal values obtained by the Monte-Carlo update scheme in Ref. [22] where $J_{\rm R} \approx 0.36$, $J_{\rm SF} \approx 0.31$ under the same conditions. All these show that the self-organized optimization mechanism is more efficient compared with the man-made Monte-Carlo update scheme.

In Fig. 2, the difference $\Delta J_{\rm o} = J_{\rm SF} - J_{\rm R}$ between the jamming values of two optimal gradient networks versus $\langle k\rangle$ is reported. For comparison, we also show the relation between the difference $\Delta J_{\rm r} = J_{\rm SF} - J_{\rm R}$ and $\langle k\rangle$ when the scale field is random. It is easy to find that the optimal scale-free networks have the jamming coefficient lower than the optimal random networks for the average degree $\langle k\rangle > 2$, which shows

that the scale-free topology can lead to efficient transport than random construction under the condition of gradient-induced flows.

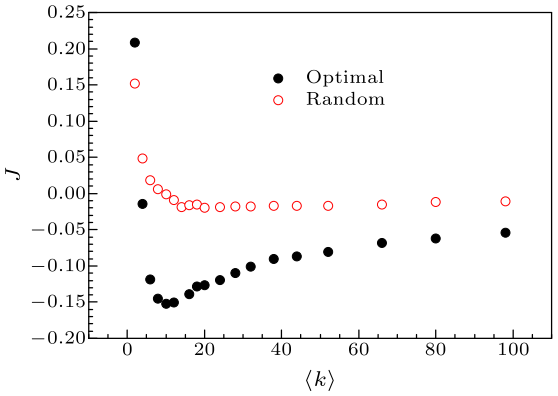

Fig. 2. Difference $\Delta J = J_{\rm SF} - J_{\rm R}$ between the jamming values of scale-free and random networks versus the average degree $\langle k\rangle$ when the gradient field is random (open circle) and optimal (solid circle). The simulations are averaged over 10^3 configurations.

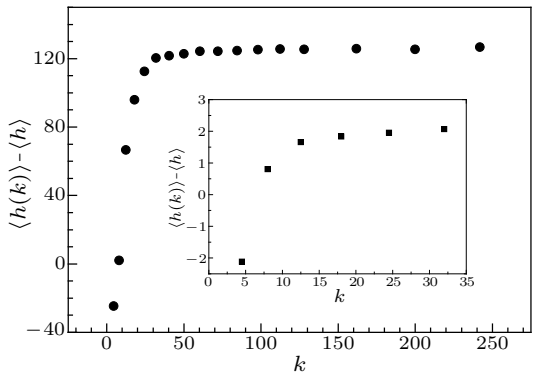

Fig. 3. Linear–linear plot of $\langle h(k)\rangle - \langle h\rangle$ as a function of k for optimal scale-free network (main plot) and optimal random network (inset). The average degree of the substrate network is $\langle k\rangle = 4$. The simulations are averaged over 10^6 configurations.

The transport capability of network is determined by the topology structure of the gradient network which depends on the structure of the substrate network and the properties of scale field. The self-organized optimization mechanism must change the structure of the gradient network by altering the properties of gradient field. Thus it is necessary to study the effects of the self-organized mechanism on the properties of scale field, and on the structure of the gradient network in the optimal steady state.

Firstly, we adopt the approach introduced in Ref. [21] to understand the correlations between the scale field and the local topology by evaluating $\langle h(k)\rangle - \langle h\rangle$ as a function of the degree k in the optimal steady state. The results are shown in Fig. 3. We can see that the height increases with the degree, which means that higher degree nodes have higher scalar values in the optimal steady state. In fact, if a node with higher degree has a smaller potential, this node will receive more gradient in-links from its neighbors, and thus it will contribute to higher jamming, thus to reduce jamming, the node with larger degree must have a larger potential. The same conclusions have been found by Piontti *et al.*[21] and Gulbahce.[22]

On the other hand, for the nodes with the same degree, their potentials should not be the same, and be likely to depend on the degree of their neighbors. To investigate the problem, for each node i, we define j to represent the neighbors of the node i. In Fig. 4, we report the dependence of $h_i - \langle h\rangle$ on M_i for the different nodes i with the same degree for two optimal networks. It can be seen that the potentials have obvious difference for different nodes although their degrees are the same, and the potential h_i increases with M_i. It can be analyzed that this correlated property is beneficial to reduce congestion. The degree of a node is relatively smaller if all its neighbors have larger degree, thus it should have a smaller scalar value to receive the gradient in-link. However, if any of its neighbors has a smaller degree, its degree is relative higher and then it has a larger scalar value to avoid the gradient in-link.

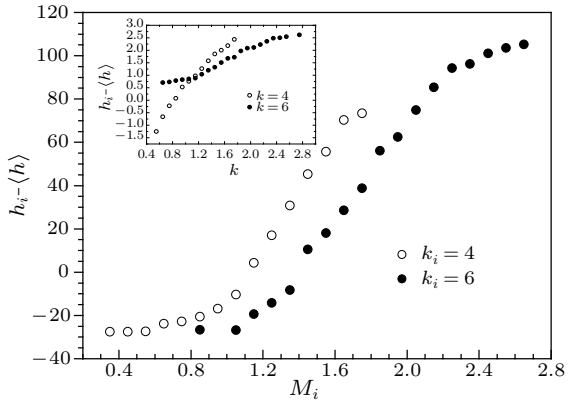

Fig. 4. Linear–linear plot of $h_i - \langle h\rangle$ as a function of M_i for the optimal scale-free network (main plot) and the optimal random network (inset). The degree of the nodes i considered is $k_i = 4$ (open circle) and $k_i = 6$ (solid circle), respectively. The average degree of the substrate network is $\langle k\rangle = 4$. The simulations are averaged over 10^6 configurations.

The correlations between the scale field and the local topology must change the topology structure of the gradient networks. The degree distribution of the network is a main property of network structure. In the following we discuss how the correlations affect it and how the transport capability depends on it. In Fig. 5, log–log plot of the in-degree distributions $L(k)$ of the optimal and random gradient networks based on scale-free and random networks are reported. We have found that the degree distributions of the optimal gradient networks have major changes compared with the random gradient network. The scaling becomes steeper and the cutoff degree of the gradient network becomes smaller. In particular, for scale-free substrate network, the cutoff degree of the optimal gradient network is reduced by an order of magnitude from roughly 200 to 20 compared with the random one. That is, there are higher in-degree nodes in random gradient networks and they will contribute to higher jamming,

while the nodes with higher in-degree are very few in the optimal gradient network, which is caused by the fact that the higher degree nodes have higher potentials. These observations offer evidence for the fact that the optimal gradient network is less congested than the random gradient network.

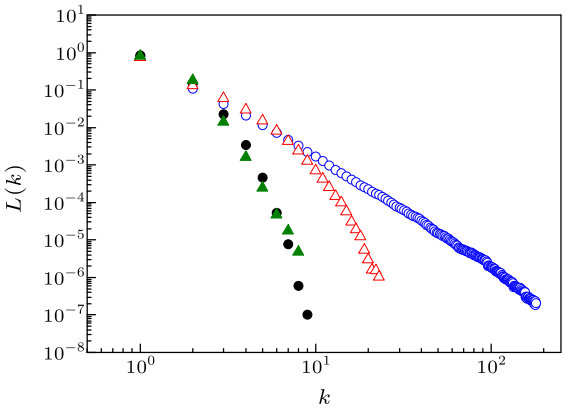

Fig. 5. Log–log plot of in-degree distributions $L(k)$ for the gradient networks based on random (triangle) and scale-free (circle) networks when the gradient field is random (open) and optimal ((solid). The average degree of the substrate network is $\langle k\rangle = 4$. The simulations are averaged over 10^3 configurations.

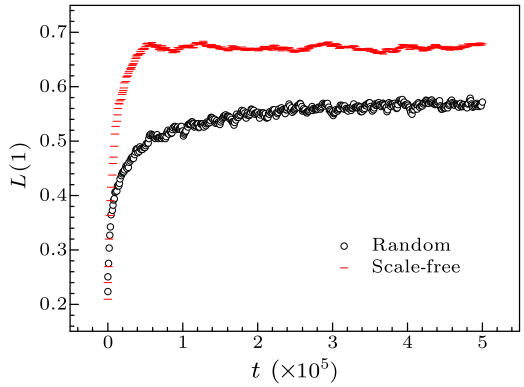

Fig. 6. The relation between the probability $L(1)$ that a randomly selected node has a gradient in-link and the evolutionary time t for the gradient networks based on scale-free and random networks. The average degree of the substrate network is $\langle k\rangle = 4$.

Another important way to investigate the effects of the structure of the gradient network on the transport capability is to evaluate the probability $L(1)$ that a randomly selected node has an incoming link in the gradient network. The nodes with a gradient in-link play an important role in network transport due to the fact that they offer their maximal transport capacity. Thus increasing the number of nodes with a gradient in-link is a very effective way to decrease jamming. In Fig. 6, we report the evolution of the probability $L(1)$ as a function of evolutionary time t for $\langle k\rangle = 4$. We find that the probability $L(1)$ becomes larger and larger and reaches its maximum when the system reaches its optimal steady state, which offers more evidence for which the optimal gradient network is less congested than the random gradient network. Specifically, $L(1)$ of scale-free network is larger than $L(1)$ of random network when the gradient field is optimal, although the contrary case happens when the gradient field is random, which gives an explanation why the scale-free network is less congested than a random network after the optimization is completed.

In summary, we have proposed a self-organized optimization mechanism to improve the transport capacity of complex gradient networks. We find that, regardless of network topology, the congestion pressure can be strongly reduced by the self-organized optimization mechanism, and the scale-free topology is more effective to reduce congestion under the optimization mechanism. Furthermore, the self-organized optimization mechanism is more efficient to reduce congestion compared with the Monte-Carlo optimization scheme. We study the effects of our optimization mechanism on the properties of scale field, and on the structure of the gradient network, and find that the optimization mechanism introduces the correlations between the gradient field and the local topology of the substrate network, and then strongly reduces the cutoff degree of the gradient networks and increases the number of the nodes exerting their maximal transport capacity. This work shows that the complex systems can improve their transport capacity by the self-organized mechanism which is usually more effective compared with the man-made mechanism.

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