CHIN. PHYS. LETT. Vol. 33, No. 3 (2016) 034301

Bayesian Tracking in an Uncertain Shallow Water Environment *

Qian-Qian Li(李倩倩)[1,2]**

[1]Key Laboratory of Marine Surveying and Mapping in Universities of Shandong, Shandong University of Science and Technology, Qingdao 26690

[2]State Key Laboratory of Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190

(Received 5 November 2015)

A Bayesian source tracking approach is developed to track a moving acoustic source in an uncertain ocean environment. This approach treats the environmental parameters (e.g., water depth, sediment and bottom parameters) at the source location and the source parameters (e.g., source depth, range and speed) as unknown random variables that evolve as the source moves. To track a target with low signal-to-noise ratio (SNR), acoustic signals from a series of observations are treated in a simultaneous inversion. This allows real-time updating of the environment and accurate tracking of the moving source. The noise signals radiated from a surface ship target are processed and analyzed. It is found that the Bayesian source tracking method could enhance the localization accuracy in an uncertain water environment and low SNR.

PACS: 43.60.Kx, 43.60.Jn DOI: 10.1088/0256-307X/33/3/034301

Localization and tracking of a moving source are important problems in underwater acoustics. Matched-field processing (MFP) is based on finding the best match between measured fields and simulated acoustic fields.[1] In a realistic application of matched field processing for source localization, the ability to localize an acoustic source is strongly affected by the uncertainty of the ocean environment.[2−11] The method of focalization[3] addresses environmental uncertainty in matched-field localization by including the environmental parameters as additional unknowns. The integration in the Bayesian focalization[4,6] provides an efficient sampling in a multi-dimensional parameter space. However, any link error in actual implementation may cause lower efficiency. Tracking involves a moving source and hence the ocean environment and the path between the source and receiver are changing with time. Tantum *et al.* produced the optimum uncertain field tracking algorithm (OUFTA).[7] The OUFTA is a sequential algorithm, and it takes the source movement as a Markov process. The focalization tracking algorithm[9−11] applies track constraints by the Viterbi algorithm and each location of the source is influenced by all acoustic data. However, the dimension of its estimated parameters is very large.

For these reasons, this study proposes a new source tracking algorithm based on a Bayesian algorithm. The Bayesian algorithm developed here is a simultaneous inversion, which calculates acoustic data from a series of observations in the same time. Further, let m represent the unknown model, m includes the environmental parameters, the source location at the first observation, and the source speed. Finally, by applying the source speed as interparameter information in simultaneous inversion, the inversed location of the source is influenced by all acoustic data, treating all source locations in an equivalent manner.

In a Bayesian formulation,[9] the solution to an inverse problem is characterized by its PPD. Before measurement, the information about the models is reflected in the *a priori* distribution $\rho(m)$, and after the experiment, the information about the models is reflected in the posteriori distribution $\sigma(m)$. These distributions are related by the likelihood function $L(m)$, which is a measure of the fitness between the observed data and the data generated by using a physical model and the environment m (Bayesian theorem),

$$\sigma(m) = L(m)\rho(m). \tag{1}$$

Due to the multi-dimensionality, the posteriori distribution is not suitable for graphic display. Therefore, the main integral properties of this distribution are of interest, such as the maximum posteriori (MAP) solution and the marginal probability distribution. Marginal probability distributions are particularly useful for understanding parameter uncertainties by reducing the multidimensional PPD to representative one-dimensional (1D) or two-dimensional (2D) probability distributions according to

$$m^{\mathrm{MAP}} = \arg(\max(\sigma(m))), \tag{2}$$

$$\sigma^i(m^i) = \int \sigma(m) dm^1 \ldots dm^{i-1} dm^{i+1} \ldots dm^M, \tag{3}$$

$$\sigma^{i,j}(m^i, m^j) = \int \sigma(m) dm^1 \ldots dm^{i-1} \cdot dm^{i+1} \ldots dm^{j-1} dm^{j+1} \ldots dm^M, \tag{4}$$

where M is the dimension of the estimated parameters. The Bayesian tracking algorithm is based on the MAP solution.

Knowing that the likelihood function is usually related to the objective function $\Phi(m)$ through an ex-

*Supported by the National Natural Science Foundation of China under Grant Nos 11434012, 10974218, and 11174312, the State Key Laboratory of Acoustics of Chinese Academy of Sciences under Grant No SKLA201407, the Key Laboratory of Marine Surveying and Charting in Universities of Shandong of Shandong University of Science and Technology under Grant No 2013A02, the Scientific Research Foundation of Shandong University of Science and Technology for Recruited Talents under Grant No 2014RCJJ004, the Project of the Public Science and Technology Research of Ocean under Grant No 201305034, and the National Key Technology R&D Program under Grant No 2012BAB16B01.

**Corresponding author. Email: lqq@mail.ioa.ac.cn

© 2016 Chinese Physical Society and IOP Publishing Ltd

ponential $L = \exp(-\Phi(m)/\hat{v})$,[12] where $\hat{v}$ is the estimated noise power, we use the following scaling

$$L_{\text{emp}} = \exp(-[\Phi(m) - \Phi(m_0)]/T), \tag{5}$$

where $\Phi(m)$ is the objective function mentioned in Eq. (1), m_0 is the estimated parameter vector corresponding to the optimal value of the objective function, and T is the 'temperature'. Experimentally, it was found that a good value for T is the average of the 50 best objective functions obtained during the optimization, minus the best value of the objective function. The advantage of this scheme is that it works irrespective of the stochastic model for the data or likelihood function.

To define the objective function $\Phi(m)$, we consider complex acoustic-field data due to a (moving) source at S locations as measured at an array of N sensors at F frequencies, i.e., $p^{\text{e}} = \{p^{\text{e}}_{ik} : i = 1, F; k = 1, S\}$.

Therefore, for the Bayesian tracking approach, the objective function may be written as

$$\Phi(m) = 1 - \frac{1}{SF}\sum_{k=1}^{S}\sum_{i=1}^{F} \cdot \frac{|\sum_{j=1}^{N} p^{\text{e}*}_{ijk} p^{\text{c}}_{ijk}(m)|^2}{\sum_{j=1}^{N}|p^{\text{e}}_{ijk}|^2 \sum_{j=1}^{N}|p^{\text{c}}_{ijk}(m)|^2}, \tag{6}$$

where p^{e}_j is the experimental complex pressure field received by the jth hydrophone, p^{c}_j is the modeled complex pressure field, and the asterisk represents the complex conjugation. Assuming the observation time is sufficiently small compared with the temporal variability of the ocean, the environment will remain constant over the observations. The Bayesian tracking algorithm is based on maximizing the PPD over unknown parameters to obtain a sequence of the best estimation, i.e. Eq. (2). The genetic algorithm (GA) parameters are taken as quotation:[12] the reproduction rate is 0.5, the mutation rate is 0.05, and the crossover rate is 0.8.

The algorithm is applied in the following to determine the most probable track. The three source parameters (i.e., the source depth, range and radial speed) are grouped as $S_k = [z_{\text{s}} r_{\text{s}} v_{\text{s}}]^T_k$, where S_1 are the parameters related to the first acoustic signal. The kth parameters require a state equation that models the connection with S_1. Using a constant velocity (CV) track model for the source, the state equation is given by

$$\begin{bmatrix} z_{\text{s}} \\ r_{\text{s}} \\ v_{\text{s}} \end{bmatrix}_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & (k-1)t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_{\text{s}} \\ r_{\text{s}} \\ v_{\text{s}} \end{bmatrix}_1, k = 2, 3, \ldots, s, \tag{7}$$

where t is the time between successive measurements, and $S1 = \begin{bmatrix} z_{\text{s}} \\ r_{\text{s}} \\ v_{\text{s}} \end{bmatrix}_1$.

The whole acoustic data are divided into N segments. A one-minute time window (containing S observations) which slides over the time domain with an increment of t seconds (one observation) applied. Meanwhile the second position (r_{i+1}, zs_{i+1}) and (D_i, cb_i) in the ith time window are regarded as the a priori estimate (m^{apri}) for the $(i+1)$th time window based on the Tikhonov regularization. That is, given *a priori* estimate $\boldsymbol{m}^{\text{apri}}$ solution, it seems natural to minimize the difference between the solution $\boldsymbol{m}$ and the estimate $\boldsymbol{m}^{\text{apri}}$,

$$\Phi_{\text{reg}} = \|L(\boldsymbol{m}^{\text{apri}} - \boldsymbol{m})\|. \tag{8}$$

Normally, L would be the identity matrix or a discrete approximation to a derivative operator. Due to different physical dimensions of the parameters, an ad hoc regularization could be

$$\Phi_{\text{reg}}(\boldsymbol{m}^{\text{apri}}, \boldsymbol{m}) = 1/M \sum \frac{|m_i - m_i^{\text{apri}}|^\gamma}{|m_i^{\max} - m_i^{\min}|^\gamma}, \tag{9}$$

where m_i^{apri} is the *a priori* expected value, $m_i^{\max} - m_i^{\min}$ is the search interval for the ith parameter, and the exponent γ controls the shape of the penalization. We use $\gamma = 1$. The objective function combining the *a priori* knowledge and the data reads

$$\Phi(\boldsymbol{m}) = \Phi_{\text{B}}(\boldsymbol{m}) + \lambda \Phi_{\text{reg}}(\boldsymbol{m}^{\text{apri}}, \boldsymbol{m}), \tag{10}$$

where λ is a Lagrange multiplier. The objective function consists of two terms, i.e., Φ_{B} measuring the match between observed and synthetic data (identical to Eq. (6) and Φ_{reg} analyzing the deviation from the *a priori* model (using Eq. (9). We use $\lambda = 1$ and thus place equal emphasis on the data and the *a priori* information. Therefore, the adjacent time windows are linked up, and the estimated results are kept stable.

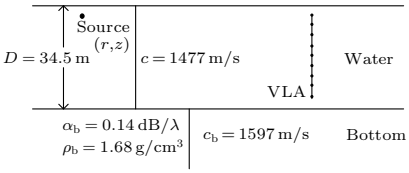

Fig. 1. Experimental configuration and shallow water environment.

The experiment was conducted in the Yellow Sea. Figure 1 shows the measured ocean environment at the receiver ship and the inverted bottom model.[13] The sound speed profile was measured near the position of the vertical line array (VLA), which is about 1477 m/s.

Data were recorded by a VLA with 7 elements at 3.5-m spacing, sampling the water column with depths of 4.9–25.7 m. Throughout the experiment, the target ship moved towards the VLA at a radial speed of about 7.5 m/s. Acoustic data are collected for 25 min in all, corresponding to the source-receiver range of 14.4–2.8 km. The noise from the ship propeller is taken as the target. Therefore, the source depth is less than 7 m.

The measured acoustic signal over a range of 100–300 Hz is Fourier transformed to produce a frequency-domain experimental complex pressure field, with frequency resolution equal to 1 Hz, which puts the frequency number F in Eq. (6) equal to 201. The whole acoustic data is divided into $N = 150$ segments and the time interval is $t = 10$ s. Six acoustic data sets are computed in a simultaneous inversion ($S = 6$). Figure

2 shows a received time series for frequency band 100–300 Hz from the source at 14.4 km, where the receiver depth is 22 m.

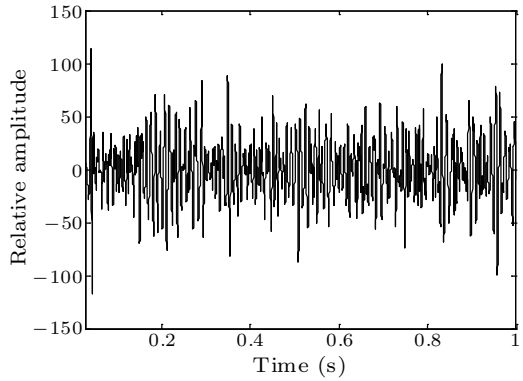

Fig. 2. Received time series of ship noise from 14.4 km, where the frequency band is in the range of 100–300 Hz and the receiver depth is 22 m.

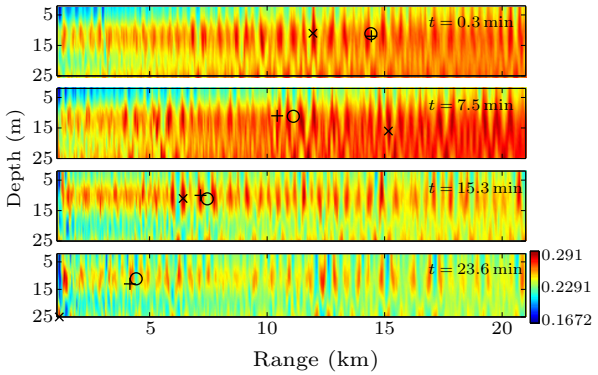

Fig. 3. The Bayesian tracking (+) and the conventional MFP (x) at $t = 0.3$, 7.5, 15.3, 23.6 min respectively. Background is the conventional MFP ambiguity surface for the environment shown in Fig. 1. Here circles denote the GPS source locations.

When there is a mismatch between the assumed environment and its actual value, the source appears at a different location. The bathymetry changes as the ship moves towards the VLA. Ignoring this variation results in a great localization error. Figure 3 shows such a mismatched MFP estimator by using a normalized Bartlett mismatch. One can also see from Fig. 3 that the estimated source range computed by the Bayesian tracking approach is near the GPS measured ranges, while the conventional MFP breaks down.

The Bayesian tracking treats source parameters (source range, depth and speed) and 2 environmental parameters, the water depth D and the sound speed of bottom c_b, as unknown parameters. Considering that the bottom density is less sensitive to the wave field, we utilize the empirical relationship to invert it implicitly,[5]

$$c_\mathrm{b} = 2330.4 - 1257.0\rho_\mathrm{b} + 487.7\rho_\mathrm{b}^2. \tag{11}$$

The bottom attenuation coefficients at different frequencies use the nonlinear relationship[5]

$$\alpha_\mathrm{b} = 0.29 f^{1.91} dB/\lambda. \tag{12}$$

When carrying out the optimization by SAGA,[14] sampling 4000 models with a population size of 120 are used.

Inversion results from two approaches to Bayesian tracking of increasing complexity and information content are shown in Figs. 4 and 5. In each case, six acoustic measurements of the moving source are computed by searching the maximum of the posteriori distribution, as described in Eq. (2).

Figure 4 shows the inversion results computed from simultaneous inversion of six acoustic data sets for the moving source. This constitutes a more informative formulation than independent inversions, since all six data sets constrain a single set of environmental parameters, which should in turn better constrain source locations. However, it is also much more computationally demanding, since it involves 14 unknown parameters including six source ranges and depths. Figure 4 shows inversion results tend to be generally less diffuse than those for the MFP inversions, with some concentration of probability near the correct source depth. However, Fig. 4 indicates that the information content of the noisy data and wide environmental priors is not sufficient to localize or track the source with any certainty.

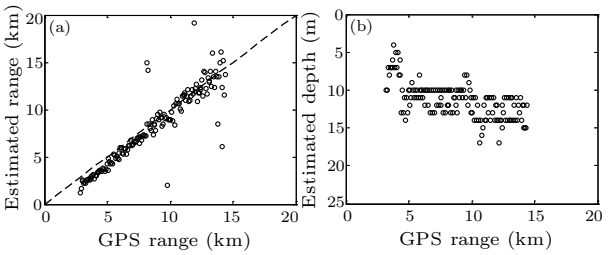

Fig. 4. Simultaneous inversion without the CV track model: (a) range results and (b) depth results.

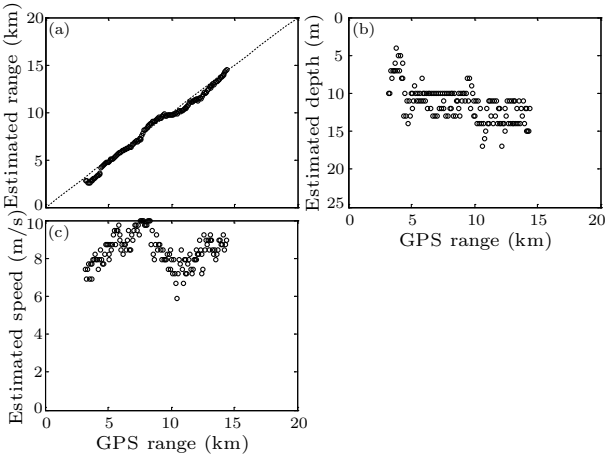

Fig. 5. The Bayesian tracking algorithm performance (with the CV track model and the Tikhonov regularization) (a) range results, (b) depth results and (c) speed results.

Figure 5 shows the inversion results computed from simultaneous inversion of six acoustic data, together with additional prior information in the form of the CV track model that the speed of the source is inverted and the Tikhonov regularization that the adjacent time windows are linked up.

Figure 5(a) shows that the estimated range of every signal is consistent with the GPS measured ranges.

Especially for the signals which are not properly located shown in Fig. 4, the Bayesian tracking method gives a relatively good range localization results. The mean absolute error for the Bayesian tracking from the GPS range is 6.0%. Figure 5(b) shows the depth estimation at different ranges, which are distributed at about 11 m. None of the estimated source depth reaches the search bound anymore. The source speed results are given in Fig. 5(c), and the mean value is 8.4 m/s, which is consistent with the actual result. Figure 5 indicates that, with the additional prior information provided by the CV track model and the Tikhonov regularization, the source can be localized and tracked with a high degree of certainty despite the poor environmental knowledge and low SNR. The CV track model greatly reduces the parameter search space from 14 parameters to 5 parameters, which include only one source range, depth and source speed.

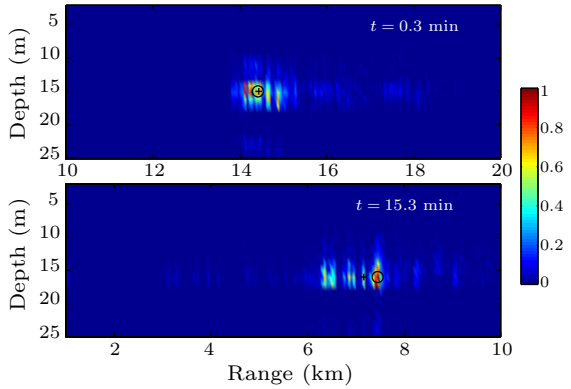

Fig. 6. The 2D depth-range PASs of the moving source at time $t = 0.5$ and 15.3 min. Here circles represent the true source location, and the pluses represent the MAP value.

Integrating the PPD over uncertain environmental parameters, it can produce a joint marginal probability distribution over source range and depth, called PAS. The peak of the PAS represents the most probable source location, and the distribution itself provides a quantitative measure of localization uncertainty. The PASs of the source depth/range at two time segments is given in Fig. 6. We can see that the MAP solution makes less error than the 2D solution at the first-track according to the GPS value. The results at the second track are just opposite. The comparison of these three methods is not considered here, and the work will be studied in the next step.

To display the inversion uncertainty, the marginal distribution is evaluated from Eq. (3). The result of this integration at time $t = 7.5$ min is shown in Fig. 7. The horizontal axes show the search spaces, and the vertical axes indicate probability distributions with the maximal value of one. The ability to track both the geo-acoustic parameters and the source enables the Bayesian tracking algorithm to track accurately the source with a small variance in range. It is seen in Fig. 7 that the estimated source range converges to the global optimum with high probabilities, and it is very close to the GPS value 11.1 km (relative error is 6%). The other four estimated parameters are relatively less sensitive than source range. It is also seen that the estimated source depth is 14 m, which is more reasonable than the result by MFP in Fig. 3. The estimated source speed is 7.2 m/s, which is in accordance with the GPS data.

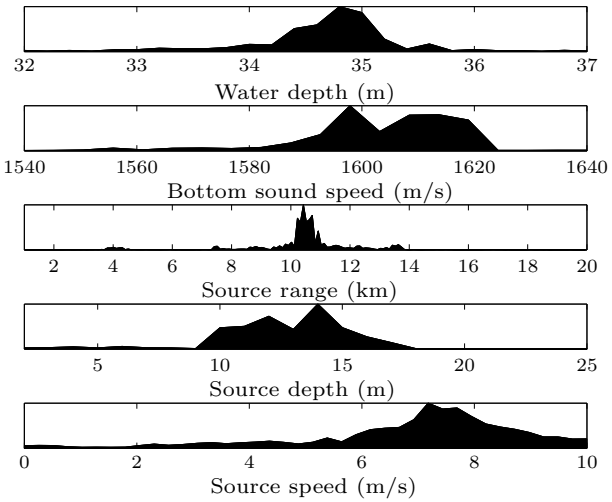

Fig. 7. Marginal probability density distribution for the ship noise at 11.1 km.

In summary, the Bayesian tracking algorithm developed here treats acoustic data from a series of observations together in a simultaneous inversion. This provides the most informative solution, since data from multiple source locations are brought to bear simultaneously on the environmental unknowns, which in turn better constrain source locations. The source locations are treated implicitly by the source speed, which substantially reduce the dimensionality and complexity of the problem. Tracking results of the ship noise indicate that simultaneous inversion of multiple acoustic observations with the CV track model and the Tikhonov regularization provides a better solution than sequential independent inversions. It is indicated that the Bayesian tracking method is learning the uncertain environment as more observations become available.

References

[1] Baggeroe A B et al 1993 *IEEE J. Oceanic Eng.* **18** 401
[2] Li W et al 2015 *Chin. Phys. Lett.* **32** 064302
[3] Collins M D and Kuperman W A 1991 *J. Acoust. Soc. Am.* **90** 1410
[4] Dosso S E and Wilmut M J 2009 *J. Acoust. Soc. Am.* **125** 717
[5] Li Q Q et al 2013 *Chin. Phys. Lett.* **30** 024301
[6] Richardson A M and Nolte L W 1991 *J. Acoust. Soc. Am.* **89** 2280
[7] Tantum S L and Nolte L W 1998 *J. Acoust. Soc. Am.* **103** 362
[8] Yardim C, Gerstoft P and Hodgkiss W S 2010 *J. Acoust. Soc. Am.* **128** 75
[9] Dosso S E and Wilmut M J 2008 *J. Acoust. Soc. Am.* **124** 82
[10] Dosso S E and Wilmut M J 2010 *J. Acoust. Soc. Am.* **128** 66
[11] Dosso S E and Wilmut M J 2013 *J. Acoust. Soc. Am.* **133** EL274
[12] Gerstoft P 1998 *J. Acoust. Soc. Am.* **104** 808
[13] Li Q Q 2015 *Tech. Acoust.* **34** 490
[14] Gerstoft P 1997 *SAGA Users Guide 2. 0, an inversion software package* (Italy La Spezia SACLANT Undersea Research Center)