Characterization of Elastic Modulus of Granular Materials in a New Designed Uniaxial Oedometric System *

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A simple uniaxial oedometric system is developed to test the elastic modulus of granular materials. The stress–strain relationship is first measured under conditions of uniaxial compression with additional lateral stress and strain, then the elastic modulus of the material is determined by the linear fitting method. It is found that the modulus is positively correlated to the grain size and negatively correlated to the container size. Arching and dragging are revealed to be the mechanism of such correlations by using the digital image correlation method and the pressure film technology based on the statistical method.

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Granular materials, which is also called granular media, exhibits mechanical behavior different from liquids and solids owing to their discrete nature.[1,2] In particular, the deformation property such as the elastic modulus under static load should be evaluated because it is one of the critical parameters for material physics and engineering designs. For example, the pebble bed core is well known to be a key component of the high temperature reactor in nuclear engineering, and its structure should be strong enough to make sure the fuel spheres cannot escape.[3] Therefore, the elastic modulus, which tells us how much it deforms when it is subjected to a given increase of stress, becomes a valuable parameter for safe designing to prevent the structural failure.

It is due to the importance of application background, Young's modulus of granular materials is of interest to researchers in the past few decades.[4–9] In fact, the previous studies mainly focused on the experiments which seem to be an effective way as far as the precise value for modulus is concerned. Kiyota *et al.* found that Young's modulus appears as an increasing trend with the density at the same stress level by measuring three kinds of granular media (Toyoura sand, Hime gravel and glass beads) through a triaxial test.[4] For the coarse granular materials, the modulus of the specimen with the large particle size is higher than that of the small one in the large-scale triaxial testing. It is inferred that the higher modulus of the larger grain samples is caused by particle chains constructed by direct contact between large aggregates.[5] The stress–strain relationship obtained by a uniaxial oedometric test for sand is used to determine the elastic moduli of non-cohesive particulate materials from the linear stage of unloading.[6] Following this work, the stress–strain relationship of cereal grain and rapeseed is modeled and fitted to the experimental data during loading and unloading performed on the material with several moisture content levels.[7] Different from the traditional triaxial tests and uniaxial oedometric tests, the acoustic method has been used to measure the elastic modulus in recent years. The elastic modulus of a bulk of cereal grains and rapeseeds is found to increase with an increase in hydrostatic pressure and to decrease with increasing moisture content by measuring compression wave velocities.[8] However, Young's modulus evaluated from the acoustic method is slightly larger than that from the measurement in large-scale triaxial tests.[9]

Notice that most notable achievements on granular materials provide interesting findings on the deformation behaviors, while the apparatus used in the triaxial and uniaxial tests are complex and expensive. Furthermore, these works study mechanical properties only limited to the measurement, but still lack adequate investigations for the mechanism which needs to be fundamentally understood.

In this Letter, a simple uniaxial oedometric system is first designed to measure the stress–strain relationship with additional lateral stress and strain, and then to determine the elastic modulus of the granular materials using the linear fitting method. Finally, the mechanism of deformation behaviors is investigated under the size effect by the digital image correlation

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(DIC) method and the pressure film technology.

An experimental system for the uniaxial oedometric test is constructed as shown in Fig. 1(a).

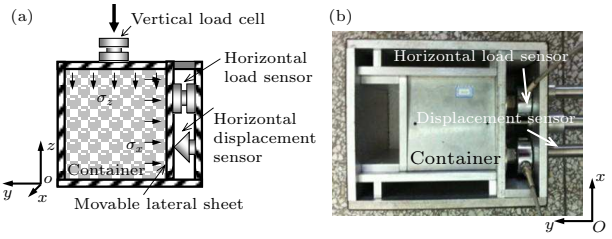

Fig. 1. Schematic and photograph of the oedometric uniaxial compression system: (a) lateral view of diagrammatic sketch, and (b) top view of the apparatus.

The system consists of two parts: a length adjustable cubic container, load cells and sensors. The container is made of a 10-mm-thick aluminum plate to ensure enough rigidity under the applied load. One of the four lateral sheets of the container is movable. The sensors are allowed to measure the vertical and horizontal loads and displacements, thus the mean pressure acting on the top and lateral sheets can be observed. The advantage of the apparatus can directly measure the lateral force and displacement to obtain the value of elastic modulus based on the new mechanical model. Compared with the other devices reported, it greatly simplifies the measuring process. Moreover, the size of the container can be adjusted to suit for different sizes of particles conveniently, thus the apparatus is more general.

Figure 2 is a typical stress–strain curve of granular media. Considering its nonlinearity, the loading process is divided into k ($k = 0, 1, 2, \ldots$) linear steps to facilitate the analysis. According to the theory of elastic mechanics, the modulus E_k and Possion's ratio v_k for the step k can be expressed as[10]

$$E_k = \frac{(1-v_k^2)(\Delta\sigma_z)_k - v_k(1+v_k)(\Delta\sigma_x)_k}{(\Delta\varepsilon_z)_k}, \quad (1)$$

$$\nu_k = \frac{(\Delta\varepsilon_z)_k(\Delta\sigma_x)_k - (\Delta\varepsilon_x)_k(\Delta\sigma_z)_k}{[(\Delta\varepsilon_z)_k - (\Delta\varepsilon_x)_k][(\Delta\sigma_z)_k + (\Delta\sigma_x)_k]}, \quad (2)$$

where ε_x and ε_z are the strain in the x and z directions, and σ_x and σ_z are the stress in the x and z directions, respectively. All these parameters can be obtained from experiments. Further, $(\Delta\sigma_x)_k = (\sigma_x)_k - (\sigma_x)_{k-1}$ and $(\Delta\sigma_z)_k = (\sigma_z)_k - (\sigma_z)_{k-1}$ are the stress increments, $(\Delta\varepsilon_x)_k = [(u_x)_k - (u_x)_{k-1}]/L$ and $(\Delta\varepsilon_z)_k = [(u_z)_k - (u_z)_{k-1}]/L$ are the strain increments, L is the length of specimen, and u_x and u_z are the mean displacements in x and z directions, respectively. Generally, granular materials are considered to be the continuous solids without side limits in engineering.[10] The stress–strain relationship on the loading direction of the granular media can be rewritten as

$$\sigma_k = \sigma_{k-1} + E_k\Delta\varepsilon_k$$
$$\varepsilon_k = \varepsilon_{k-1} + \Delta\varepsilon_k, \quad (3)$$

where the initial value of strain, i.e., ε_0, is set as 0.

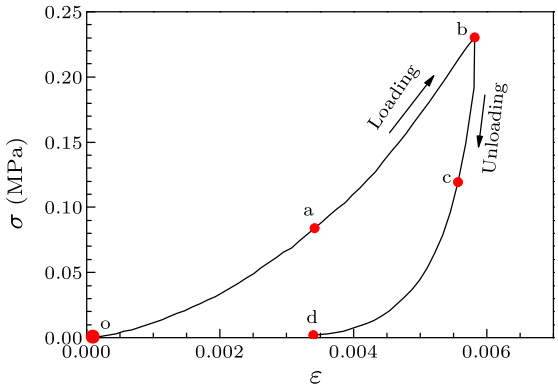

Fig. 2. Typical stress–strain curve: the range from point b to point c stands for the linear range, which is fitted as E.

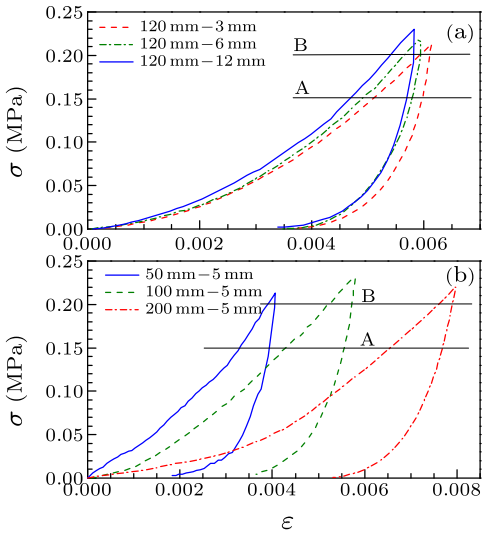

Fig. 3. The stress–strain curves for three sizes of (a) particles and (b) containers.

In fact, the stress–strain relationship shown in Fig. 2 performs two typical stages, i.e., loading and unloading. In the loading stage, the path o–a indicates the translation and rotation movements of particles, and a–b represents an increase in plastic and elastic stresses in the media. In the unloading stage, the path c–d is the nonlinear part of unloading with the vertical stress decrease, and b–c reflects the linear characteristic of the elastic deformation, which is used to determine the elastic modulus E generally.

The experiment to measure the elastic modulus for dry, noncohesive and monodispersed glass beads is performed by using our apparatus. Three different sizes of particles ($\phi = 3$, 6 and 8 mm) and cubic containers ($L = 50$, 100 and 200 mm) are especially

selected to observe the size effect on the elastic modulus. The sample is first poured into the test chamber by using the raining method without any vibration, then a pre-loading process is given so as to compact the granular assembly. Secondly, the media is loaded from 0 to 0.23 MPa by using a uniaxial compression testing machine with the speed of 0.5 mm/min. Next, unloading takes place with the same speed of deformation until 0 MPa. Finally, the stress–strain curve shown in Fig. 3 is measured, in which the linear range of the unloading (see Fig. 3: line A to line B, range from 0.15 MPa to 0.20 MPa) is fitted as the elastic modulus E of the sample. The values of elastic modulus measured in the experiment are listed in Table 1.

Table 1. The elastic modulus for different sizes of particles and containers.

Container size unchanged			Grain size unchanged		
Container size L (mm)	Particle diameter d (mm)	E (MPa)	Container size L (mm)	Particle diameter d (mm)	E (MPa)
120	3	267	50	5	288
	6	287	100		265
	12	298	200		225

Obviously, the elastic modulus E increases with the grain size in one kind of container, i.e., the grains with larger particle size have larger elastic modulus. However, the elastic modulus E decreases with the increase of the container size for one size of grain, i.e., the grains have smaller elastic modulus in a larger container. Thus it can be concluded that the variation of elastic modulus is dependent on the size of particle and container.

Why is the modulus positively correlated to the grain size and negatively correlated to the container size? The arching and the friction are taken into account for the reason of such a function. Due to the fact that the large grains have fewer contact points than the small grains in the assembly, the larger interaction forces of large grains make their movement difficult, thus the arches are easily constructed. On the other hand, the friction between the particles and the container hinders the motion of glass beads, thus the smaller container will suffer more dragging actions. To verify this prediction, two specially designed experiments with the digital image correlation (DIC) method and the pressure film technology are used to investigate the arching and friction effects, respectively.

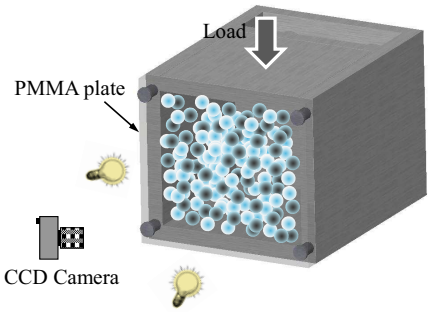

Fig. 4. Schematic diagram of the test configuration.

The schematic diagram of the DIC method is shown in Fig. 4. The front sheet of containers is replaced by a transparent polymethyl-methacrylate (PMMA) plate, and the glass beads are especially painted into white and black for image recognition. The distribution of particles are captured by a CCD camera (BASLER A641f with a resolution of 1624 × 1236 pixel). Although the image captured is of two dimensions, it can still reflect the arching effect from the strain field of granular assembly.

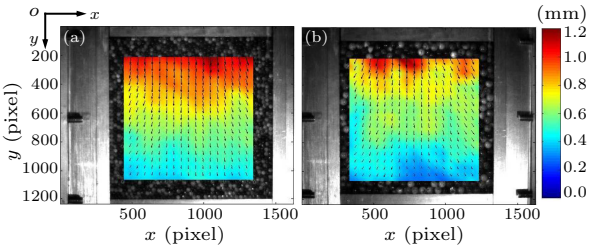

Fig. 5. Vertical displacement field for granular assembly of 3 mm (a) and 6 mm (b) glass beads.

The DIC method is an important photomechanical technique widely used for deformation measurements.[11] By comparing and matching the images captured before and after loading, it is able to resolve the displacement field through the correlation algorithm. The so-called displacement field reflects the distribution of moving quantities for all points on the surface of the specimen. To analyze the arching phenomenon of two kinds of glass beads (ϕ = 3 mm and 6 mm), which have smaller size difference, the vertical displacement fields are calculated based on the DIC method. Figures 5(a) and 5(b) are front views of the displacement field under 3.25 kN. The deeper the red color is, the larger the displacement is. On the contrary, the lighter the blue color is, the smaller the displacement is. Obviously, the granular assemblies are both compressed under the downward load, while the displacement field of 6 mm granular assembly is more non-uniform than that of 3 mm granular assembly.

To easily understand the deformation distribution,

we differentiate with respect to the space variable y, i.e., du/dy, to obtain the strain field[12] as shown in Fig. 6. It is clear that the value of vertical strain field in 6 mm granular assembly is much larger than that of 3 mm granular assembly. Accordingly, this case means that the larger particle is more difficult to move, thus the arch is easily constructed.

To quantitatively evaluate the strain inhomogeneity that reflects the arching in assembly, the variance of the strain field S is calculated based on the statistical method according to[13]

$$S = \sum_{i=1,j=1}^{m\times n} (\varepsilon(i,j) - \overline{\varepsilon})^2, \tag{4}$$

where m and n are the length and the width of the strain field matrix, (i,j) represent the position of a point, $\varepsilon(i,j)$ is the value of strain at the point (i,j), $\overline{\varepsilon}$ is the average value of strain field, and the result is illustrated in Fig. 7.

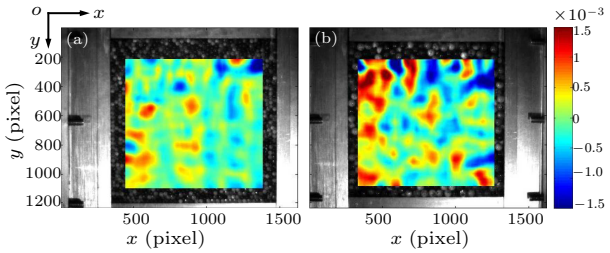

Fig. 6. Vertical strain field (relative) for granular assembly of 3 mm (a) and 6 mm (b) glass beads.

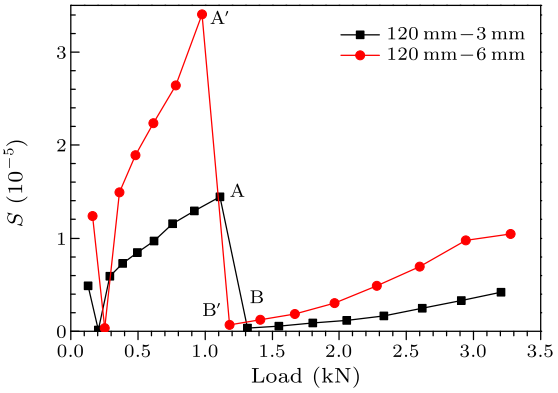

Fig. 7. Variances of the vertical strain field.

Clearly, the variance of the larger particles' assembly is larger than that of the smaller particles' assembly, which means that the strain field of the larger particles assembly is more inhomogeneous. Moreover, the process of sudden falling (point A to point B, or point A′ to point B′) can be considered as the collapse of the arch, thus the strain field becomes comparatively homogeneous. After the collapse, there is a slight increase of the variance again, and the value of larger particles' assembly is larger with the load increase. This validates that the large size of grains easily arch and thereby enhances the elastic modulus of the granular material.

Figure 8(a) is the schematic diagram of using the pressure film technology. The films (Fuji LLLW, measuring range: 0.2–0.6 MPa) S1 and S2 are respectively placed at the middle and bottom parts in granular assembly to measure the pressure in the z-direction, and then the indentation will occur on the film under the pressure, and the color intensity changes with the load. To investigate the influence of friction between the particles and the lateral sheet of container to the pressure distribution in one horizontal plane, three different sizes of cubic containers (L = 80, 100 and 120 mm) are poured by glass beads (ϕ = 4 mm), respectively. Figures 8(b) and 8(c) are the scanned images (pixels) of S1 and S2 for these containers.[14] To analyze the pressure distribution quantitatively, a statistical method based on the image processing is developed as follows.

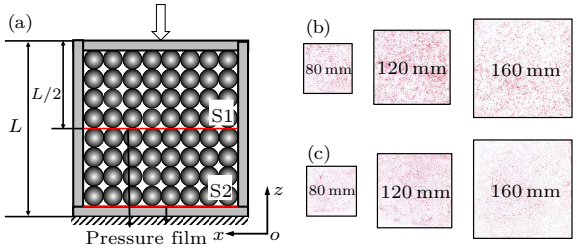

Fig. 8. Diagram of test configuration and results: (a) front view of the film position. (b) Scanned images of film S1. (c) Scanned images of film S2.

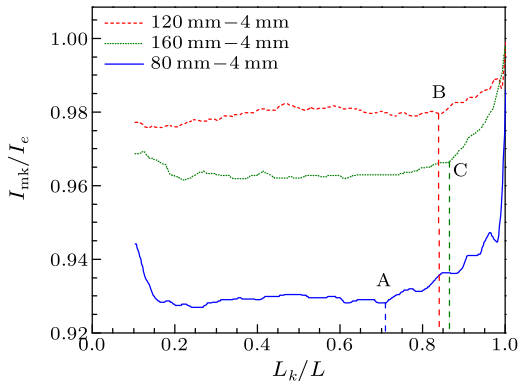

Fig. 9. Normalized gray distribution from the center to the edge of horizontal plane for three sizes of containers.

Firstly, the scanned image is converted to the gray image in which the less the gray value is, the larger the pressure is, and then the image is divided into k square rings with the length of $L_k = L/k$. The average gray value I_{mk} for each square ring can be calculated by using the equation

$$I_{mk} = \frac{I_k}{N_k}, \tag{5}$$

where I_k is the sum of gray values in each square ring, N_k is the number of pixels. Actually I_{mk} represents

the average pressure in one horizontal plane. Figure 9 shows the curves of pressure distribution from the center to the edge of the horizontal plane. To be convenient for comparing the pressure especially near the edge, the dimensions and pressure of the three containers are all normalized. Let I_{mk} be divided by the mean gray value $I_{\rm e}$ at the edge as the relative pressure, and L_k be divided by the length L of the container itself as the relative dimension. Note that the relative pressure increases in the right hand (the area near the edge) of the figure. For the sake of description, the start points of large variation are marked as A, B and C for three containers. From the point of view of the distance from the boundary, point A is relatively far from the boundary, i.e., the influence of friction force is larger for the smaller container.

This means that the smaller container will suffer more dragging actions under load. That is why the elastic modulus of the smaller container is larger than that of the larger ones.

In summary, we have designed a new type of uniaxial oedometric system. It simplifies the elastic modulus of measuring the granular materials greatly and it is conveniently size adjustable. Through the test of different sizes of grains and containers selected especially, the modulus of granular materials is found to be positively related to the grain size and negatively to the container size. Using the DIC method, it is observed that the strain field of the larger-particle assembly is more inhomogeneous. Thus the movement of larger particles is expected to be more difficult, so that the arch is easily formed, and thereby the elastic modulus of granular material is finally enhanced. By using the pressure film technology, it is revealed that the influence range of friction force between the grains and the container boundary is smaller for the larger container. For this reason, it is suggested that the larger container will suffer less dragging action and therefore the elastic modulus of the larger container becomes smaller.

References

[1] Mehta A 2007 *Granular Physics* (Cambridge: Cambridge University Press)
[2] Gennes P G D 1999 *Rev. Mod. Phys.* **71** s374
[3] Shamsa A, Roelofs F, Komen E M J, Baglietto E 2015 *Nucl. Eng. Des.* **290** 51
[4] Kiyota T 2007 *Soil Stress-Strain Behavior: Measurement, Modeling and Analysis* (Berlin: Springer Netherlands)
[5] Lee S J, Choo Y W, Lee J W and Sagong M 2013 *Geotech. Test. J.* **36** 1
[6] Sawicki A and Świdziński W 1998 *Powder Technol.* **96** 24
[7] Molenda M and Stasiak M 2002 *Int. Agrophys.* **16** 61
[8] Stasiak M, Molenda M and Horabik J 2007 *J. Food Eng.* **82** 51
[9] AnhDan L Q, Koseki J and Sato T 2002 *Geotech. Test J.* **25** 349
[10] Timoshenko S P and Goodier J A 1970 *Theory of Elasticity* (New York: McGraw Hill Higher Education)
[11] Chu T C, Ranson W F, Sutton M A and Peters W H 1985 *Exp. Mech.* **25** 232
[12] Pan B, Qian K M and Xie H M 2009 *Meas. Sci. Technol.* **20** 062001
[13] Strang G 1991 *Calculus* 2nd edn (Wellesley: Wellesley Cambridge Press)
[14] Kim H, Ulan-Kvitberg C and Daley C 2014 *Int. J. Nav. Archit. Ocean Eng.* **6** 578