A New Solution in Understanding Massive White Dwarfs *

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The observed high over-luminous type-Ia supernovae imply the existence of super-Chandrasekhar limit white dwarfs, which raises a challenge to the classical white dwarf theories. By employing the Eddington-inspired Born–Infeld (EiBI) gravity, we reinvestigate the structures and properties of white dwarfs, and find out that the EiBI gravity provides a new way to understand the observations. It is shown that by choosing an appropriate positive Eddington parameter κ, a massive white dwarf with mass up to $2.8M_\odot$ can be supported by the equation of state of free electron gas. Unlike the classical white dwarf theory, the maximum mass of the white dwarf sequence in the EiBI gravity is not decided by the mass–radius relations, but is decided by the central density, $\rho_c = 4.3 \times 10^{14}\ kg/m^3$, above which neutronization cannot be avoided and the white dwarf will transform into a neutron star. On the other hand, if the gravity in the massive white dwarf really behaves as the EiBI gravity predicts, then one can obtain a constraint on the Eddington parameter in the EiBI gravity, that is, $8\pi\rho_0\kappa G/c^2 \geq 80$ (where $\rho_0 = 10^{18}\ kg/m^3$) to support a massive white dwarf with mass up to $2.8M_\odot$. Moreover, we find out that the fast Keplarian frequency of the massive white dwarf raises a degeneration between the two kinds of compact stars, that is, one cannot distinguish whether the observed massive pulsar is a massive neutron star or a massive white dwarf only through the observed pulse frequency and mass.

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It is generally believed that when the mass of a white dwarf in the binary system approaches the Chandrasekhar mass limit $1.44M_\odot$,[1] where $M_\odot$ is the mass of the sun, type-Ia supernovae (SN Ia) will happen. Due to this unique feature, SN Ia can be regarded as standard candles for measuring far distances and thus in understanding the expansion history of the universe. However, some SN Ia dwarfs with exceptionally higher luminosity, such as SN 2003fg, SN 2006gz, SN 2007if and SN 2009dc,[2,3] have been observed in the past decade. The luminosity of this group of SN Ia implies that their progenitors ought to be white dwarfs with super-Chandrasekhar masses lying in a range of 2.1–$2.8M_\odot$.[2–7] Obviously, in the classical theoretical framework one cannot give a satisfactory explanation for the new observations as the equation of state (EOS) of free electron gas only can support a white dwarf with a maximum $1.44M_\odot$ in Newtonian gravity.[1] One question is naturally raised: do new mechanisms exist to support such a massive white dwarf? To answer this question, several models have been proposed in the past few years, including binary evolution of accreting differentially rotating white dwarfs,[8] rapidly rotating white dwarfs,[9] highly magnetized white dwarfs,[10,11] electronic charged white dwarfs,[12] using modified Einstein's gravity on white dwarfs,[13,14] and so on. However, all of these models have their own short slabs.[13,15] In fact, there is a similar problem for another compact star, neutron star. To the recent discovery of two massive neutron stars: PSR J1614+2230 (with mass of $1.97\pm0.04M_\odot$)[16] and PSR J0348+0432 (with mass of $2.01\pm0.04M_\odot$),[17] on the one hand, it can be explained by rebuilding the EOS of the super dense matters;[18–21] on the other hand, considering the larger uncertainties of the nuclear matters EOS at very high densities, many other mechanisms are also proposed,[22–25] such as employing the new gravity theories.[23–25]

Similar to the neutron star, for the massive white dwarf, considering its strong gravity, the Newtonian gravity theory should not be suitable anymore for investigating its structures and properties. On the other hand, although the most successful contemporary gravity theory is general relativity (GR), which has been well tested in the weak gravity case, it is still an open problem whether gravity also behaves as predicted by GR in strong gravity situations or at cosmological scales. Moreover, in both the Big Bang model and the black hole model, GR cannot avoid the singularity problems. In 2010, one modified Einstein's gravity theory was proposed by Bañados *et al.*,[26] which was called the Eddington-inspired Born–Infeld (EiBI) gravity theory and has been gaining great attention.[24,27–31] This new theory can avoid the Big Bang singularity and can reduce to GR in a vacuum.[26] In GR, the gravity couples to matters linearly as the Einstein tensor $G_{\mu\nu}$ is proportional to the stress-energy tensor $T_{\mu\nu}$ in the Einstein field equations, while in EiBI it is nonlinear coupling between the matters and the gravity, and thus a significant de-

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viation between the GR and EiBI should happen at very high densities. Due to this characteristic, the EiBI theory has been used successfully in explaining the massive neutron stars.[24] Hence, we expect that the EiBI gravity theory can provide us with a new way to understand the massive white dwarfs.

The EiBI gravity, an alternative to GR, was proposed by Bañados *et al.* The theory is described by the action

$$S = \frac{1}{16\pi}\frac{2}{\kappa}\int d^4x(\sqrt{-|g_{\mu\nu}+\kappa R_{\mu\nu}|} - \lambda\sqrt{-g}) + S_{\rm M}[g, \Psi_M], \quad (1)$$

where g_{ab} is the metric, R_{ab} is the Ricci tensor, and the Eddington parameter κ is the only new parameter of the theory through setting $\lambda = 1$. When $\kappa \to 0$, the theory is identical to GR. The matter action $S_{\rm M}$ depends on the matter field $\Psi_{\rm M}$ and metric $g_{\mu\nu}$.

Considering the metric $g_{\mu\nu}$ and the connection $\Gamma^{\alpha}_{\beta\gamma}$ as independent fields in the EiBI theory, variation of the action Eq. (1) leads to

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}, \quad (2)$$

$$\sqrt{-q}q^{\mu\nu} = \sqrt{-g}g^{\mu\nu} - 8\pi\kappa\sqrt{-g}T^{\mu\nu}, \quad (3)$$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}q^{\alpha\sigma}(\partial_\gamma q_{\sigma\beta} + \partial_\beta q_{\sigma\gamma} - \partial_\sigma q_{\beta\gamma}), \quad (4)$$

where $q_{\mu\nu}$ is an auxiliary metric. Then the line element for the spacetime metric $g_{\mu\nu}$ and for the auxiliary metric $q_{\mu\nu}$ can be written as[29]

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(r)}c^2dt^2 + e^{\lambda(r)}dr^2 + f(r)d\theta^2 + f(r)sin^2\theta d\phi^2, \quad (5)$$

$$q_{\mu\nu}dx^\mu dx^\nu = -e^{\beta(r)}c^2dt^2 + e^{\alpha(r)}dr^2 + r^2d\theta^2 + r^2sin^2\theta d\phi^2. \quad (6)$$

Combining the above equations with Eq. (3), one can obtain

$$e^\beta = e^\nu b^3 a^{-1},\ e^\alpha = e^\lambda ab,\ f = r^2a^{-1}b^{-1}, \quad (7)$$

where $a = \sqrt{1+\frac{8\pi G}{c^2}\kappa\rho}$ and $b = \sqrt{1-\frac{8\pi G}{c^4}\kappa p}$.

According to Eqs. (2) and (3), the modified Einstein equation can be written as[29]

$$G^\mu_\nu \equiv R^\mu_\nu - \frac{1}{2}R\delta^\mu_\nu = 8\pi\tau T^\mu_\nu - \Big(\frac{1-\tau}{\kappa} + 4\pi\tau T\Big)\delta^\mu_\nu, \quad (8)$$

where $\tau = \sqrt{g/q}$, $R = R^\mu_\mu$ and $T = T^\mu_\mu$. The stress-energy tensor T^μ_ν of a static neutron star can be described by a perfect fluid, that is,

$$T_{\mu\nu} = (p + \rho c^2)u_\mu u_\nu + pg_{\mu\nu}. \quad (9)$$

According to the modified Einstein equation, the gravitational field equations describing the structure of a compact star can be obtained as[29]

$$\frac{d}{dr}(re^{-\alpha}) = 1 - \frac{1}{2\kappa}\Big(2 + \frac{a}{b^3} - \frac{3}{ab}\Big)r^2, \quad (10)$$

$$e^{-\alpha}\Big(1 + r\frac{d\beta}{dr}\Big) = 1 + \frac{1}{2\kappa}\Big(\frac{1}{ab} + \frac{a}{b^3} - 2\Big)r^2. \quad (11)$$

On the other hand, the conservation of the stress-energy tensor gives

$$0 = \nabla_\nu T^\mu_\nu = \frac{dp}{dr} + \frac{\nu'}{\rho c^2 + p}. \quad (12)$$

Combining Eqs. (10)–(12), one can obtain the q-metric generalization of standard TOV equation as[29]

$$\frac{db}{dr} = \Big\{ab(a^2-b^2)\Big[\frac{1}{2\kappa}\Big(\frac{1}{ab} + \frac{a}{b^3} - 2\Big)r^3 + \frac{2Gm}{c^2}\Big]\Big\}/\Big\{r^2\Big(1 - \frac{2Gm}{c^2r}\Big)[4ab^2 + 3a(a^2 - b^2) - b(a^2-b^2)c_q^2]\Big\}. \quad (13)$$

Substituting $\frac{db}{dr} = -\frac{4G\pi\kappa}{c^4 b}\frac{dp}{dr}$ and $c_q^2 = -\frac{bc^2}{a}\frac{d\rho}{dp}$ into Eq. (13), we can obtain

$$\frac{dp}{dr} = -\Big\{\frac{c^4}{4\pi G\kappa}ab(a^2-b^2)\Big[\frac{1}{2\kappa}\Big(\frac{1}{ab} + \frac{a}{b^3} - 2\Big)r^3 + \frac{2Gm}{c^2}\Big]\Big\}/\Big\{r^2\Big(1 - \frac{2Gm}{c^2r}\Big)\Big[4ab^2 + 3a(a^2 - b^2) + \frac{b^2c^2(a^2-b^2)}{a}\frac{d\rho}{bp}\Big]\Big\}, \quad (14)$$

where m is given by integrating Eq. (10), that is,

$$e^{-\alpha} = 1 - \frac{2Gm(r)}{c^2r}, \quad (15)$$

with

$$\frac{dm}{dr} = \frac{c^2}{4G\kappa}\Big(2 + \frac{a}{b^3} - \frac{3}{ab}\Big)r^2. \quad (16)$$

The physical mass $M(r)$ of the compact star is defined as

$$e^{-\lambda} = 1 - \frac{2GM(r)}{c^2r} = \Big(1 - \frac{2Gm(r)}{c^2r}\Big)ab. \quad (17)$$

Thus it is easy to obtain the relation between the physical masses $M(r)$ and $m(r)$ in the physical g and auxiliary q metrics,

$$M(r) = \frac{c^2r}{2G}(1-ab) + m(r)ab. \quad (18)$$

Theoretically, giving an EOS $p = p(\rho)$, the structures of compact stars in the EiBI gravity can be calculated by solving Eqs. (14) and (16) under appropriate boundary conditions, $e^{\lambda(r)} = 1$ and $\rho(r) = \rho_{\rm c}$ at $r = 0$; $p(r) = 0$, $a(r) = b(r) = 1$ and $M(R) \equiv m(R)$ at $r = R$.

However, as the auxiliary metric $q_{\mu\nu}$ contains both $g_{\mu\nu}$ and $T_{\mu\nu}$ and thus the Ricci tensor $R_{\mu\nu}$ involves second derivatives of the matter field,[31] it becomes more difficult to solve the structure equations in the

EiBI theory than in general relativity. Recently, Delsate *et al.*[32] found that the field equations in EiBI can be rewritten in a form similar to the Einstein field equations in general relativity, that is,

$$G^{\mu}_{\nu} \equiv R^{\mu}_{\nu} - \frac{1}{2}R\delta^{\mu}_{\nu} = 8\pi\tau T^{\mu}_{\nu} - \Big(\frac{1-\tau}{\kappa} + 4\pi\tau T\Big)\delta^{\mu}_{\nu} \equiv 8\pi\tilde{T}^{\mu}_{\nu}, \quad (19)$$

where the tensor G^{μ}_{ν} is defined by the auxiliary metric $q_{\mu\nu}$, and $\tilde{T}^{\mu}_{\nu}$ is defined as the apparent stress-energy tensor in the form of

$$\tilde{T}^{\mu}_{\nu} = (\tilde{\rho}c^2 + \tilde{p})\upsilon^{\mu}\upsilon_{\nu} + \tilde{p}\delta^{\mu}_{\nu}, \quad (20)$$

in which the apparent mass density $\tilde{\rho}$ and pressure $\tilde{p}$ are defined by

$$\tilde{\rho}c^2 = \tau\rho c^2 - \bar{p}, \quad (21)$$

$$\tilde{p} = \tau p + \bar{p}, \quad (22)$$

where $\bar{p} \equiv (\tau-1)c^4/(8\pi\kappa G) - \tau(3p-\rho c^2)/2$. Moreover, the apparent four velocities υ^{μ} satisfy the conditions

$$\upsilon^{\mu}\upsilon^{\nu}q_{\mu\nu} = -1, \ \upsilon^{\mu}\upsilon_{\nu} = u^{\mu}u_{\nu}. \quad (23)$$

It is worth noting that the indices of υ^{μ} and u^{μ} should be lowered by $q_{\mu\nu}$ and $g_{\mu\nu}$, respectively.

Now we can numerically calculate the structure of a compact star in a conversant way, that is, by employing the apparent EOS given by Eqs. (21) and (22), we can simply solve the familiar TOV equations in general relativity to obtain the structures of a compact star in the EiBI gravity.

Recent observations suggest that some peculiar type-Ia supernovae have massive progenitors, white dwarfs in the binary system with mass up to 2.1–2.8$M_{\odot}$, which is far higher than the classical Chandrasekhar limit.[2,10,33] In the classical theoretical framework, one cannot give a satisfactory explanation for the new observations, that is, there is a discrepancy between the new observations and the classical white dwarf theories. This discrepancy presages that some new physics may be stored in the massive white dwarfs.

In the following we use the EiBI gravity to investigate the structures and properties of white dwarfs and it demonstrates that this new gravity theory can give a self-consistent explanation for the massive white dwarfs.

Normally, the EOS of white dwarfs can be described by the free electron gas,

$$k_{\rm F} = \hbar\Big(\frac{3\pi^2\rho}{m_p\mu}\Big)^{\frac{1}{3}}, \quad (24)$$

$$p = \frac{8\pi c}{3(2\pi\hbar)^3}\int_0^{k_{\rm F}} \frac{k^2}{(k^2 + m_{\rm e}^2c^2)^{\frac{1}{2}}}k^2dk, \quad (25)$$

where k is the momentum of electron, and μ is the ratio of nucleon numbers to electron numbers.

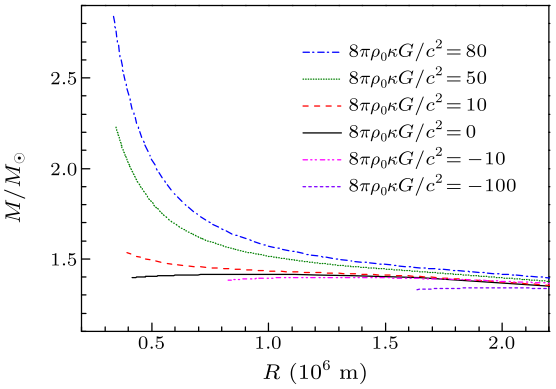

Fig. 1. (Color online) Mass-radius relations in the EiBI gravity with different values of the coupling parameter κ, where $\rho_0 = 10^{18}\,{\rm kg/m^3}$. The black solid line $\kappa = 0$ denotes the GR case, in which the classical Chandrasekhar result $M_{\rm max} \sim 1.4M_{\odot}$ recovers.

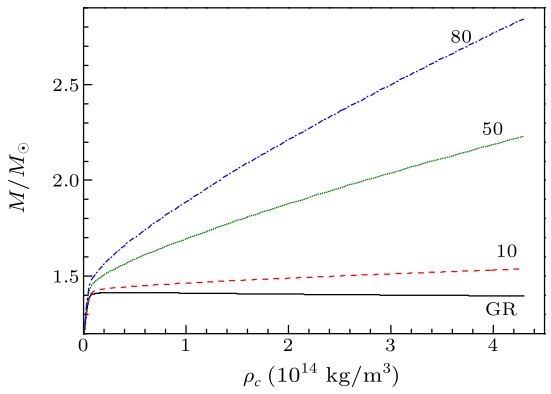

Fig. 2. (Color online) Mass-density relations for different values of the coupling parameter κ, where the numbers beside the lines are the values of $8\pi\rho_0\kappa G/c^2$ with $\rho_0 = 10^{18}\,{\rm kg/m^3}$, and GR represents the case of $8\pi\rho_0\kappa G/c^2 = 0$, which reduces to the general relativity gravity. Hereafter we only focus on $\kappa > 0$.

Employing the EOS of free electron gas, we numerically calculate the structures and properties of white dwarfs in the EiBI gravity. In Fig. 1, the mass–radius relations of white dwarfs in EiBI are presented, where a dimensionless parameter $8\pi\rho_0\kappa G/c^2$ is adopted to denote the Eddington parameter, in which $\rho_0 = 10^{18}\,{\rm kg/m^3}$, and the line $8\pi\rho_0\kappa G/c^2 = 0$ denotes the case in GR. It is shown that a positive value of the parameter $8\pi\rho_0\kappa G/c^2$ can augment the maximum mass of the white dwarf sequence, while a negative parameter will lessen the maximum mass. Adopting an appropriate parameter, that is, $8\pi\rho_0\kappa G/c^2 = 80$, a massive white dwarf with mass up to 2.8$M_{\odot}$ can be obtained before the central density reaches $\rho_{\rm c} = 4.3\times10^{14}\,{\rm kg/m^3}$, an upper limit density to avoid neutronization. This result can be seen more clearly in Fig. 2. It is worth noting that in the case in GR, there is a maximum mass in the mass–radius sequence,

while in the case $8\pi\rho_0\kappa G/c^2 \neq 0$, there is no maximum mass in the mass-radius relation. How can we obtain the maximum mass of the white dwarf sequence? As shown in Fig. 2, the maximum mass of white dwarf sequence in the EiBI gravity should be decided by the upper central density limit $\rho_c = 4.3 \times 10^{14}\,\mathrm{kg/m^3}$, above which electron capture will happen and the white dwarf will transform into a neutron star. In the massive white dwarfs in EiBI, they have a relative higher $\frac{GM}{c^2R}$. For example, in the case $8\pi\rho_0\kappa G/c^2 = 80$ with $M = 2.8M_\odot$, $\frac{GM}{c^2R} \approx 1.2 \times 10^{-2}$; while in the case of GR with $M = 1.4M_\odot$, $\frac{GM}{c^2R} \approx 2.3 \times 10^{-3}$ (as shown in Fig. 1), this means that there is a relatively stronger gravity in a massive white dwarf. This is just the reason why the Newtonian gravity is not proper to the massive white dwarf anymore.

On the other hand, if the gravity in massive white dwarfs really behaves as the EiBI gravity predicts, a constraint on the Eddington parameter in the EiBI gravity can be obtained, that is, to support a massive white dwarf with a mass up to $2.8M_\odot$, we must have $8\pi\rho_0\kappa G/c^2 \geq 80$.

It is worth pointing out that though the work of Pani *et al.*[24] has given the mass–radius relation of white dwarf in the EiBI gravity, there is no constraint on the Eddington parameter and the mass of the star can be over $1000M_\odot$ (see Fig. 7 in Ref. [24]). Moreover, there is also no constraint on the central density and even the central density can be larger than the saturation nuclear density (see the right panel of Fig. 7 in Ref. [24]). To seek a valid mechanism to support the massive white dwarf, we carry out a detailed calculation of the white dwarf structure in the EiBI gravity theory and obtain the constraint on the Eddington parameter through the possible upper limit mass of white dwarf and the possible maximum central density.

As has been pointed out, if we use the apparent EOS described by Eqs. (21) and (22), the structures and properties of white dwarfs in the EiBI gravity can be directly calculated by the structure equations in GR. Thus we will employ the apparent EOS to calculate the moment of inertia of the white dwarf in the EiBI gravity. In the framework of GR, ignoring the rotational deformation, the moment of inertia for the rotating compact star can be written as[34]

$$I = \frac{8\pi}{3}\int_0^R drr^4 \frac{\rho(r) + p(r)/c^2}{\sqrt{1 - \frac{2Gm(r)}{c^2 r}}} e^{-\beta(r)/2}. \tag{26}$$

The Kepler frequency of a compact star satisfies the relation[35]

$$\Omega_\mathrm{k} \approx \left[1 + \frac{\omega(R)}{\Omega_\mathrm{k}} - 2\left(\frac{\omega(R)}{\Omega_\mathrm{k}}\right)^2\right]^{(-1/2)} \sqrt{\frac{GM}{R^3}}. \tag{27}$$

Considering $\omega(R)/\Omega_\mathrm{k} = 2GI/c^2r^3$,[35] we can write the Kepler frequency as

$$\Omega_\mathrm{k} = \left[1 + \frac{2GI}{c^2r^3} - 2\left(\frac{2GI}{c^2r^3}\right)^2\right]^{(-1/2)} \sqrt{\frac{GM}{R^3}}. \tag{28}$$

Normally, in the classical theory, white dwarfs with mass of $1.4M_\odot$ have fiducial parameters $R = 10^6$ m and $I = 10^{42}\,\mathrm{kg{\cdot}m^2}$.[36] For the massive white dwarf, due to its stronger gravity and thus a relatively smaller figure, it is expected that the massive white dwarf should have a smaller moment of inertia and thus can support a faster spin frequency. The moments of inertia of the massive white dwarfs in the EiBI gravity are plotted in Fig. 3. It is shown that for the massive white dwarfs with mass $>2.0M_\odot$, its moment of inertia is of the order of $I = 10^{41}\,\mathrm{kg{\cdot}m^2}$, which is lower in an order of magnitude compared with the classical white dwarf model with mass of $1.4M_\odot$. On the other hand, in the lower mass region ($<1.4M_\odot$), the effect of the coupling parameter κ can be ignored, while to the massive white dwarf, there is an obvious effect of κ on the moment of inertia, as shown in Fig. 3.

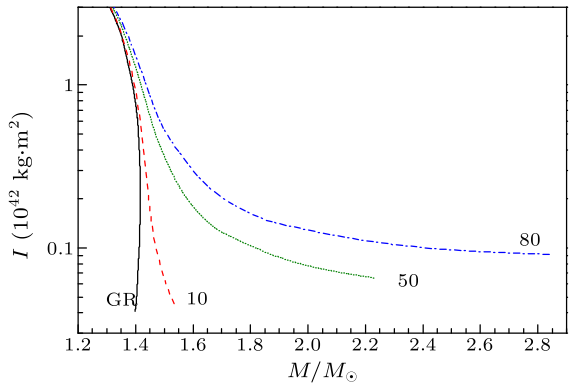

Fig. 3. (Color online) Moment of inertia as a function of the stellar mass for different κ, where the numbers beside the lines are the values of $8\pi\rho_0\kappa G/c^2$ with $\rho_0 = 10^{18}\,\mathrm{kg/m^3}$, and GR represents the case of $8\pi\rho_0\kappa G/c^2 = 0$.

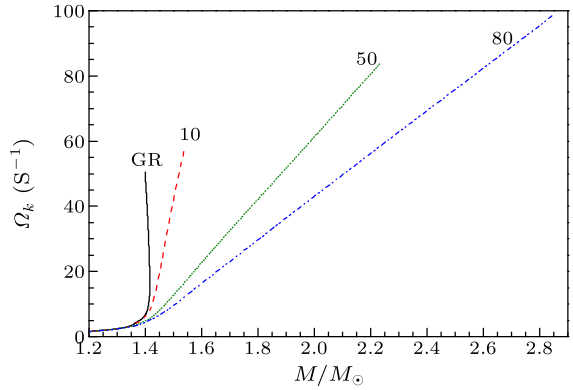

Fig. 4. (Color online) Kepler frequency Ω_k as a function of the stellar mass for different parameter κ.

One natural consequence of the above results is that the massive white dwarf can support a faster

Keplerian frequency, as shown in Fig. 4. The numerical results show that for a white dwarf with mass up to $2.8M_{\odot}$, its Keplerian spin frequency can be up to $98\,\mathrm{s}^{-1}$, that is, this massive white dwarf can spin once in about 0.064 s. Such a small spin period makes an overlap between the spin periods of neutron stars and white dwarfs. As has been argued in Ref. [36], some observed pulsars, such as anomalous x-ray pulsars (AXPs) and soft gamma ray repeaters (SGRs) (e.g., SGR 0418+5729 and 1E 2259+586), which were considered as low magnetic field magnetars, also can be explained by the massive fast rotating highly magnetized white dwarfs. Similar degeneration is also raised: to a massive compact star in a binary system located very far away from our earth, if we only can observe its spin pulsar and stellar mass, then both the neutron star models and the white dwarf models can describe the observed massive pulsar. Obviously, the best way to break this degeneration is to obtain the observed radius for the massive pulsar. Unfortunately, nowadays researchers cannot give a compelling and accurate data on the radius of an observed pulsar. In this sense, we will face a new challenge to distinguish whether the massive pulsar is a massive neutron star or a massive white dwarf.

In summary, we have introduced an effective mechanism to explain the observed over-luminous type-Ia supernovae which prefer massive white dwarfs. By employing the Eddington-inspired Born–Infeld (EiBI) gravity to investigate the structures and properties, we provide a new way to understand the observed massive white dwarfs. It is shown that by choosing an appropriate positive Eddington parameter, a massive white dwarf with mass up to $2.8M_{\odot}$ can be obtained. On the other hand, if the gravity in a massive white dwarf really behaves as the EiBI gravity predicts, then one can obtain a constraint on the Eddington parameter in the EiBI gravity. Moreover, we find out that the small spin period of massive white dwarfs in the EiBI gravity makes an overlap between the spin periods of neutron stars and white dwarfs. This result raises a challenge to distinguish whether the observed massive pulsar is a massive neutron star or a massive white dwarf only through the observed pulse frequency and mass.

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