CHIN. PHYS. LETT. Vol. 33, No. 12 (2016) 120501

Effect of Network Size on Collective Motion of Mean Field for a Globally Coupled Map with Disorder *

Jing-Hui Li(李静辉)**

Faculty of Science, Ningbo University, Ningbo 315211

(Received 13 June 2016)

We investigate the effect of the network size (or the elements number) on the collective motion of the mean field for a globally coupled map with disorder. It is shown that, with the increasing network size, the collective motion of the mean field of the globally coupled map can be shrunk periodically. In the absence of disorder or in the presence of disorder while without the coupling, this phenomenon is absent. Our result means that disorder can make the globally coupled map tame itself for certain numbers of network size. In addition, we discuss the possible application of our result to the network for action potential wave block at-a-distance in the heart.

PACS: 05.45.Pq, 05.45.−a, 05.45.Xt DOI: 10.1088/0256-307X/33/12/120501

For coupled maps (including the locally coupled map lattices and the globally coupled maps, here the word 'globally' means that every element is connected with all the other elements and the word 'locally' means that every element is connected with the nearest elements), the stochastic variables, such as noise (or random) and disorder (generally, noise is stochastic with respect to the time, while disorder is stochastic with respect to the space), have some important and even sometimes dominant effects on the collective motion of these systems.[1−10] It was reported that noise leads the mean square deviation of the mean field to decrease with the increasing number of units of the coupled map.[2] It was also reported that noise can sharpen the peak in the power spectrum of the collective motion when the noise strength is less than a certain value.[3] In Ref. [4], Shibata *et al.* studied the collective motion of the globally coupled logistic map, and showed that, when the elements number is infinite, the collective motion can be successively simplified with the increase of the noise strength. We found that, when the number of units of the coupled map is finite, a simplest collective motion for the globally coupled map can be obtained by controlling the noise strength, and the dimension of the collective motion (a method for calculating it has been proposed by Shibata *et al.* in Ref. [5]) can exhibit a minimum as a function of the noise strength.[6] Restrepo *et al.* investigated the effect of the coupling strength on the collective motion of a globally coupled map driven by the additive temporal-spatial noise, and found that the critical coupling strength at which a transition to synchrony takes place depends separately on the dynamics of the individual uncoupled systems and on the largest eigenvalue of the adjacency matrix of the coupling network (their theory directly generalizes the Kuramoto model of equal strength all-to-all coupled phase oscillators to the case of oscillators with more realistic dynamics coupled via a large heterogeneous network).[7] Chen *et al.* studied the globally coupled excited map subject to the Gaussian disorder, and found the enhancement of resonance coherence caused by disorder.[8] In Ref. [9], Parra-Rojas *et al.* investigated the effect of the intrinsic noise on the evolution of quasi-cycles, quasi-periodicity, and chaos for a locally coupled Markov chain map. In Ref. [10], Nag *et al.* studied a random threshold bistable locally coupled map and observed that the random threshold coupling can provide an annihilation method of the coexisting chaotic attractors of the locally coupled map and stabilize it to some fixed point (or periodic attractor).

For the above investigations of the collective motion of the coupled maps,[2−10] except for the one in Ref. [8], they mainly focused on the effect of noise (or random).[2−7,9,10] However, disorder is always present in the natural systems.[11] In the following we give some examples for the appearance of disorder. (1) When we deal with the problems of fluid systems or the movement of some objects with friction, the internal friction (viscosity) or the external friction can cause disorder.[12] (2) If the temperature of the system studied is stochastic with respect to the space, disorder connected with the temperature can appear.[13] (3) When dealing with the problems of condensed matter systems, we always run into some sorts of disorder,[14] such as the space distribution for the direction of electron spin. (4) Any matter inevitably contains a foreign substance and the space distribution of the foreign substance is wholly disordered.[15] (5) For the natural lattice systems, the actual lattice does not actually have the ones determined theoretically, and all the points of the natural lattice can contain the errors in comparison with the ones determined theoretically, and these errors can be stochastic with respect to the space. In addition, the disorder behavior is common and easily observed in social, ethnic, religious and political groups, cartel of industries, countries, herds of animals, flocks of birds, etc. For these systems, in some circumstances, we can use coupled maps (including locally coupled map lattices or globally coupled maps) to describe the collective motion of the elements.

*Supported by the K. C. Wong Magna Fund of Ningbo University of China.
**Corresponding author. Email: lijinghui@nbu.edu.cn
© 2016 Chinese Physical Society and IOP Publishing Ltd

In this study, we investigate the collective motion of the mean field of a globally coupled map subject to disorder. Now, because of the presence of disorder, the collective motion of the mean field of the coupled map will become more complicated. Our large number of the numerical simulations of the collective motion of the mean field of the globally coupled elements will reveal an undiscovered phenomenon. With the increasing network size (or the elements number), the collective motion of the mean field of the system can be shrunk periodically. In the absence of disorder, this phenomenon disappears. In the presence of disorder while without the coupling, this phenomenon also disappears.

Let us consider a network iterated by a function $f(y)$ with global coupling and disorder. The dynamical equation of the globally coupled map is

$$y_x(t+1) = (1-e)f(y_x(t)) + \frac{e}{N}\sum_{x'=1}^{N} f(y_{x'}(t)) + \varepsilon\xi(x+x_0), \quad (1)$$

where x is the space that equals $1, 2, 3, 4, \ldots, N$, N is the network size (or the space length of the network), t is the time that equals $1, 2, 3, 4, \ldots$, $y_x(t)$ is the dynamical variable at the space x at the time t, e is the coupling constant, and $\xi(x)$ is the disorder. For the function $f(y)$, we adopt the logistic map $f(y) = 1 - ay^2$.[4,6] For disorder, here for convenience, we set it as $\xi(x+1) = 1 - b\xi^2(x)$ with $b = 1.86$,[2–6] from which we can obtain the chaotic disorder after neglecting the transients. In Eq. (1), $\xi(x+x_0)$ is made to start from $x = x_0$ that is taken to be large enough for the appearance of steady chaotic disorder. Here as in Refs. [2–6], we also use the mean field of the system to describe the collective motion of the coupled map. The mean field of the system is[2–6]

$$h_t = \frac{1}{N}\sum_{x=1}^{N} f(y_x(t)). \quad (2)$$

Substituting Eq. (2) into Eq. (1), we can obtain another form for the expression of the mean field as follows:

$$h_t = \frac{1}{N}\sum_{x=1}^{N} y_x(t+1). \quad (3)$$

In the absence of disorder, if the coupling constant e is zero or small enough, the behavior of each unit is independent or almost independent of the others and the collective motion of the system with a large number of units will move only or almost only according to the large number theorem,[2] if the coupling constant is not small enough, in addition to the aspect of the movement for the collective motion of the system with a large number of units according to the large number theorem, the other aspect of movement for the collective motion will emerge depending on the effect of the coupling, for which some sorts of coherence among units have been observed.[2–6,16,17]

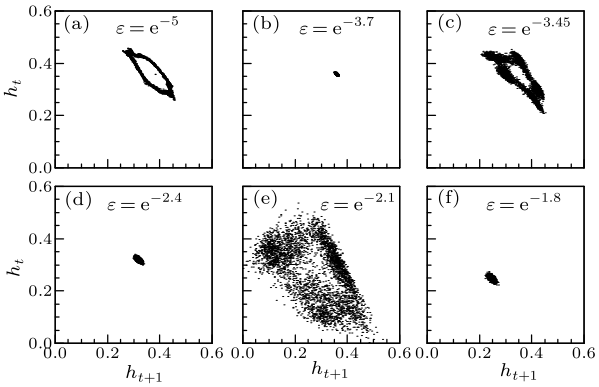

Fig. 1. The return maps of the mean field for different values of the disorder strength ε, with $a = 1.86$, $e = 0.1$ and $N = 100000$.

In the presence of disorder, the situation is different. Now, the coupled map described by Eq. (1) is subjected to disorder, the appearance of coherence among units[2–6,16,17] can be destroyed by disorder, and the evolution of Eq. (1) will exhibit very complex behavior. Our large numbers of the numerical simulations for the collective motion of the mean field of the coupled map show that, now, disorder tremendously affects the movement of the collective motion of the mean field (see Figs. 1(a)–1(f)). In Figs. 1(a)–1(f), we plot some results for the return maps of the collective motion of the mean field. These figures exhibit an example of the collective motion of the mean field for different values of the disorder strength ($\varepsilon = \exp(-5)$, $\exp(-3.7)$, $\exp(-3.45)$, $\exp(-2.4)$, $\exp(-2.1)$ and $\exp(-1.8)$, respectively) with the other parameters $a = 1.86$, $e = 0.1$ and $N = 100000$. In this progression of the collective motion of the mean field with the increasing disorder strength, Figs. 1(b), 1(d) and 1(f) have the shrunk structures, which represent that the return map of the collective motion of the mean field can be shrunk (or simplified) by disorder for several times and somewhat resembles the results found in Refs. [4,6]. However, our result is different for the ones found in Refs. [4,6]. First, our result is caused by disorder, while the latter ones are induced by noise. Secondly, ours emerges several times, while the latter ones appear only once.

We have investigated in detail the effect of disorder on the collective motion of the mean field of the globally coupled map in Ref. [18]. Here to avoid the repetition, we do not give the figures that are somewhat similar to Figs. 4 and 5 in Ref. [18]. In addition, we should give the main difference between this work and Ref. [18]. In this study, we consider the effect of the network size on the collective motion of the mean field, while we only consider the effect of disorder on the collective motion of the mean field in Ref. [18]. In addition, the kind of disorder in the present study is different from the one in Ref. [18] (in Ref. [18], disorder is a kind of uniform disorder).

Let us consider the effect of the network size on the collective motion of the mean field. In Ref. [18], we have demonstrated in detail that the variance of the mean field of the globally coupled map fully deno-

tes the shrunk extent of the collective motion of the mean field of the system. The variance σ^2 of the mean field is[18]

$$\sigma^2 = (1/N)\langle\sum_x [y_x(t+1) - \langle y\rangle]^2\rangle_t$$
$$= \langle(h_t - \langle h_t\rangle)^2\rangle_t$$
$$= (1/M)\sum_{t=1}^{M}[h_t - (1/M)\sum_{t'=1}^{M} h(t')]^2, \quad (4)$$

where $\langle\rangle_t$ denotes the average over the time t, and M is the number of the time steps, which is taken to be so large that σ^2 is wholly independent of M.

Our large numbers of the numerical simulations show that, in the presence of disorder, the variance σ^2 of the mean field can be periodically shrunk with the increasing number of the network size. Here for convenience, we call this phenomenon the collective motion period shrink with size increasing caused by disorder (CMPSSCD). In Fig. 2, we plot one of our results of the variance of the mean field versus the natural logarithm of N, i.e. $\ln N$, with the other parameters $a = 1.86$, $e = 0.1$ and $\varepsilon = e^{-2.3}$. From this figure, we can find that, with the increasing network size, the collective motion of the mean field of the system can be periodically shrunk. At the wells (not peaks) in Fig. 2, the collective motion of the mean field was mostly shrunk, which means that some kinds of synchronization or coherence may be present (see Ref. [7]). This property implies that the system can tame itself from some numbers of the network size. In addition, our large numbers of the numerical simulations also show that, in the absence of disorder (i.e., $\varepsilon = 0$), no CMPSSCD phenomenon appears. If the coupling is absent while the disorder is present, the system with many uncoupled units (i.e., Eq. (1) with $e = 0$) will evolve according to the large number theorem. In this case, our study shows that no CMPSSCD phenomenon emerges. Thus it is clear that disorder and coupling are ingredients for the appearance of the CMPSSCD phenomenon in Fig. 2.

We have depicted the appearance of the CMPSSCD phenomenon for a globally coupled map with finite globally coupled logistic maps subjected to the chaotic disorder of a logistic map. Now, disorder determines the collective dynamics of the mean field of the system, leading to the emergence of a wide range of new collective behavior of the mean field in which the coupled units system in the absence of disorder and the uncoupled units system with disorder are incapable of producing. To determine whether our result has the universality rather than only dependence on the specific model (1), we also investigate several other maps (including the circle map, tent map, etc) and some different networks (including the random network, the bipartite network, and so on), and other types of disorder (including uniform disorder,[18] dichotomous disorder, Gaussian disorder, and so on), for which we could find the similar behavior (namely, we are able to obtain the periodic-shrink of the collective motion of the mean field induced by disorder with the increase of the network size). It is probably a general result that, with the increase of the network size, disorder can produce the CMPSSCD phenomenon for the globally coupled maps. Of course, it is remained to verify if, with the increasing network size, disorder can cause the CMPSSCD phenomenon for all sorts of globally coupled maps. In addition, in this work, we only find the emergence of the CMPSSCD phenomenon in the globally coupled map. It remains to be studied whether the CMPSSCD phenomenon can emerge in other coupled systems, such as the locally coupled maps, the coupled system with finite or infinite coupled oscillators (including globally coupled and locally coupled),[19] the random walk with the coupled different equations of the master equations.[20]

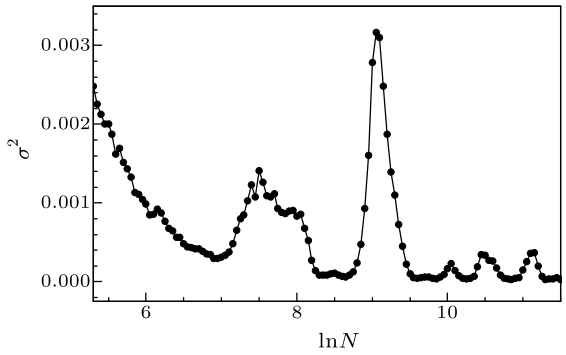

Fig. 2. The variance of the mean field versus the natural logarithm of N with $a = 1.86$, $e = 0.1$ and $\varepsilon = e^{-2.3}$.

In the following we give some explanations for the appearance of the CMPSSCD phenomenon induced by disorder for our model. Actually, the above simulation result in Fig. 2 can give some mechanisms responsible for the appearance of the CMPSSCD phenomenon, though we have not performed any analytical explanations above. Figure 2 reveals some evident representations that, in the presence of disorder, the system dynamics can show some 'resonant' behaviors with some optimal levels of the network size, similar to the effects of the signal frequency or other determined terms (not noise) of the nonlinear systems on stochastic resonance (or stochastic multi-resonance)[21] and resonant activation.[22] Thus in the presence of disorder, the network size can play two roles. On the one hand, it increases the coherent motion with the increase of the symmetry of the system, and this motion does not exist in the absence of disorder and in the uncoupled case. On the other hand, it reduces the coherent motion with the decrease of the symmetry of the system. The competition of these two opposite roles can lead to the 'resonant' behaviors of the system for some optimal levels of the network size. In addition, we can understand the appearance of the CMPSSCD phenomenon in Fig. 2 from another point of view. Firstly, in the absence of disorder, the network of the globally coupled map generally occupies a large high-dimensional parameter space and every neighborhood in the parameter space is associated with numerous dynamic attractors. Then, after we add disorder, with

the increasing network size, for some numbers of the network size, disorder can shift the system's dynamics from the spatiotemporal attractors to the shrunk attractor nearby. The appearance of the periodicity of the shrink in Fig. 2 can be understood in the following way. With the increasing network size, the system can present several different intrinsic frequencies that depend on the disorder strength and the network size. Thus with the increase of the network size, the system can present several resonances that correspond to some different intrinsic frequencies of the system.

Our result can be applied to the real systems, if these systems can be physically regarded as the globally coupled maps. We give an example in the following. For the action potential wave block at-a-distance in the heart in Refs. [23,24], the time intervals between the nearest action potential wavefronts were regarded as the globally coupled map. According to our result in the present study, the real collective motion of the mean field of the time intervals between stimuli could be tamed by lethal and nonlethal heart rhythm disorders for some numbers of the network size of the globally coupled map, which will lead to the change of the theory of action potential wave block at-a-distance in the heart proposed by Otani.[24]

We noted that, in Ref. [25], Huang *et al.* investigated the dynamics-based scalability for some networks (including the globally coupled network, the random network, the locally coupled network and the scale-free network). By focusing on network synchronizability, they found that, with the increase of the network size, the globally coupled network and random network can be scalable, while the locally coupled regular network cannot, and the scale-free network can also be scalable for certain types of node dynamics. Here we found that, for the globally coupled map with finite coupled logistic maps, in the presence of disorder, the collective motion of the mean field of the system can be scalable for several times with the increase of the network size (see Fig. 2). The main differences between our work and Ref. [25] are that (i) we consider the disorder effect on the dynamics-based scalability of a globally coupled map with the increase of the network size, while disorder was not considered in Ref. [25], and (ii) the dynamics-based scalability for our model can appear periodically with the increase of the network size (i.e., the variance of the mean field versus the network size has several wells), while, in Ref. [25], the dynamics-based scalability, if the network can be scalable, emerges only once (i.e., the synchronization probability versus network size has only one peak).

In conclusion, we have investigated the effect of the network size on the collective motion of the mean field for a globally coupled map with disorder, and found that, with the increasing network size, the collective motion of the mean field can be shrunk periodically. In the absence of disorder, or in the presence of disorder while without coupling, this phenomenon is absent. Our result has been possible to apply to the network for action potential wave block at-a-distance in the heart.[23,24] Here our model is a system with finite identical maps globally coupled with the mean field subjected to disorder, namely, our model is the globally coupled identical maps with disorder. As for the corresponding coupled non-identical maps with disorder, such as the case when a is a noise (or disorder) for Eq. (1), and the case when e is a noise (or disorder) for Eq. (1), we will consider them elsewhere. For the coupled non-identical maps, we note that, in Ref. [26], Martí *et al.* investigated the globally coupled non-identical maps with inhomogeneous delayed interactions, and found the emergence of the synchronization for the coupled elements. In addition, in the present study, we only investigate a globally coupled map with disorder. As a further investigation, the corresponding locally coupled map with disorder should be considered (we will consider it elsewhere).

References

[1] Li J H et al 1998 *Phys. Rev.* E **58** 2838
Li J H 2011 *Chaos* **21** 043115
[2] Kaneko K 1990 *Phys. Rev. Lett.* **65** 1391
Kaneko K 1995 *Physica* D **86** 158
[3] Perez G et al 1993 *Physica* D **63** 341
[4] Shibata T et al 1999 *Phys. Rev. Lett.* **82** 4424
[5] Shibata T et al 1998 *Phys. Rev. Lett.* **81** 4116
[6] Li J H 2004 *Physica* D **190** 129
[7] Restrepoet J G et al 2006 *Phys. Rev. Lett.* **96** 254103
[8] Chen H et al 2008 *Physica* A **387** 1071
[9] Parra-Rojas C, Challenger J D, Fanelli D and McKane A J 2014 *Phys. Rev.* E **90** 032135
[10] Nag M and Poria S 2014 *Phys. Scr.* **89** 095205
[11] Li J H 2007 *Physica* D **226** 209
Radons G 1996 *Phys. Rev. Lett.* **77** 4748
Popescu M N et al 2000 *Phys. Rev. Lett.* **85** 3321
[12] Herger R et al 2007 *Phys. Rev. Lett.* **98** 076102
[13] Garriga A et al 2005 *Phys. Rev.* E **72** 031505
[14] Olsson P 2007 *Phys. Rev. Lett.* **98** 097001
[15] Bakó B et al 2007 *Phys. Rev. Lett.* **98** 075701
[16] Pikovsky A S and Kurths J 1994 *Phys. Rev. Lett.* **72** 1644
Pikovsky A S and Kurths J 1994 *Physica* D **76** 411
[17] Shibata T and Kaneko K 1997 *Europhys. Lett.* **38** 417
Shibata T and Kaneko K 1998 *Physica* D **124** 163
[18] Li J H 2008 *Chin. Phys. Lett.* **25** 413
[19] Li J H and Hanggi P 2001 *Phys. Rev.* E **64** 011106
Liu Zonghua et al 2001 *Phys. Rev.* E **63** 055201
Hu B and Liu Z H 2000 *Phys. Rev.* E **62** 2114
[20] Li J H 2015 *Chin. Phys. Lett.* **32** 050502
[21] Gammaitoni L, Hä ggi P, Jung P and Marchesoni F 1998 *Rev. Mod. Phys.* **70** 223
Li J H 2002 *Phys. Rev.* E **66** 031104
Li J H 2014 *Chin. Phys. Lett.* **31** 060505
Reenbohn W L and Mangal M C 2015 *Phys. Rev.* E **91** 052151
[22] Doering C R and Gadoua J C 1992 *Phys. Rev. Lett.* **69** 2318
Li J H, Xing D Y, Dong J M and Hu B 1999 *Phys. Rev.* E **60** 1324
Li J H 2008 *Europhys. Lett.* **82** 50006
Szczepaniec K and Dybiec B 2014 *J. Stat. Mech.* **P09022**
[23] Fox J J et al 2003 *New J. Phys.* **5** 101.1
[24] Otani N F 2007 *Phys. Rev.* E **75** 021910
[25] Huang L, Lai Y C and Gatenby R A 2008 *Phys. Rev.* E **78** 045102(R)
[26] Martí A C and Masoller C 2004 *Physica* A **342** 344