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Electron-Elastic-Wave Interaction in a Two-Dimensional Topological Insulator *

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The interaction between an electron and an elastic wave is investigated for HgTe and InAs-GaSb quantum wells. The well-known Bernevig–Hughes–Zhang model, i.e., the 4 × 4 model for a two-dimensional (2D) topological insulator (TI), is extended to include the terms that describe the coupling between the electron and the elastic wave. The influence of this interaction on the transport properties of the 2DTI and of the edge states is discussed. As the electron-like and hole-like carriers interact with the elastic wave differently due to the crystal symmetry of the 2DTI, one may utilize the elastic wave to tune/control the transport property of charge carriers in the 2DTI. The extended 2DTI model also provides the possibility to investigate the backscattering of edge states of a 2DTI more realistically.

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When a two-dimensional (2D) topological insulator (TI) has a boundary with a normal insulator or vacuum, it is predicted theoretically that there exist gapless edge states.[1–3] The existence of such edge states has been confirmed experimentally.[4–7] When the system is time reversal invariant, these edge states will not be affected by an elastic back scattering center.[1–3] Thus one has a transport channel for the charge carriers, i.e., the information carriers, with no energy dissipation, and this will be of great application importance in the information technology.

The edge states resulting from the 2DTI have been intensively studied both theoretically and experimentally in recent years. There are studies on the phonon-induced back scattering in helical edge states,[8] the inelastic back scattering due to electron–electron interaction,[9] the influence of a charge puddle,[10] and the interaction between edge states and nuclear spins.[11] There are also investigations concerning the probing and tuning of the transport property of edge states, e.g., via a quantum point contact and via an artificially implemented charge puddle.[12–14]

In this Letter, we propose another mechanism to tune/control the transport property of edge states in a 2DTI. The idea behind this mechanism is simple. A 2DTI is embedded in a solid made of a crystal structure, and this solid can support an elastic wave.[15,16] The elastic wave can change the strain in the solid and thus can affect the electronic property. In fact, many TI materials are obtained by carefully manipulating the static strain in the material.[17] It is also possible to tune the strain dynamically.[18] In this study, we focus on the 2DTI system realized in a HgTe quantum well and in an InAs-GaSb quantum well. The quantum wells considered have the [001] axis as the growth direction.

A 2DTI system can be described by the well-known Bernevig–Hughes–Zhang model.[3] It is a 4×4 Hamiltonian and it reads

$$H(\boldsymbol{k}) = \varepsilon(k)I + \begin{pmatrix} h(\boldsymbol{k}) & 0 \\ 0 & h^*(-\boldsymbol{k}) \end{pmatrix},$$

with

$$h(\boldsymbol{k}) = \begin{pmatrix} M(k) & -A(k_x + ik_y) \\ -A(k_x - ik_y) & -M(k) \end{pmatrix},$$

and $\varepsilon(k) = C - Dk^2$ and $M(k) = M - Bk^2$. Here $\boldsymbol{k}$ is the in-plane wave vector of the electron. In this Hamiltonian, the upper and lower 2×2 spin blocks are not coupled.

When a symmetric quantum well is biased by an externally applied gate voltage along the growth direction, a coupling between the upper block and lower spin block is introduced. This coupling adds the following term to the Hamiltonian

$$H_1(\boldsymbol{k}) = \begin{pmatrix} 0 & h_1(\boldsymbol{k}) \\ h_1^\dagger(\boldsymbol{k}) & 0 \end{pmatrix},$$

with

$$h_1(\boldsymbol{k}) = \begin{pmatrix} A_1(k_y + ik_x) & 0 \\ 0 & 0 \end{pmatrix}.$$

For an 8-nm HgTe quantum well, one can find that $A_1 = 192 \times 10^{-5} E_z$ meV/nm. The bias electric field E_z is in the unit of V/cm. When the quantum well is asymmetric, like the case of an InAs-GaSb quantum well, the value of A_1 is determined by the specific structure parameters of the quantum well.

In a cubic crystal such as HgTe, InAs, and GaSb, an externally applied elastic wave or strain potential is characterized by six quantities $u_{\alpha,\beta} = (\partial u_\alpha/\partial x_\beta + \partial u_\beta/\partial x_\alpha)/2$ with α and β standing for x, y, z, respectively,[15,16] where u_α is the displacement, and

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$u_{\alpha,\beta}$ is the strain component.[15,16] Note that $u_{\alpha,\beta}$ can be spatially and temporally dependent. The x-axis is chosen along the [100] direction, and the y-axis is along the [010] direction. Within the elastic deformation potential approximation and the adiabatic approximation, this strain introduces an additional Hamiltonian describing the interaction between the electron and the elastic wave,

$$H_{\rm int} = \begin{pmatrix} h_{11} & h_{12} & 0 & h_{14} \\ h_{12}^{\dagger} & h_{22} & -h_{14} & 0 \\ 0 & -h_{14}^{\dagger} & h_{11} & h_{12}^{\dagger} \\ h_{14}^{\dagger} & 0 & h_{12} & h_{22} \end{pmatrix}, \qquad (1)$$

where

$$\begin{aligned} h_{11} &= a_{11}(u_{xx}+u_{yy})+b_{11}u_{zz}, \\ h_{22} &= a_{22}(u_{xx}+u_{yy})+b_{22}u_{zz}, \\ h_{12} &= a_{12}(u_{yz}-iu_{xz}), \\ h_{14} &= ia_{14}(u_{yy}-u_{xx})+b_{14}u_{xy}. \end{aligned}$$

This is the central result of this work. In Eq. (1), h_{11} describes the interaction between the electron-like component and the elastic wave, h_{22} is for the hole-like component, h_{12} couples the electron-like and hole-like components, and h_{14} couples directly two spin blocks. Note that, $u_{\alpha,\beta}$ enters the above h_{ij} terms in different ways. This is due to the symmetry of the crystal structure considered.[15,16,22]

One can generate elastic waves with various characteristics, e.g., a longitudinal wave, a shear wave, and various propagating directions.[15,16] Then, one can utilize such designed elastic waves to manipulate the transport properties in a 2DTI and of the edge states. In the absence of h_{14}, the elastic wave can be used to drive the charge carrier in the 2DTI like an ocean tide wave drives a surfer. This can be viewed as a charge pumping process, with the same pumping effect for both spin blocks. When h_{14} becomes nonzero, the elastic waves applied not only drive the charge carriers in each spin block, but also provide a channel for charge carriers transferring across two spin blocks. One can take the 2DTI model with the zero wave function boundary condition, and can deduce a low energy model for the edge states. Due to the symmetry of the solid considered, it is evident that the interaction between the edge states and the elastic wave will not just be proportional to $u_{xx}+u_{yy}+u_{zz}$. This can be easily understood, as the states in the 2DTI and in the 2DTI edge states are mixed states originated from the conduction band and valence band electronic states. This provides a possibility to tune the coupling of edge states with the elastic wave. A more detailed investigation on the influence of the electron elastic wave interaction in the 2DTI and on the transport property of edge states will be reported elsewhere. It is interesting to note that the elastic wave can be utilized to pump electrons in quantum dots, to transport optically generated excitons, and to route quantum information between quantum bits.[19–21]

Next the values of parameters a_{ij} and b_{ij} appearing in h_{ij} are provided for specific HgTe and InAs-GaSb quantum well structures, respectively. For a HgTe quantum well with well width 8 nm, enclosed by two $Hg_xCd_{1-x}Te$ barriers, it is found that

$$\begin{aligned} a_{11} &= -0.63\,{\rm eV},\ b_{11} = -1.85\,{\rm eV}, \\ a_{22} &= 0.19\,{\rm eV},\ b_{22} = 2.80\,{\rm eV}. \end{aligned}$$

They are almost independent of the bias voltage when a bias is applied along the growth direction of the symmetric quantum well. It is found that

$$\begin{aligned} a_{12} &= 1.3\times10^{-5}E_z\,{\rm eV}, \\ a_{14} &= 0.6\times10^{-5}E_z\,{\rm eV}, \\ b_{14} &= -1.3\times10^{-5}E_z\,{\rm eV}. \end{aligned}$$

They linearly depend on the bias electric field E_z in the unit of V/cm. The HgTe quantum well is enclosed by two $Hg_xCd_{1-x}Te$ barriers, and the wave function only penetrates slightly into the barriers. The corresponding a_{ij} and b_{ij} parameters for the barriers are rather small and can be safely ignored.

For an InAs-GaSb quantum well, enclosed by two AlSb barriers, the following parameters are provided as an example. The quantum well structure consists of a 12.5 nm InAs layer and a 5 nm GaSb layer. Here $H_{\rm int}$ will have two parts, one for the InAs layer and the other for the GaSb layer. For the InAs layer, one has

$$\begin{aligned} a_{11} &= -3.03,\ b_{11} = -3.64, \\ a_{22} &= 0,\ b_{22} = 0.02, \\ a_{12} &= 0.04, \\ a_{14} &= 0.01,\ b_{14} = -0.03. \end{aligned}$$

For the GaSb layer, one has

$$\begin{aligned} a_{11} &= 0.28,\ b_{11} = -0.27, \\ a_{22} &= -0.45,\ b_{22} = 2.74, \\ a_{12} &= 2.11, \\ a_{14} &= 0.85,\ b_{14} = -2.32. \end{aligned}$$

They are in the unit of eV. The InAs-GaSb quantum well is enclosed by two AlSb barriers, and the wave function only penetrates slightly into the barriers, though the wave function from the InAs layer and GaSb layer are strongly hybridized. The contribution from the barriers is small and can be safely ignored.

Next we describe briefly the derivation of Eq. (1) and the evaluation of related parameters. The approach used here is similar to that employed in deriving the well-known Bernevig–Hughes–Zhang model.[3] The calculation of one-electron energy levels of HgTe and InAs-GaSb quantum wells is based on the well-documented eight-band $\boldsymbol{k}\cdot\boldsymbol{p}$ approach. For details

about this method, e.g., the operator ordering, the influence of remote bands, the influence of strain, and the modification due to heterojunction interfaces, we refer to a partial list of publications and references therein.[22–29]

The quantum well is assumed to be parallel to the x–y plane, and the z-direction is along the growth direction of the quantum well. The HgTe quantum well structure consists of a left and a right $Cd_xHg_{1-x}Te$ barrier. Enclosed by the barriers is the HgTe quantum well, and x=0.3 is used in the calculation. The InAs-GaSb quantum well studied in this work has the following structure: a left AlSb barrier, the InAs layer, the GaSb layer, and finally a right AlSb barrier. The growth direction of the quantum wells is assumed to be [001]. The material specific parameters, i.e., the band parameters, used in the present calculation are widely used in the literature.[30–33]

When an elastic wave is applied to the quantum well, for each layer one has a correction due to the strain induced by the elastic wave. In the eight-band $\boldsymbol{k}\cdot\boldsymbol{p}$ approximation, one has the following Hamiltonian

$$\begin{pmatrix} s_1 & s_3 \\ s_3^\dagger & s_2 \end{pmatrix},$$

with s_1 given by

$$\begin{pmatrix} a'e & \sqrt{2}w & w & 0 & t^\dagger & \sqrt{2}t^\dagger \\ & p+q & \sqrt{2}q & t^\dagger & 0 & \sqrt{3/2}s^\dagger \\ & & ae & -\sqrt{2}t^\dagger & \sqrt{3/2}s^\dagger & 0 \\ & & & a'e & \sqrt{2}w & w^\dagger \\ & & & & p+q & -\sqrt{2}q \\ & & & & & ae \end{pmatrix},$$

$$s_2 = \begin{pmatrix} p-q & 0 \\ & p-q \end{pmatrix},$$

$$s_3 = \begin{pmatrix} 0 & -\sqrt{3}t \\ r^\dagger & s \\ -\sqrt{2}r^\dagger & \sqrt{1/2}s \\ -\sqrt{3}t^\dagger & 0 \\ -s^\dagger & r \\ \sqrt{1/2}s^\dagger & \sqrt{2}r \end{pmatrix}.$$

This is the wave vector independent part. There is another contribution from the spin-orbit interaction, while it will not be given here for simplicity. It can be found in the literature.[22–29] Note that, in the above equation, s_1 and s_2 are hermitian. The basis used here is $|S,\downarrow\rangle$, $i(|X,\uparrow\rangle - i|Y,\uparrow\rangle)/\sqrt{6} + i\sqrt{2/3}|Z,\downarrow\rangle$, $-i(|X,\uparrow\rangle - i|Y,\uparrow\rangle)/\sqrt{3} + i|Z,\downarrow\rangle/\sqrt{3}$, $|S,\uparrow\rangle$, $-i(|X,\downarrow\rangle + i|Y,\downarrow\rangle)/\sqrt{6} + i\sqrt{2/3}|Z,\uparrow\rangle$, $-i(|X,\downarrow\rangle + i|Y,\downarrow\rangle)/\sqrt{3} - i|Z,\uparrow\rangle/\sqrt{3}$, $i(|X,\uparrow\rangle + i|Y,\uparrow\rangle)/\sqrt{2}$, $-i(|X,\downarrow\rangle - i|Y,\downarrow\rangle)/\sqrt{2}$. This is only slightly different from the conventionally used basis in the literature.[22–29]

In the above s_i terms, one has

$$\begin{aligned} e &= u_{xx} + u_{yy} + u_{zz}, \\ w &= ib'u_{xy}/\sqrt{3}, \\ t &= b'(u_{xz} + iu_{yz})/\sqrt{6}, \\ p &= a(u_{xx} + u_{yy} + u_{zz}), \\ q &= b(u_{zz} - (u_{xx} + u_{yy})/2), \\ r &= \sqrt{3}b(u_{xx} - u_{yy})/2 - idu_{xy}, \\ s &= -d(u_{xz} - iu_{yz}), \end{aligned}$$

with a', b', a, b, and d being the deformation potential parameters. These parameters can be found in the literature for the material considered.[30–33] One can decompose the above Hamiltonian into the ones that are proportional to u_{ij}. Then Eq. (1) is obtained by taking the matrix elements for every u_{ij} term. This is almost the same approach in obtaining the well-known Bernevig–Hughes–Zhang model.[3]

It should be pointed out that the interaction Hamiltonian given by Eq. (1) also provides a more realistic model for the study of the backscattering of edge states in the 2DTI due to interaction between the charge carrier and the acoustic phonon. The acoustic phonon, as an intrinsic elementary excitation of the crystal, deforms the solid and affects the electronic properties, especially the transport properties.[15] The scattering of charge carriers by phonons is an important source of decoherence and dephasing for electrons and electron spins.[34]

In summary, the interaction between an electron and an elastic wave has been investigated theoretically for HgTe and InAs-GaSb quantum wells. The well-known Bernevig–Hughes–Zhang model, which is widely used to investigate the 2DTI properties, is further extended to include the terms that describe the coupling between the electron and the elastic wave. The influence of this interaction, which can be considered as a tuning/controlling mechanism, on the transport properties of the 2DTI and of the edge states is discussed. The extended model also provides the possibility of investigating the backscattering of edge states by acoustic phonons in a more realistic way.

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