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Tight Focusing Properties of Radially Polarized Gaussian Beams with Pair of Vortices

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The tight focusing properties of a radially polarized Gaussian beam with a nested pair of vortices having a radial wave front distribution are investigated theoretically by the vector diffraction theory. The results show that the optical intensity in the focal region can be altered considerably by changing the location of the vortices nested in a radially polarized Gaussian beam. It is noted that focal evolution from one annular focal pattern to a highly confined focal spot in the transverse direction is observed corresponding to the change in the location of the optical vortices in the input plane. It is also observed that the generated focal hole or spot lead to a focal shift along the optical axis remarkably under proper radial phase modulation. Hence the proposed system may be applied to construct tunable optical traps for both high and low refractive index particles.

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Focusing light to a focal hole and a very tight spot by varying the beam parameters is one of the most important topics for optical researchers. Recently, singular optics studying optical vortices or phase singularities has grown rapidly because optical vortices have some interesting properties[1–3] and multiple applications.[4–7] Topological interactions of vortices can lead to the creation, annihilation or even nucleation of vortices in various systems. As an example, interesting effects such as attraction followed by annihilation of a pair of vortices (POV) with opposite topological charge, or repulsion after the creation of a pair of vortices with identical topological charge have been reported widely.[9–12] Recently, Chen *et al.* investigated the properties of a pair of vortices embedded in a Gaussian beam focused by a high numerical aperture on the basis of the vector Debye integral.[13] They observed that for the incident beam with a pair of vortices with opposite topological charges, the vortices move towards each other, annihilate and revive in the vicinity of the focal plane. Fang *et al.* studied the focusing properties of the double-vortex beams through a high numerical-aperture objective based on the vectorial Debye theory.[14] They reported that the total intensity distributions in the focal region are of a double-ring structure and by adjusting certain parameters, the highly found incident double vortex beam shape of the double-ring can be changed. In the above two works vortex pairs nested in linearly polarized beams are considered. In fact, the polarization and phase wave front are very important characteristics to alter the propagating and focusing properties of beams, and have attracted a great deal of attention recently.[15–18] In high numerical aperture optical systems, the polarization characteristics of beams should be carefully considered to obtain more accurate desired focal patterns. To the best of our knowledge, the focusing properties of a radially polarized Gaussian beam nested with a pair of optical vortices have not yet been studied. Hence based on the vector diffraction theory, the present study analyzed the focusing properties of a radially polarized Gaussian beam nested with a pair of vortices. Here we also consider the phase wave front of vortices pair as the function of radial coordinates.

The schematic diagram of the focusing system for a pair of vortices nested in a radially polarized Gaussian beam is shown in Fig. 1.

In the focusing system investigated, the focusing beam has a pair of vortices of topological charge $m = 1$ located at $x = \pm d$, respectively, in the plane of the Gaussian host beam with its polarization distribution turning on radial symmetric. The electric field, in the cylindrical coordinate system (r, ϕ) can be expressed as[13,15]

$$E_0(r,\phi) = \exp\Big(-\frac{r^2}{w}\Big) \times [r\exp(i\phi) - d] \times [r\exp(i\phi) + d] \times \exp(i\psi), \quad (1)$$

where r and ϕ represent the radial distance and angle in the polar coordinates, respectively, w is a constant denoting the beam waist width, the distance between the vortex is decided by d, and ψ indicates the wave front distribution. In this work the phase wave front is the function of the radial co-ordinate and can be written as

$$\psi = g \times \frac{\sin\theta}{\sin\alpha} \times \pi, \quad (2)$$

where g is a phase parameter that characterizes the radial phase wave front.

In the case of the incident radial polarization, adopting the cylindrical coordinate's r, z, ϕ and by

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using the same analysis method as that in Refs. [16–18], the electric field in the focal region is expressed as

$$\boldsymbol{E}(\rho,\phi,z) = E_\rho \boldsymbol{e}_\rho + E_\phi \boldsymbol{e}_\phi + E_z \boldsymbol{e}_z, \tag{3}$$

where $\boldsymbol{e}_\rho$, $\boldsymbol{e}_\phi$ and $\boldsymbol{e}_z$ are the unit vectors in the radial, azimuthal, and propagating directions, respectively, and E_ρ, E_ϕ and E_z are amplitudes of the three orthogonal components, and the electric field distribution in the focal region could be expressed as follows:[19]

$$\begin{aligned} E_\rho(\rho,\phi,z) = & -\frac{iA}{\pi}\int_0^\alpha\int_0^{2\pi}\sqrt{\cos\theta}E_0\sin\theta\cos\theta \\ & \cdot\cos(\varphi-\phi)\exp\{ik[z\cos\theta \\ & +\rho\sin\theta\cos(\varphi-\phi)]\}d\varphi d\theta, \end{aligned} \tag{4}$$

$$\begin{aligned} E_\phi(\rho,\phi,z) = & -\frac{iA}{\pi}\int_0^\alpha\int_0^{2\pi}\sqrt{\cos\theta}E_0\sin\theta\cos\theta \\ & \cdot\sin(\varphi-\phi)\exp\{ik[z\cos\theta \\ & +\rho\sin\theta\cos(\varphi-\phi)]\}d\varphi d\theta, \end{aligned} \tag{5}$$

$$\begin{aligned} E_z(\rho,\phi,z) = & \frac{iA}{\pi}\int_0^\alpha\int_0^{2\pi}\sqrt{\cos\theta}E_0\sin^2\theta\exp\{ik[z\cos\theta \\ & +\rho\sin\theta\cos(\varphi-\phi)]\}d\varphi d\theta, \end{aligned} \tag{6}$$

where θ and φ denote the tangential angle with respect to the z-axis and the azimuthal angle with respect to the x-axis, respectively, k is the wave number, and $\alpha = \arcsin(\mathrm{NA})$ is the convergence angle corresponding to the radius of the incident optical aperture.

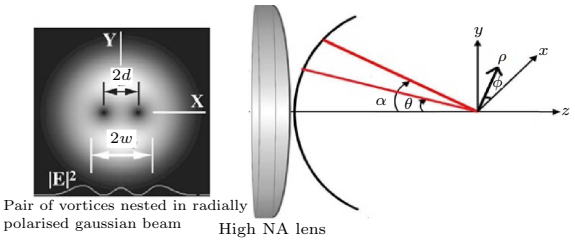

Fig. 1. Schematic diagram for a POV nested in a radially polarized Gaussian beam focused by a high NA lens.

In the investigation of tight focusing of a radially polarized pair of vortices, it is proposed for simplicity and without loss of generality, that we considered the NA of the focusing system NA = 0.95 and the waist with $w = 2\,\mathrm{cm}$, $f = 1\,\mathrm{cm}$ and $\lambda = 632.8\,\mathrm{nm}$. It should be noted that the distance unit in all figures is in the scale of λ, where λ is the wavelength of the incident beam and the intensities are normalized to unity. We consider that the topological charges of the two vortices have the same magnitude. We choose two vortices of topological charge $m = 1$, located at $x = \pm d$, respectively, in the plane of the host Gaussian beam.

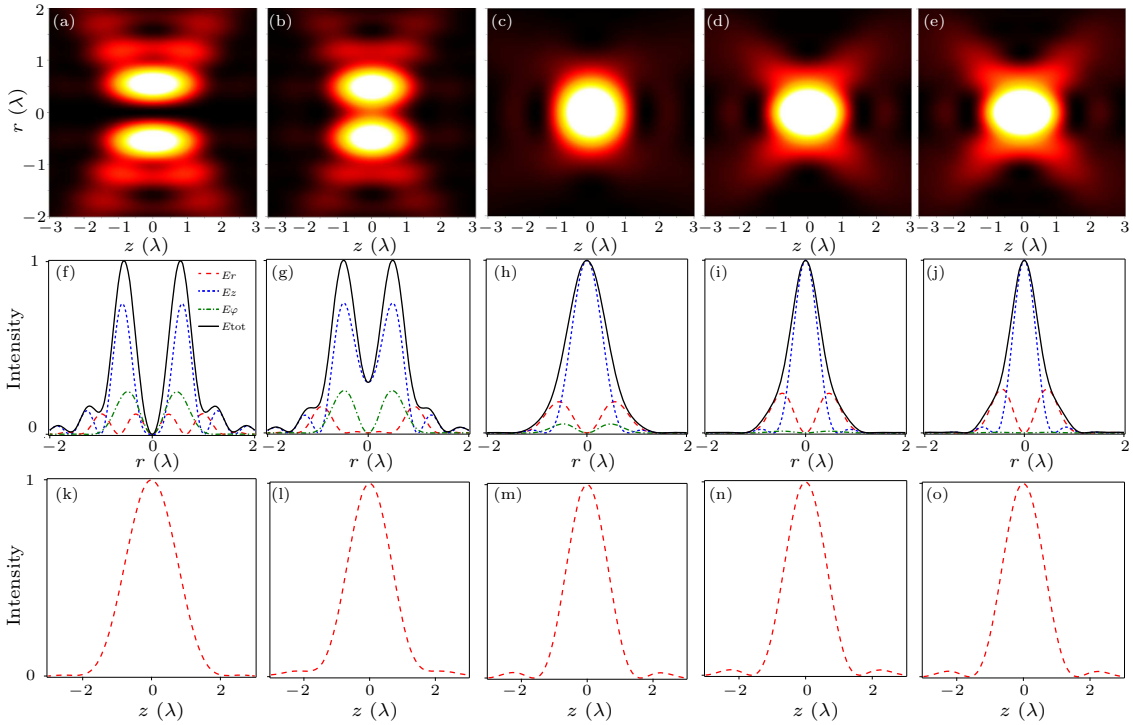

Fig. 2. (a)–(e) The 3D intensity distribution in the r–z plane corresponding to $d = 0$, $0.2w$, $0.4w$, $0.6w$ and $0.8w$. (f)–(j) The corresponding intensity measured along the radial axis at $z = 0$. (k)–(o) The corresponding axial intensity distribution. The other parameters are chosen as $\lambda = 632.8\,\mathrm{nm}$, $w = 2\,\mathrm{mm}$, $f = 2\,\mathrm{mm}$, NA = 0.95, and $g = 0$.

Figure 2 shows the influence of the location of the vortices nested in a radially polarized Gaussian beam with the radial phase parameter $g = 0$. Figures 2(a)–2(e) show the 3D intensity distribution in the r–z plane corresponding to $d = 0$, $0.2w$, $0.4w$, $0.6w$ and $0.8w$. Figures 2(f)–2(j) show the corresponding intensity measured along the radial axis at $z = 0$, and Figs. 2(k)–2(o) show the respective axial intensity dis-

tribution. It is noted from Fig. 2(a) that, when $d = 0$ which corresponds to the Gaussian vortex beam of topological charge $m = 2$, the resultant focal structure is a focal hole having FWHM of 0.628λ and focal depth of 1.739λ. It is noted from Fig. 2(f) that the longitudinal component having a central minimum dominates the focal structure. Figure 2(b) shows that increasing d to $0.2w$ results in a focal hole in which residual intensity at the center due to the central nonzero of the longitudinal component is observed. It is also noted that further increasing d to $0.4w$ the two vortices overlap in such a way that the resultant structure results in a focal spot having a FWHM of 0.918λ and a focal depth of 1.37λ. We also note that further increasing d to $0.6w$ and $0.8w$ results in a further increase of the strength of the longitudinal component which leads to further confinement of the focal spot size around 0.7λ and focal depth around 1.3λ, as shown in Figs. 2(d) and 2(e), respectively. This is due to the fact that initially only one dark core is observed in the focal plane when two vortices with equal topological charges are both located in the center of the incident beam. It then gradually disappeared and formed a sharp focal spot structure as the vortices move away from the center. Thus it is noted that by properly tuning the location of the pair of vortices, one can tune the focal structure from a highly confined longitudinally polarized focal hole to a focal spot. It is noted that the focal structure with sharp focal spots are usually used to trap particles with a refractive index higher than the ambient.[20] However, particles with a refractive index lower than the ambient can be trapped in the dark part of the doughnut-shaped beams.[21] Hence with the proposed model, by using a single focusing unit one can trap particles of both higher and lower refractive index by properly manipulating the location of the vortices.

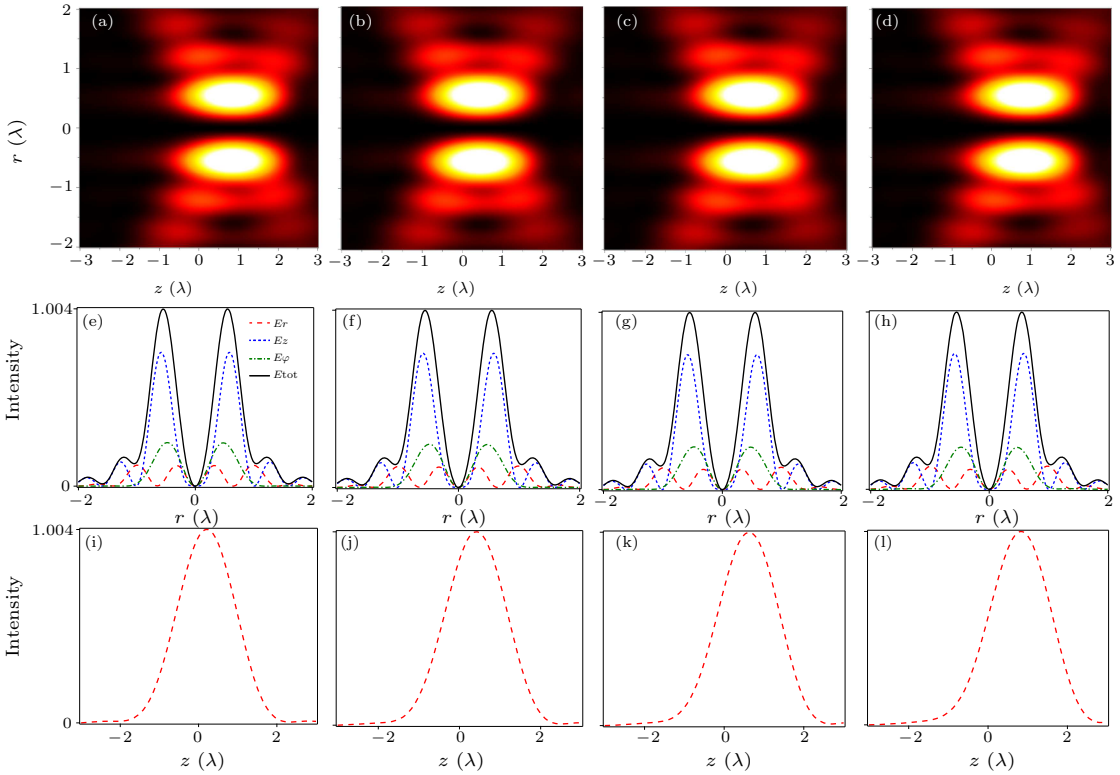

Fig. 3. The same as Fig. 2 but for $d = 0$ and for different radial phase parameters: (a) $g = 0.5$, (b) $g = 1.0$, (c) $g = 1.5$ and (d) $g = 2$.

Figure 3 shows the same as Fig. 2 while for $d = 0$ and for different radial phase constants. It is noted that the generated focal hole suffers a focal shift along the optical axis with the increase of g. Here the focal shift refers to the intensity maximum position moving away from the optical axis.[22] It should be noted here that the distance unit of the focal shift is in λ. Figure 5 shows the axial positional shift of the generated focal hole corresponding to the change in the radial phase parameter.

Figure 4 is the same as Fig. 3 but for $d = 0.8w$. It is noted from the figure that the generated focal spot also suffers a focal shift along the optical axis corresponding to the change in radial phase parameter, and Fig. 5 shows the axial positional shift of the focal spot with different g. For many applications, e.g., for moving trapped particles for scanning a specimen, one would like to change the position of the focal spot along the optical axis without moving lenses or laser beams. Hence by properly changing the radial phase parameter in the input field at the pupil plane of the lens, the generated focal structure can be shifted along the optical axis which can be used for the movement of the trapped particles towards or away from the optical aperture. Hence with the proposed method using a single focusing unit one can generate and shift a fo-

cal spot or focal hole largely used to trap and move particles of refractive index higher or lower than the ambient.

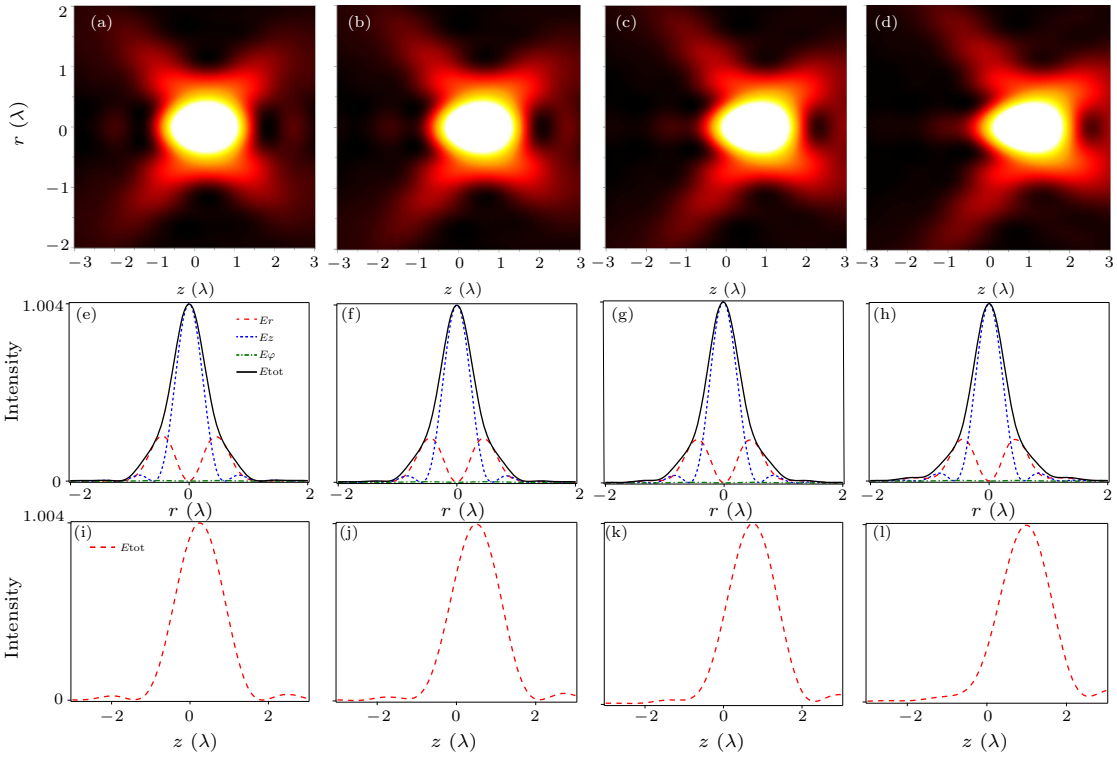

Fig. 4. The same as Fig. 3 but for $d = 0.8$ and for different radial phase parameters: (a) $g = 0.5$, (b) $g = 1.0$, (c) $g = 1.5$ and (d) $g = 2.0$.

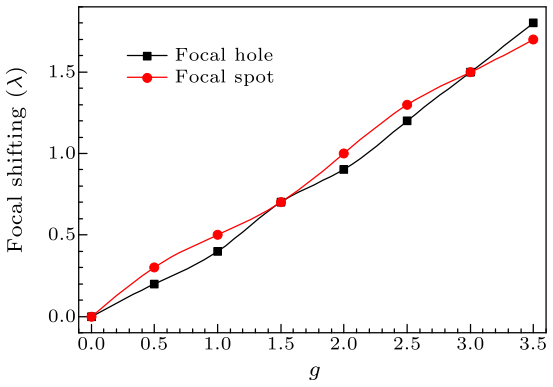

Fig. 5. Variation of focal shift on g.

In conclusion, the intensity distributions near the focal point for a radially polarized Gaussian beam nested with a pair of vortices have been calculated based on the vector diffraction theory. It is noted that by properly manipulating the location of the vortices and by properly tuning the radial phase modulation, one can tune the focal structure from focal hole to focal spot and also shift in the axial direction. Hence, with the help of a single optical setup one can trap and move particles of both higher and lower refractive index than the ambient.

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