Optimal Microwave Radiation Field Parameters for Mercury Ion Microwave Frequency Standards *

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We propose a method to determine the optimal power of the microwave resonance transition that simultaneously improves the signal-to-noise ratio and reduces line width based on saturation broadening theory and experiment. Saturation broadening spectra of the ground state hyperfine transition of trapped $^{199}Hg^{+}$ ions are measured and analyzed. The value of the optimal microwave power is obtained by using the proposed method and is verified. Rabi oscillations decay spectra of trapped $^{199}Hg^{+}$ ions are observed and the optimal microwave irradiation time for the maximum transition signal intensity is determined. This work will help to improve the short-term frequency stability of the mercury ion microwave frequency standard.

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A trapping mercury ion microwave clock that has a long coherence time[1] and a relatively low second-order Doppler shift,[2,3] as well as being insensitive to environmental perturbations[4,5] and suitable for miniaturization,[6] is regarded as one of the main candidates as a space clock and for timekeeping standards.[7,8] The short-term stability of the helium buffer-gas-cooled linear trap mercury ion microwave frequency standard is extraordinary and has not yet been significantly surpassed by any passive frequency standard.[9] Short-term frequency stability as an important indicator used to characterize the performance of frequency standards, is usually defined by using the Allen variance, $\sigma_y(\kappa)$, where κ is the sampling time. In the case of the passive atomic clock, the Allen variance can be approximated as

$$\sigma_y(\kappa) \propto \frac{1}{\pi Q}\frac{1}{\mathrm{SNR}}\frac{1}{\sqrt{\kappa}}, \tag{1}$$

where $Q = \nu/\Delta\nu$, with $\Delta\nu$ being the line width of the clock transition, and SNR is the signal-to-noise ratio. It is necessary to improve Q (i.e., reduce the line width) and SNR to improve the short-term frequency stability of the mercury ion microwave clock.

Attempts have been made to improve Q and SNR of the $^{199}Hg^{+}$ trapped ions microwave clock by using a novel linear ion trap as a replacement for the traditional Paul trap to improve the number of the trapped ions, which improves SNR too,[3,10] and time-separated oscillatory fields to incentivize hyperfine transitions which reduces the line width.[11,12] However, there have been few specialized research studies on the optimization of the microwave radiation field to improve the performance of the mercury ion clock. Optimizing the microwave radiation field is important due to the fact that it is the source of the hyperfine transition of $^{199}Hg^{+}$ ions and affects the signal intensity and line width. We report the successful observation of the saturation broadening spectrum and Rabi oscillation decay spectrum of the hyperfine transition of trapped $^{199}Hg^{+}$ ions. A method to determine the optimal microwave power of the microwave resonance transition is proposed based on the saturation broadening theory and experiment, and the value of the optimal microwave power of trapped $^{199}Hg^{+}$ hyperfine transition is calculated and verified. The optimal microwave irradiation time to obtain the maximum of transition signal is also obtained. Using the optimal parameters for the microwave resonance transition experiment could lead to improved performance of the mercury ion clock.

Due to the fact that the microwave radiation field couples with only the ground state hyperfine levels $^2S_{1/2}(F = 0, m_F = 0)$ and $^2S_{1/2}(F = 1, m_F = 0)$ of trapped $^{199}Hg^{+}$ ions, the $^{199}Hg^{+}$ ion can be viewed as a two-level model system.[13,14] In typical steady-state resonance absorption spectroscopy experiments, when ions interact with the radiation field, the absorbed power, P_S, can be expressed as[15,16]

$$P_S = \frac{N_0 - N_1}{2\tau}\cdot\frac{|2b|^2\hbar\omega}{(\omega-\omega_0)^2 + |2b|^2 + (1/\tau)^2}, \tag{2}$$

where N_0 and N_1 are the ion numbers of hyperfine levels $F = 0$ and $F = 1$ at the thermal equilibrium state, respectively, τ represents the decay time of the upper transition level, $F = 1$, $\nu = \frac{\omega}{2\pi}$ is the microwave frequency, $\nu_0 = \frac{\omega_0}{2\pi}$ is the frequency of the clock transition, $b = \frac{\mu_0 H_z}{2\hbar}$, with H_z being the projection of the applied magnetic field on the static magnetic field direction, $\hbar$ being the reduced Planck constant and μ_0 being the Bohr magneton, and $|2b|^2$ is proportional to the power of the applied microwave radiation field.

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The absorbed power increases when the applied microwave radiation field power increases; the resonance transition signal intensity, which is proportional to the absorbed power, also increases. However, at higher input power levels, the saturation of population densities broadens the transition spectra.[17,18] The full width at half maximum (FWHM) of the intensity including the saturation factor of the Lorentz profile is given by

$$\delta\omega = 2\sqrt{(1/\tau)^2 + |2b|^2} = \frac{2}{\tau}\sqrt{1+S}, \tag{3}$$

where S is the saturation factor, which describes the degree of saturation of the population densities. Due to the fact that $|2b|^2$ is proportional to the power of the applied microwave radiation field, P, it can be assumed that $P = C|b|^2$, where C is the scale factor and the relation between $\delta\omega$ and the microwave radiation field power becomes

$$\delta\omega^2 = 4\tau^{-2} + 16C^{-1}P. \tag{4}$$

To ensure that the transition signal is strong enough and the resonance line is as narrow as possible, we divide Eq. (2) (signal intensity) by Eq. (3) (line width), and then obtain the optimal condition of radiation field by maximizing about $|b|$ using the condition

$$|b|_{\rm opt} = \frac{1}{\sqrt{2}}\frac{1}{\tau}. \tag{5}$$

The optimal microwave power is obtained by taking both signal intensity and line width into account, and can be obtained by

$$P_{\rm opt} = C|b|^2_{\rm opt}. \tag{6}$$

To obtain a specific value for the optimal microwave power of the microwave resonance transition of trapped $^{199}Hg^+$ ions with this method, experimental saturation broadening data are required.

The experimental setup has been described in detail in our previous studies and we provide a brief introduction here.[19,20] As shown in Fig. 1, in our experiment the quadrupole linear ion trap was arranged in an ultra-high vacuum chamber with a pressure of 1×10^{-10} Torr. Neutral mercury atoms were created by heating HgO powder in an oven. Electron pulses emitted by an electronic gun bombarded neutral Hg atom vapor to generate ions. Hot mercury ions were trapped in the potential well by the radio-frequency field added to the four poles. To prevent ions from escaping along the axial direction, a direct current bias voltage was applied to the electrodes on both ends of the cap. The 194 nm state selection/interrogation light emitted from the $^{202}Hg^+$ discharge lamp was focused onto the center of the trap, and 40.5 GHz microwave radiation was fed into the trap by a horn to drive the hyperfine transition from $F = 0$, $m_F = 0$ to $F = 1$, $m_F = 0$ of $^{199}Hg^+$ ions. The fluorescence from the stored $^{199}Hg^+$ ions was collected by a photomultiplier tube (PMT).

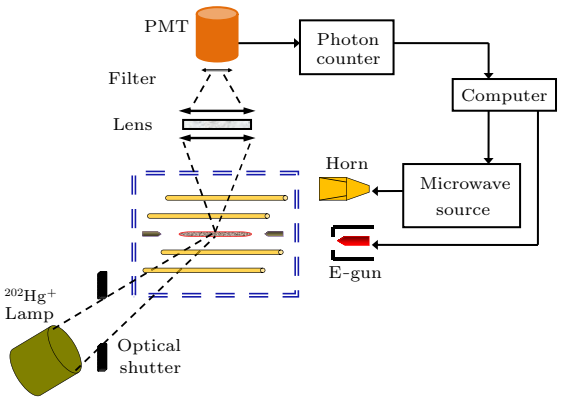


Fig. 1. (Color online) Experimental setup.

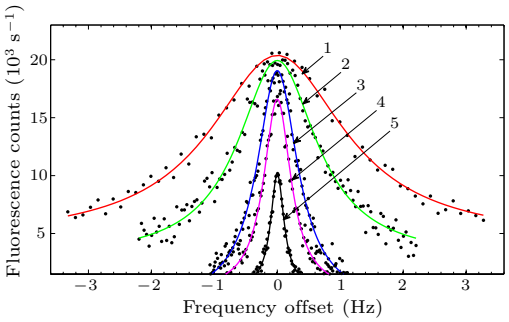


Fig. 2. (Color online) Saturation broadening spectra of the ground state hyperfine transition of the trapped $^{199}Hg^+$ ions. The microwave power increases successively from curve 5 (−50 dBm) to curve 1 (−30 dBm). The solid lines are the Lorentz fits of the experimental data, which are indicated by dots. Each set of experimental data is an average of 15 scans.

The saturation broadening spectra of the ground state hyperfine transition of the trapped $^{199}Hg^+$ ions are shown in Fig. 2. The spectra are obtained by high-resolution scanning near the 40.5 GHz microwave frequency with various microwave powers. The microwave power increases successively from curve 5 (−50 dBm) to curve 1 (−30 dBm). The deviation of the scattered data from the fitting curve is caused by the irregular shakes of the background stray light. The saturation effect gradually increases the line width of the transition signal as the power increases. Although the signal peak increases, signal broadening effects unfortunately made it difficult to differentiate between the transition signal and the background noise as the power increased.

The SNRs of the experimental spectra are determined by using the signal size measured at the half maximum of the resonance, $S_{\rm half}$, and the shot noise in the number of detected fluorescence, $\sqrt{S_{\rm half} + B}$, where B is the amount of collected background light that does not contribute to the signal, which gives SNR $= S_{\rm half}/\sqrt{S_{\rm half} + B}$. The results are shown in Fig. 3(a). As the microwave power increases, the SNR initially increases, then remains unchanged before finally dropping gradually. This result can be explained by considering the effect of the radiation field power on the signal. When the radiation field power is less than −47 dBm (2×10^{-8} W), the signal $S_{\rm half}$ increases rapidly with the increasing power, as

the SNR. Sufficiently large radiation intensity (greater than $-30\,\mathrm{dBm}$ ($1\times10^{-6}\,\mathrm{W}$)) causes a saturation of the population densities of the absorbing level, which dims the resonance phenomenon.[13] That is, as the radiation field power increases, the transition signal increases slowly while the saturation broadening effect and background noise B increase rapidly, which lead to the SNR remaining unchanged before finally decreasing. Therefore, the power of the microwave radiation field enhances the SNR in the low-power range and reduces the SNR when the power is too high. This phenomenon provides further evidence to support the assertion that the microwave power used in the resonance experiment can be neither too large nor too small.

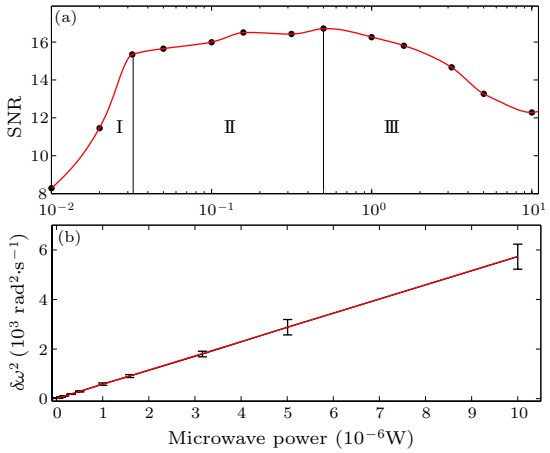


Fig. 3. (Color online) Variation of (a) SNR and (b) FWHM of the microwave resonance hyperfine transition with the microwave power.

The FWHM of the saturation broadening spectra under the corresponding microwave power, P, can be obtained by using a Lorentz fit, as shown in Fig. 3(b). The error bars in Fig. 3(b) are obtained from the statistical error of the Lorentz fit. As mentioned above, the line width increases as the applied radiation field power increases. The function related to the FWHM and microwave power in Fig. 3(b) can be fitted with a linear equation as

$$\delta\omega^2 = 7.889(\pm4.3) + 5.73(\pm0.013)\times10^8 P. \quad (7)$$

Comparing Eq. (4) and Eq. (7), the values of the decay time τ and the scale factor C can be deduced as $\tau = 0.7121(\pm0.1941)\,\mathrm{s}$ and $C = 2.7923(\pm0.0063)\times10^8$. Substituting the calculated values of τ and C into Eqs. (5) and (6) gives the optimal radiation field condition $|b|_{\mathrm{opt}} = 0.993(\pm0.2706)\,\mathrm{dBm}^{-1/2}$, corresponding to an optimal microwave power of $-45.6(\pm2.36)\,\mathrm{dBm}$.

Figure 4 presents the short-term frequency stabilities of the mercury ion microwave clock under different microwave powers, which are estimated from the SNR and line width shown in Fig. 3. The hyperfine clock transition frequency is 40507347996.8 Hz. As shown in Fig. 4, the lowest value of the short-term frequency stability is calculated near the optimal microwave power (approximately $4.47\times10^{-13}\kappa^{-1/2}$ at $-45.6\,\mathrm{dBm}$). If the microwave power deviates from the optimal value, low SNR or increasing line width increases the short-term stability, resulting in poor performance of the clock. This verifies that the proposed method for calculating the optimal microwave power can be used to improve the short-term frequency stability of the mercury ion microwave clock. This method could also be used in other similar microwave resonance transition experiments to search the optimal microwave power and to improve SNR and Q.

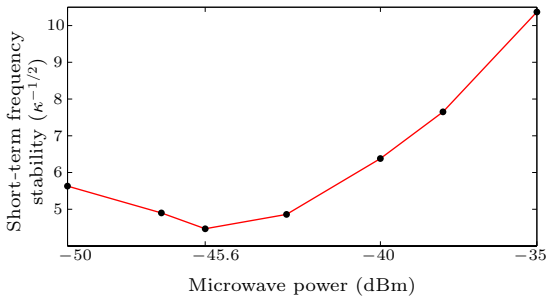


Fig. 4. Short-term frequency stabilities of the mercury ion microwave clock under different microwave powers.

To obtain the optimal microwave irradiation time of the ground state hyperfine transition of trapped $^{199}\mathrm{Hg}^+$ ions, the Rabi oscillation experiment is performed by scanning microwave irradiation time at a fixed microwave frequency, $\nu = \nu_0$. The Rabi oscillation decay spectrum observed is shown in Fig. 5. The measured data are fitted to

$$\rho(t) = \frac{e^{-2\gamma t} - 2e^{-\gamma t}\cos(\Omega t) + 1}{2(e^{-2\gamma t} + 1)}, \quad (8)$$

where $\rho(t)$ is the resonance transition probability of ions from level $F = 0$ to level $F = 1$ when interacting with the microwave radiation field for a time period t, and is proportional to the transition signal intensity, $\Omega = \sqrt{(\omega-\omega_0)^2 + |2b|^2}$ is the Rabi flopping frequency, and γ is the decay factor caused by relaxation mechanisms of the two-level system, such as spin charge exchange collision and finite ion storage time. Equation (8) was obtained by solving the time-dependent Hamiltonian equation[13] under the assumption that all ions are in the lower state $F = 0$ at $t = 0$ and equally populate in the two levels when $t = \infty$. As shown in Fig. 5, the resonance transition probability oscillates periodically as the microwave irradiation time t increases. However, due to the influence of the decay factor, γ, the probability amplitude tends to decrease overall. In the case of $\Omega t = \pi$, the periodic oscillation reaches the first peak and the degree of attenuation is relatively small. The transition signal intensity at this time, which is proportional to transition probability, is the strongest and the corresponding t is the optimal microwave irradiation time, t_{opt}, which can be calculated as $t_{\mathrm{opt}} = \frac{\pi}{\Omega}$. Due to the dependence of Rabi flopping frequency Ω on microwave radiation field intensity $|2b|^2$, the optimal microwave irradiation time t_{opt} also changes with the

microwave field power P, as shown in Fig. 6 and as described by

$$t_{\rm opt} = \frac{\pi}{\sqrt{c_1 + c_2 P}}, \tag{9}$$

where $c_1 = 2.151(\pm 0.374)$ and $c_2 = 7.7(\pm 1.508) \times 10^7$.

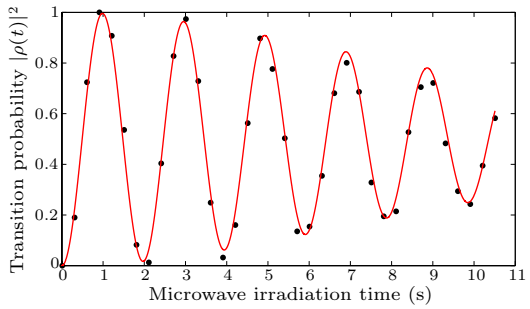


Fig. 5. (Color online) Rabi oscillation decay spectrum of trapped $^{199}Hg^+$ ions.

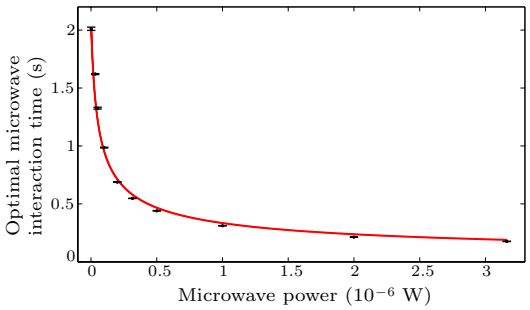


Fig. 6. (Color online) Dependence of the optimal microwave irradiation time on the microwave power.

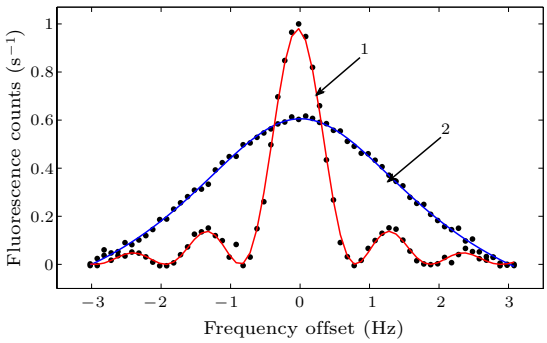


Fig. 7. (Color online) Comparison of the Rabi resonance transition signal of trapped $^{199}Hg^+$ ions with different radiation field parameters. Curve 1 is measured with the optimal radiation field parameters $P = -45.6$ dBm and $t = 1.5087$ s, while curve 2 is measured with $P = -37$ dBm and $t = 0.4$ s.

The optimal microwave irradiation time corresponding to the microwave field power $-45.6(\pm 2.36)$ dBm is $t_{\rm opt} = (1.5087 \pm 0.1395)$ s according to Eq. (9). A comparison of the Rabi resonance signals measured with and without the radiation field satisfying these optimal radiation field parameters is shown in Fig. 7. The Rabi resonance signal is obtained by scanning the microwave frequency with a fixed microwave irradiation time. Normalized fluorescence signal intensity is used to compare the signals. The signal intensity and line width of curve 1, which is measured with the optimal radiation field parameters, are better than those of curve 2.

In conclusion, we have proposed a method to determine the optimal microwave power of the microwave resonance transition to improve the SNR and to reduce the line width of the measured signal. The saturation broadening spectra of the ground state hyperfine transition of trapped $^{199}Hg^+$ ions are measured and analyzed. The SNR and FWHM of the transition signal are described in terms of the microwave field power. The developed equations are used to calculate the upper transition level decay time τ and the optimal radiation field condition $|b|_{\rm opt}$. By using the proposed method, the optimal microwave power to make the clock transition signal stronger and line width narrower is calculated to be $-45.6(\pm 2.36)$ dBm. The short-term frequency stabilities of the mercury ion microwave clock at different microwave powers are estimated. These calculations demonstrate that the short-term stability is improved significantly under the optimal microwave power. The Rabi oscillation decay spectrum of trapped $^{199}Hg^+$ ions is also observed. The optimal microwave irradiation time for the maximum transition signal intensity under specific microwave power is defined theoretically and measured experimentally. These results will be able to improve the transition signal size and the short-term stability of the mercury ion microwave clock.

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