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Anomalous Variation of Beat Frequency in a Dual Frequency He–Ne Laser *

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The beat frequency in a dual frequency He–Ne laser varies while the resonant cavity length is tuned. As to the laser with two longitudinal modes, the variation amplitude is commonly less than 500 kHz, proven by experiments and theories. This study reveals an anomalous variation of the beat frequency when a piece of element is put into the cavity and is aligned with the laser axis. Consequently the variation amplitude could reach 22 MHz, several dozen times larger than that without the intra-cavity element. This cannot be explained only by laser mode pulling and pushing effects. Some influencing factors are investigated experimentally, including the tilted angle of the element and the distance between its surface and cavity mirror. The qualitative analysis is discussed, which agrees with the experimental results.

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The output characteristics of a laser are altered when an optical element is inserted into the intra-cavity or external cavity, of which the physical principles attract the interests of many researchers.[1–4] Some outstanding achievements, based on those effects of laser, have been made in precision measurement.[5–8] In addition to the modulated intensity, the laser frequency variation is important,[9–12] especially in some novel applications.[13–15] Owing to the intra-cavity anisotropies of the inserted element, each laser longitudinal mode would split into two frequencies. The so-called laser frequency splitting technique has been employed in metrological applications with excellent performance.[16–19]

Variation of beat frequency in the dual frequency laser, i.e. the beat frequency of two longitudinal modes within gain bandwidth, plays an important role in the study of laser physics.[20–22] As to the He–Ne laser, it was reported that the relative variation of beat frequency changing with cavity length has the order of 10^{-3} for the standing wave laser and 10^{-5} for the ring laser.[23,24] There are some nonlinear effects, known as abnormal dispersion, hole burning, hole repulsion, or in general mode pushing and pulling, contributing to variation of beat frequency. According to Lamb's theory, the variation amplitude is commonly several hundreds of kHz.[25,26]

In some applications, the beat frequency in a dual frequency He–Ne laser serves as the basis of precision measurement.[17,27,28] Its variation, resulting in systematic error, should be investigated thoroughly in both theory and experiment. In this Letter, we find the beat frequency varying abnormally when the intra-cavity element is collimated along the laser axis. The variation amplitude could even reach 22 MHz, which is much larger than that data expected in the literature. The experimental results of the so-called anomalous variation of beat frequency are demonstrated. Some influencing factors are investigated, including the tilted angle of the element and the distance between its surface and resonator mirror. We attribute this phenomenon to the weak reflection of intra-cavity element surfaces, which has effects on laser frequency and finally influences the beat frequency.

The experimental setup is shown in Fig. 1. A half-external cavity He–Ne laser at wavelength of 632.8 nm is employed. The discharge tube T is filled with a mixture of helium and neon gas. The total pressure is about 460 Pa, and the fractional ratio is $He:Ne^{20}:Ne^{22}=7:0.5:0.5$. The resonant cavity, about 190 mm in length, consists of a concave mirror M_1 (1 m in curvature radius, 99.5% in reflectivity) and a plane mirror M_2 (98.9% in reflectivity). M_2 can be driven by piezoelectric transducer (PZT) to change the cavity length. The optical element (OE), coated with antireflection film, is placed into the cavity. The output laser beam from M_1 is divided into two orthogonally polarized components by Wollaston prism BS and recorded by two photoelectric detectors PD. Controlling board CB could receive light intensity and drive PZT along the laser axis. The other beam from M_2 generates the light beat of two laser modes after a polarizer P, which is detected by an avalanche photodiode APD. A frequency counter FC (FC-7013, 1 kHz resolution) is used to read out the beat frequency.

In experiments, two types of optical elements, quartz wave plate and K9 glass plate with residual stress, were used separately. The former could

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make each longitudinal mode split into two, and the beat frequency is large enough to conquer mode competition.[17,29] The latter has too small a stress birefringence and only the beat frequency of two adjacent longitudinal modes could be obtained. Additionally, both the principal axes should be parallel to the orthogonal polarization of laser. A quarter wave plate (QWP) in thickness 1.5 mm is placed into resonant cavity. The beat frequency is about 444.6 MHz, while the longitudinal mode spacing approximates 798 MHz. When PZT is driven to change the cavity length, the beat frequency varies as illustrated in Fig. 2(a). Among the coexistent region of two modes, the beat frequency changes from 444.79 MHz to 444.54 MHz, i.e., the variation amplitude is about 250 kHz.

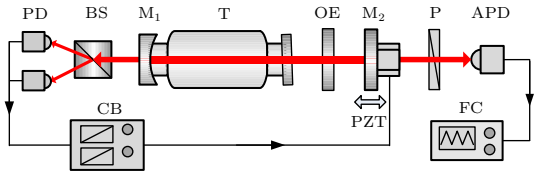

Fig. 1. Block diagram of the experimental setup. M_1, M_2: resonator mirrors; T: discharge tube; OE: optical element; PZT: piezoelectric transducer; BS: beam splitter; PD: photoelectric detectors; P: polarizer; APD: avalanche photodiode; FC: frequency counter; and CB: controlling board;

As a dual frequency He–Ne laser, the beat frequency could be calculated from the semi-classical theory. Lamb's scalar theory is capable of describing the drift of frequency for a linearly polarized laser (e.g., a gas laser with Brewster window). However, it is not suitable for an orthogonally polarized laser. One way to deal with such laser is the vectorial extension of Lamb's theory in three-order approximation,[29,30] which takes the vector characteristics of electromagnetic field and degeneracy of atomic energy levels into account. Therefore under the effects of mode pushing and pulling, the frequency of two modes with orthogonal polarization could be expressed as

$$v_{\parallel} = \Omega_{\parallel} + \sigma_{\parallel} - \rho_{\parallel} I_{\parallel} - \tau_{\parallel} I_{\perp}, \tag{1a}$$

$$v_{\perp} = \Omega_{\perp} + \sigma_{\perp} - \rho_{\perp} I_{\perp} - \tau_{\perp} I_{\parallel}, \tag{1b}$$

where $\Omega_{\parallel}$ and $\Omega_{\perp}$ are the resonant frequencies of two modes (the subscript $\parallel$ and $\perp$ represent two orthogonal polarizations of laser respectively), $\sigma_{\parallel}$ and $\sigma_{\perp}$, $\rho_{\parallel}$ and $\rho_{\perp}$, $\tau_{\parallel}$ and $\tau_{\perp}$ are the coefficients denoting linear mode pulling, self-pushing and cross-pushing of the lasing modes, which all are functions of oscillating frequencies v_1 and v_2. Then the beat frequency is

$$\Delta v = \Delta v_0 + (\sigma_{\perp} - \sigma_{\parallel}) - (\rho_{\perp} I_{\perp} - \rho_{\parallel} I_{\parallel}) - (\tau_{\perp} I_{\parallel} - \tau_{\parallel} I_{\perp}), \tag{2}$$

where $\Delta v_0 = \Omega_{\perp} - \Omega_{\parallel}$ is determined by the anisotropy modified optical path length of the resonator. In Eqs. (1) and (2), intensities of two oscillating modes with arbitrary unit read

$$I_{\parallel} = (\alpha_{\parallel}\beta_{\perp} - \alpha_{\perp}\theta_{\parallel})/(\beta_{\parallel}\beta_{\perp} - \theta_{\parallel}\theta_{\perp}), \tag{3a}$$

$$I_{\perp} = (\alpha_{\perp}\beta_{\parallel} - \alpha_{\parallel}\theta_{\perp})/(\beta_{\perp}\beta_{\parallel} - \theta_{\perp}\theta_{\parallel}), \tag{3b}$$

where $\alpha_{\parallel}$ and $\alpha_{\perp}$, $\beta_{\parallel}$ and $\beta_{\perp}$, $\theta_{\parallel}$ and $\theta_{\perp}$ denote the coefficients of the unsaturated net gain, self-saturation and cross-saturation of two laser polarizations, respectively. Compared with Lamb's theory, the above coefficients should be modified considering the vector characteristics of laser. According to Eqs. (2) and (3), the beat frequency is calculated, as shown in Fig. 2(b), in which the variation amplitude is about 260 kHz. Thus the theoretical result is consistent with that in Fig. 2(a). In general, depending on the laser semi-classical theory, the variation amplitude is not more than 500 kHz as to a beat frequency of several hundred megahertz.

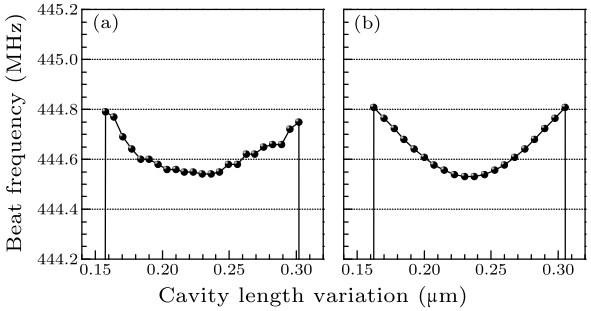

Fig. 2. Variation of beat frequency (444.6 MHz in average) with resonant cavity length changed: (a) experiments, and (b) the calculated results.

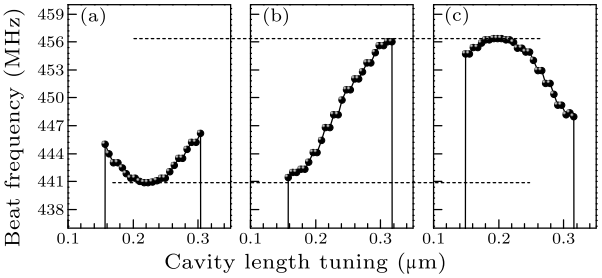

Fig. 3. Anomalous variation of beat frequency with an intra-cavity wave plate aligned. Here (a), (b), (c) are the beat frequency variation measured at different times.

However, once the QWP is collimated along the laser axis thoroughly, the beat frequency varies very differently from the result shown in Fig. 2. In this situation, the variation curve with respect to cavity length changes occasionally as given in Fig. 3. Obviously the variation amplitude, about 5.3 MHz in Fig. 3(a), is much larger than 260 kHz. It also reveals that those curves alter in several tens of minutes. We associate this shift with the thermal extension of laser cavity. Due to the fact that the two modes have a limited coexisting region, those curves should be at

different portions of the whole variation of beat frequency. If the maximum change of beat frequency is viewed as the actual variation amplitude, it is therefore about 15.5 MHz as marked between the dashed lines in Figs. 3(a)–3(c).

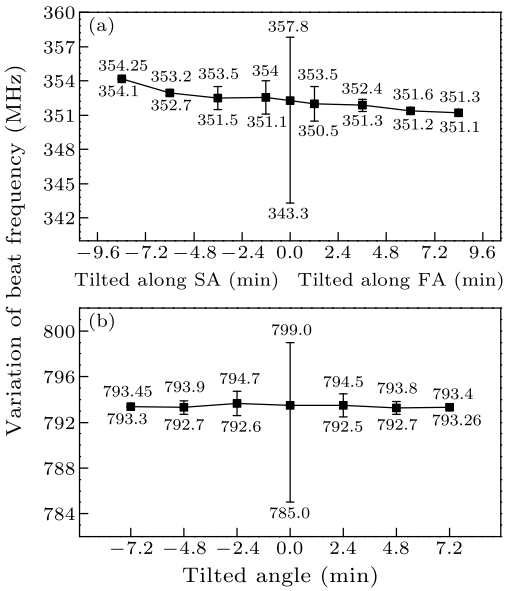

Fig. 4. Variation of beat frequency versus the tilted angle of (a) the quarter wave plate and (b) the glass plate. (a) The wave plate is tilted along both the fast axis (FA) and the slow axis (SA). (b) The glass plate is tilted along orthogonal directions.

Comparing Fig. 2 with Fig. 3, it could be found that the misaligned angle of QWP affects beat frequency variation greatly. Study of this effect is carried out by using another QWP and a glass plate. With different tilted angles of QWP, the variation amplitude is examined as demonstrated in Fig. 4(a). The positive section of x-coordinate represents that QWP is tilted along its fast axis (FA) while the negative section denotes that it is tilted along the slow axis (SA). The maximum variation amplitude of 14.5 MHz is obtained at 0°. The more the QWP deviates from the laser axis along FA or SA, the smaller the variation amplitude is. If the misaligned angle comes to 8′, the amplitude decreases to 150–200 kHz remarkably. It is comparable to the theoretical analysis. In Fig. 4(a), it is noticed that the average value of beat frequency decreases with the QWP tilted along FA, and increases with the QWP tilted along SA. This follows the fact that the actual phase retardation becomes smaller or larger when the wave plate rotates about FA or SA, separately. Similar results are acquired in Fig. 4(b) by using the glass plate. The maximum variation amplitude is about 14 MHz if the glass plate is collimated along the laser axis. When its tilted angle becomes 7.2′, the variation amplitude reduces to about 200 kHz or less. Owing to the small residual anisotropy of the glass plate, no obvious change of average beat frequency appears.

Then it could be concluded that very large variation amplitude of beat frequency, tens of times larger than that calculated by Lamb's theory, is introduced by the collimated element. We obtain that this anomalous variation comes from the weak reflection of element surfaces, considering the proof that the variation amplitude reduces rapidly to the normal as the element tilted angle exceeds 7.2′. After the element is inserted into the cavity, a tiny portion of laser is reflected back to the opposite mirror to form multi-reflection. If the reflected beams are coupled with the oscillating modes, it would cause disturbances in mode frequency and finally influence the beat frequency. The more the reflected beam is coupled, the more the output laser suffers from intra-cavity element. In this way, the misaligned angle of the element can be estimated. According to the laser configuration in Fig. 1, the Gaussian beam has its waist on the plane mirror M_2, while the radius of the laser beam spot on the concave mirror M_1 is

$$\omega = (\lambda^2 L R^2)^{1/4}/[\pi^2(R-L)]^{1/4}, \tag{4}$$

where L and R are the cavity length and the curvature radius of M_1, respectively, λ denotes the laser wavelength. Then the average radius of the laser beam of the fundamental transverse mode is $\omega = 0.3$ mm, and the reflected beams by the element have the same radius. It is assumed that the overlapped volume of the two beams decreases to $1/e$ when the variation of the beat frequency returns to normal. The critical angle θ_c of the tilted element should satisfy

$$\int_0^{x_0\theta} \left[\arctan\left(\sqrt{\frac{\omega-h}{\omega+h}}\right) - \frac{h}{2\omega^2}\sqrt{\omega^2-h^2}\right] dh \le \frac{\pi\theta}{4e}L, \tag{5}$$

where $h = x\cdot\theta$, x is the distance between the element and the integral section. Given that the distance from the element to M_1 is l, the upper limit of integration x_0 is

$$x_0 = \min\{\omega/\theta, l\}. \tag{6}$$

Through Eq. (5), θ_c is calculated to be 7.0′, close to that obtained in Fig. 4. Beyond this critical angle, the element reflecting beam is almost not coupled with the laser beam and thus leads to no effect on variation of beat frequency. Additionally, there are two surfaces of each element and only the first order reflection beams are taken into account because the reflectivity is very low.

To verify those analyses, we investigate the variation altering of beat frequency when the collimated element is moved along the laser beam. As illustrated in Fig. 5(a), the distance l' is changed, making the glass plate locate at different positions in the cavity (e.g., α and β points), then the maximum variation amplitude

is recorded. The experimental results are presented in Fig. 5(b). The variation amplitude decreases with the plate moving towards M_2, which ranges from 4.4 MHz to 22 MHz with a slope of about 0.88 MHz/mm. Due to the fact that the laser gain medium occupies most of the cavity space (in Fig. 1 the discharge tube T is 156 mm long), the largest variation amplitude of 22 MHz is obtained as l' is 28 mm.

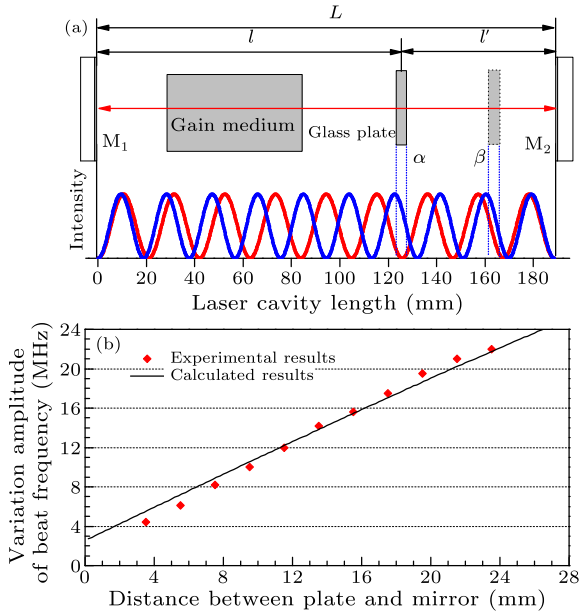

Fig. 5. (a) Diagram of the laser resonator with a glass plate inserted. The plate can change its position along the beam, e.g., positions α and β. Sinusoidal curves in red and blue represent schematically the intensity distribution of a standing wave in the cavity. (b) Variation amplitude of beat frequency when the distance between the plate and M_2 is changed. Both the experimental and the calculated results are presented.

Since the intra-cavity element influences the laser frequency through surface reflection, we assume that its effect is proportional to the intensity of standing wave laser in the resonator. In Fig. 5(a) the intensities of two adjacent longitudinal modes are schematically denoted in sinusoidal distribution. Due to the fact that the two modes have a total of π phase difference in the whole cavity, the intensity distributions have certain phase difference at different positions. Then the beat frequency, associated with the intensity difference, can be expressed as

$$\Delta v = A[\cos^2(2\pi v_{\parallel} l'/c) - \cos^2(2\pi v_{\perp} l'/c)] + \Delta, \quad (7)$$

where A is the proportional coefficient, $v_{\parallel}$ and $v_{\perp}$ are the frequencies of two modes, Δ is the laser longitudinal modes spacing, and c denotes the speed of light in a vacuum. By choosing the value of A, we calculate the variation amplitude of Δv versus l' in Fig. 5(b). The analysis agrees with the experimental results well, and the variation amplitude changes greatly within the whole cavity length. Its maximum value of 50 MHz could be expected at the center of the laser cavity.

In conclusion, the anomalous variation of beat frequency is induced in a dual frequency He–Ne laser by an intra-cavity element. The more the element is collimated with the laser axis, the more greatly the beat frequency varies. This is applicable to some precision measurements, especially when the proper laser frequency difference or frequency stabilization is essential, such as interferometer, phase retardation measurement utilizing frequency splitting. One way to diminish such an unwanted effect is making the intra-cavity element misaligned. In other applications, the beat frequency variation may work as a starting point for measuring or sensing. Of course, detailed investigations should be carried out in further research to reveal the mechanism of this phenomenon, and we hope it could benefit some other applications in laser physics fields.

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