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Wigner-Matrix-Based Normality Test and Application to Weak Signal Detection in SISO/SIMO Systems *

Jun Chen(陈军), Fei Wang(汪飞), Jian-Jiang Zhou(周建江)**

Key Laboratory of Radar Imaging and Microwave Photonics (Ministry of Education), Nanjing University of Aeronautics and Astronautics, Nanjing 210016

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Based on the asymptotic spectral distribution of Wigner matrices, a new normality test method is proposed via reforming the white noise sequence. In this work, the asymptotic cumulative distribution function (CDF) of eigenvalues of the Wigner matrix is deduced. A numerical Kullback–Leibler divergence of the empirical spectral CDF based on test samples from the deduced asymptotic CDF is established, which is treated as the test statistic. For validating the superiority of our proposed normality test, we apply the method to weak 8PSK signal detection in the single-input single-output (SISO) system and the single-input multiple-output (SIMO) system. By comparing with other common normality tests and the existing signal detection methods, simulation results show that the proposed method is superior and robust.

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The white Gaussian noise property of observations is one of the important theoretical bases in statistical inference, signal detection, estimation theory, and so on. The normality test has been well studied,[1] such as Pearson's χ^2 test, the D'Agostino test, the Lilliefors test and the Jarque–Bera test. Although these classical test methods are simple and matured, they are just suitable for cases of high signal-to-noise ratios (SNRs), and will miss low-SNR signals. Since it is logical to compare random data sets by way of the associated probability density function (PDF), and the Kullback–Leibler divergence (KLD) has been confirmed to be a powerful and accurate tool to measure the information of multivariate data,[2] Lu *et al.*[3] attempt to find a novel normality measure which uses the KLD between the fitted PDF and the generalized Gaussian PDF to test the white Gaussian noise property. However, this method needs some complex parameter estimations and is only suitable for high false alarm circumstances, and the detection performance still needs to be further improved. With the wide use of the random matrix theory (RMT),[4–10] it not only solves the problem of invalidation of the classical limiting theorems for large dimensional data, but also provides a completely new way of signal detection.[11] Therein, the maximum-minimum eigenvalue (MME) spectrum sensing technology[12] for cognitive radio is a recent and classical research result. However, the MME method only uses the ratio of the maximum eigenvalue to the minimum eigenvalue to detect the presence of signals, while it does not make use of the KLD information of PDFs.

To further improve the detection performance, this study takes full advantage of KLD and RMT, and uses a numerical KLD of the empirical spectral cumulative distribution function (CDF) based on observations from the asymptotic spectral CDF of the Wigner matrices to test the white Gaussian noise property. Under the white Gaussian noise assumption, we reform observations into a Wigner random matrix and regard the asymptotic spectral CDF of the Wigner matrices as the inherent feature of the white Gaussian noise. Simulation results verify that the proposed detection method outperforms all other compared methods in the single-input single-output (SISO)/single-input multiple-output (SIMO) system. It should be noted that the proposed weak signal detection method in this work is just a general case. It can be adopted for most large dimensional systems under the white Gaussian noise background.

Let the test sequence be a sample of $N \times 1$ white Gaussian noise random vector $\boldsymbol{x}$ with independent entries. Split the random vector $\boldsymbol{x}$ into M non-overlapping sub-vectors $\boldsymbol{x}_m = [x(k \cdot (m-1)+1), \ldots, x(k \cdot m)]^{\mathrm{T}}$, $m = 1, \ldots, M$, where $M = \lfloor \frac{N}{k} \rfloor$, $\lfloor o \rfloor$ denotes the floor of o, and the symbol $(\cdot)^{\mathrm{T}}$ denotes the transpose. Thus $\boldsymbol{x}_m \in \mathbb{R}^{k \times 1}$, $m = 1, \ldots, M$, are independent and identically distributed (IID) random vectors drawn from a k-dimensional normal population $\mathcal{N}_k(\boldsymbol{\mu}, \boldsymbol{R})$ with mean $\boldsymbol{\mu} = \mu \cdot \boldsymbol{I}_{1\times k}^{\mathrm{T}}$ and covariance matrix $\boldsymbol{R} = \sigma^2 \cdot \boldsymbol{I}_k$, where $\boldsymbol{I}_{1\times k} = [1, \ldots, 1]_{1\times k}$, and $\boldsymbol{I}_k$ denotes the $k \times k$ identity matrix. Normalize these random vectors with the maximum likelihood estimates (MLE) $\widehat{\mu}$ and $\widehat{\sigma}^2$ of parameters μ and σ^2. The normalized random vectors are $\boldsymbol{x}_m^{\dagger} = \frac{\boldsymbol{x}_m - \widehat{\mu} \cdot \boldsymbol{I}_{1\times k}^{\mathrm{T}}}{\widehat{\sigma}}$, $m = 1, \ldots, M$, where $\widehat{\mu} = \frac{1}{M \cdot k} \sum_{m=1}^{M} (\boldsymbol{I}_{1\times k} \cdot \boldsymbol{x}_m)$, $\widehat{\sigma}^2 = \frac{1}{M \cdot k} \sum_{m=1}^{M} [(\boldsymbol{x}_m - \widehat{\mu} \cdot \boldsymbol{I}_{1\times k}^{\mathrm{T}})^{\mathrm{T}} \cdot (\boldsymbol{x}_m - \widehat{\mu} \cdot \boldsymbol{I}_{1\times k}^{\mathrm{T}})]$ and $\boldsymbol{x}_m \in \mathbb{R}^{k\times 1}$, $m = 1, \ldots, M$ are the samples of random vectors $\boldsymbol{x}_m$.

The normalized random vectors $\boldsymbol{x}_m^{\dagger}$, $m = 1, \ldots, M$, are independent random vectors with the same k-dimensional normal distribution $\mathcal{N}_k(\boldsymbol{0}_{k\times 1}, \boldsymbol{I}_k)$, as the value of $M \cdot k$ is large enough. The covariance matrix of vectors $\boldsymbol{x}_m^{\dagger}$, $m = 1, \ldots, M$ can be defined as $\boldsymbol{C}_M = \frac{1}{M-1} \sum_{m=1}^{M} (\boldsymbol{x}_m^{\dagger} \boldsymbol{x}_m^{\dagger T})$.

As $M \to \infty$, we have $\boldsymbol{C}_M \to \boldsymbol{I}_k$. For notation con-

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**Corresponding author. Email: zjjee@nuaa.edu.cn

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venience, $\sqrt{M}\cdot(\boldsymbol{C}_M-\boldsymbol{I}_k)$ is denoted as $\boldsymbol{W}_k$. When $M\to\infty$, it can be easily seen that $E[\boldsymbol{W}_k(i,j)]=0$, for all i,j, where $\boldsymbol{W}_k(i,j)$ is the element in the ith row and the jth column of matrix $\boldsymbol{W}_k$. For $i=j$, we have $E[\boldsymbol{W}_k(i,j)^2]=\frac{M}{(M-1)^2}\sum_{m=1}^{M}\mathrm{Var}[\boldsymbol{x}_m^{\dagger 2}(i)]=\frac{2M^2}{(M-1)^2}\to 2$. For $i\neq j$, we have $E[\boldsymbol{W}_k(i,j)^2]=\frac{M}{(M-1)^2}\sum_{m=1}^{M}\mathrm{Var}[\boldsymbol{x}_m^{\dagger}(i)\boldsymbol{x}_m^{\dagger}(j)]=\frac{M^2}{(M-1)^2}\to 1$. Thus we obtain that $\boldsymbol{W}_k$ is the Wigner matrix, and $\frac{1}{\sqrt{k}}\boldsymbol{W}_k$ is the normalized Wigner matrix, as $M\to\infty$.

Here the constructed matrix $\boldsymbol{T}:=\sqrt{\frac{M}{k}}\cdot(\boldsymbol{C}_M-\boldsymbol{I}_k)$ is what we need in the end. It has the property that $\lim_{M\to\infty}F^{\boldsymbol{T}}(y)=F^{\boldsymbol{G}_k}(y)$, where $F^{\Delta}(y)$ is the spectral CDF of the random matrix Δ, and it can be defined as $F^{\Delta}(y)=\frac{1}{k}\#\{j\le k:\lambda_j\le y\}$, where $\#\Psi$ denotes the cardinality of the set Ψ, and $\Lambda=\{\lambda_j,j=1,\ldots,k\}$ is the eigenvalue set of $k\times k$ matrix Δ.

According to Wigner's semicircular law,[4] the asymptotic spectral distribution of the normalized Wigner matrix $\boldsymbol{G}_k$ is $\lim_{k\to\infty}\frac{dF^{\boldsymbol{G}_k}(y)}{dy}=\frac{\sqrt{4-y^2}}{2\pi}\mathbf{1}_{[-2,2]}(y)$. Thus the asymptotic spectral CDF $F^{\dagger}$ of $\boldsymbol{G}_k$ can be calculated as

$$\begin{aligned}F^{\dagger}(y)&=\lim_{k\to\infty}F^{\boldsymbol{G}_k}(y)\\&=\begin{cases}0, & y<-2,\\ \frac{1}{2\pi}\int_{-2}^{y}\sqrt{4-u^2}du, & -2\le y<2,\\ 1, & y\ge 2.\end{cases}\end{aligned}\tag{1}$$

Therein, when $-2\le y<0$,

$$\begin{aligned}F^{\dagger}(y)&=\frac{1}{2\pi}\Big(\int_{-2}^{0}\sqrt{4-u^2}du+\int_{0}^{y}\sqrt{4-u^2}du\Big)\\&=\frac{1}{\pi}\Big(\int_{0}^{1}v^{-\frac{1}{2}}(1-v)^{\frac{1}{2}}dv\\&\quad+\int_{\frac{y^2}{4}}^{0}v^{-\frac{1}{2}}(1-v)^{\frac{1}{2}}dv\Big)\\&\quad\text{(by setting } u=2\sqrt{v})\\&=\frac{B\left(\frac{1}{2},\frac{3}{2}\right)}{\pi}\Big\{1-I_{\frac{y^2}{4}}\Big(\frac{1}{2},\frac{3}{2}\Big)\Big\}.\end{aligned}\tag{2}$$

When $0\le y<2$,

$$\begin{aligned}F^{\dagger}(y)&=\frac{1}{2\pi}\Big(\int_{-2}^{0}\sqrt{4-u^2}du+\int_{0}^{y}\sqrt{4-u^2}du\Big)\\&=\frac{1}{\pi}\Big(\int_{0}^{1}v^{-\frac{1}{2}}(1-v)^{\frac{1}{2}}dv\\&\quad+\int_{0}^{\frac{y^2}{4}}v^{-\frac{1}{2}}(1-v)^{\frac{1}{2}}dv\Big)\\&\quad\text{(by setting } u=2\sqrt{v})\\&=\frac{B\left(\frac{1}{2},\frac{3}{2}\right)}{\pi}\Big\{1+I_{\frac{y^2}{4}}\Big(\frac{1}{2},\frac{3}{2}\Big)\Big\}.\end{aligned}\tag{3}$$

Thus the property of the constructed matrix $\boldsymbol{T}$ can be further expressed as $\lim_{k\to\infty}\lim_{M\to\infty}F^{\boldsymbol{T}}(y)=F^{\dagger}(y)$.

The test for the white Gaussian noise hypothesis can be instinctively transformed into a test of whether the unknown CDF $F(y)$ of eigenvalues $\{\lambda_j,j=1,\ldots,k\}$ of the random matrix $\boldsymbol{T}$ resembles the specified CDF $F^{\dagger}(y)$. It can be formulated as $F(y)=F^{\dagger}(y)$, under H_0; $F(y)\neq F^{\dagger}(y)$, under H_1.

As $k,M\to\infty$, there will be a very slight difference between the spectral CDF of the constructed matrix $\boldsymbol{T}$ and CDF $F^{\dagger}(y)$ if the observation is the white Gaussian noise. On the contrary, under the alternative hypothesis H_1, the difference will be enlarged. Hence, the KLD $D_{\mathrm{KL}}(F^{\dagger}\|F^{\boldsymbol{T}})$ between the empirical spectral CDF $F^{\boldsymbol{T}}(y)$ of matrix $\boldsymbol{T}$ and the asymptotic spectral CDF $F^{\dagger}(y)$ is used as the measure of normality test statistic. The KLD of $F^{\boldsymbol{T}}(y)$ from $F^{\dagger}(y)$ is defined as

$$D_{\mathrm{KL}}(F^{\dagger}\|F^{\boldsymbol{T}})=\int_{-\infty}^{+\infty}f^{\dagger}(y)\log\Big(\frac{f^{\dagger}(y)}{f^{\boldsymbol{T}}(y)}\Big)dy,\tag{4}$$

where $f^{\dagger}$ and $f^{\boldsymbol{T}}$ denote the densities of $F^{\dagger}$ and $F^{\boldsymbol{T}}$, respectively.

Since the interval of eigenvalues of matrix $\boldsymbol{T}$ is indeterminate, for the KLD calculation convenience, the probability integral transform $\lambda_j^{\dagger}=F^{\dagger}(\lambda_j)$ is used to transform the original eigenvalues to an equivalent set $\Lambda^{\dagger}=\{\lambda_j^{\dagger},\ j=1,\ldots,k\}$. Under the null hypothesis H_0, as $M\to\infty$ and $k\to\infty$, the entries of set $\Lambda^{\dagger}$ come from the uniform distribution $\boldsymbol{U}[0,1]$. Therefore, as set $\Lambda\Rightarrow\Lambda^{\dagger}$, we have the density function $f^{\dagger}(y)\Rightarrow u^{\dagger}(y)=\mathbf{1}_{[0,1]}(y)$ and $F^{\boldsymbol{T}}(y)\Rightarrow U^{\boldsymbol{T}}(y)=\frac{1}{k}\#\{j\le k:\lambda_j^{\dagger}\le y\}$, where $\Rightarrow$ denotes mapping.

Thus the KLD $D_{\mathrm{KL}}(F^{\dagger}\|F^{\boldsymbol{T}})$ can be transformed into

$$D_{\mathrm{KL}}(U^{\dagger}\|U^{\boldsymbol{T}})=-\int_{0}^{1}\log(u^{\boldsymbol{T}}(y))dy,\tag{5}$$

where $U^{\dagger}(y)$ is the CDF of the uniform distribution in [0,1], $U^{\boldsymbol{T}}(y)$ is the empirical CDF of eigenvalue set $\Lambda^{\dagger}$, and $u^{\dagger}(y)$ and $u^{\boldsymbol{T}}(y)$ are the corresponding PDFs.

We denote c_p, $p=0,1,\ldots,P$ as the partition points in the interval [0,1], and $0=c_0<c_1<\ldots<c_P=1$. When $P\to\infty$ and $\max_{1\le p\le P}\mid c_p-c_{p-1}\mid\to 0$, the test statistic $D_{\mathrm{KL}}(U^{\dagger}\|U^{\boldsymbol{T}})$ can be numerically recast to

$$\begin{aligned}D_{\mathrm{KL}}(U^{\dagger}\|U^{\boldsymbol{T}})\approx&-\sum_{p=1}^{P}(c_p-c_{p-1})\\&\cdot\log\Big(\frac{U^{\boldsymbol{T}}(c_p)-U^{\boldsymbol{T}}(c_{p-1})}{c_p-c_{p-1}}\Big).\end{aligned}\tag{6}$$

Since KLD is always non-negative, the proposed detection method is a one-tailed test. Therefore, if the value of test statistic $D_{\mathrm{KL}}(U^{\dagger}\|U^{\boldsymbol{T}})$ is no less than the critical value $C(\alpha)$, one may reject the null hypothesis H_0 at the pre-specified significance level α. Therein, the critical value $C(\alpha)$ is the $(1-\alpha)$-quantile of the distribution of the test statistic under the null hypothesis. Nevertheless, the null distribution of $D_{\mathrm{KL}}(U^{\dagger}\|U^{\boldsymbol{T}})$ is unavailable for specific M and

k. Thus critical values can only be determined as the corresponding percentage points of the empirical distribution obtained by Monte Carlo simulations.

The major complexity of the proposed method comes from the computation of the covariance matrix and the eigenvalue decomposition, thus its complexity can be written as $O(k^3 + k^2M)$, which is almost the same as the classical MME method.

For the SIMO system with a frequency flat channel, the signal detection problem can be formulated as a binary hypothesis test between null ($H_0 : \boldsymbol{X} = \boldsymbol{W}$) and alternative ($H_1 : \boldsymbol{X} = \boldsymbol{S}^\dagger + \boldsymbol{W}$) hypotheses, where $\boldsymbol{S}^\dagger = (\boldsymbol{h} \cdot \boldsymbol{s}^{\mathrm{T}}) \otimes \boldsymbol{I}_{1\times Q}$, $\boldsymbol{s} \in \mathbb{R}^{L\times 1}$ is the transmitted signal vector with entries $s[i] := s[iT_{\mathrm{b}}]$, $i = 1, \ldots, L$, T_{b} is the duration of a single symbol, $\boldsymbol{h} = [h_1, \ldots, h_M]^{\mathrm{T}}$, h_m is the channel coefficient from the transmitter to the mth receive antenna, $\otimes$ denotes the Kronecker product, $\boldsymbol{I}_{1\times Q} = [1, \ldots, 1]_{1\times Q}$, and $\boldsymbol{W}$ is an $M \times LQ$ additive white Gaussian noise (AWGN) matrix with independent entries. Note oversampling is used in each receive antenna with Q samples per T_{b}, and the received signal matrix can be denoted by $\boldsymbol{X} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M]^{\mathrm{T}}$, where $\boldsymbol{x}_m \in \mathbb{R}^{LQ\times 1}$ is the observation vector at the mth receive antenna. As a special case, the above detection model is also suitable for the SISO system, when the receive antenna number $M = 1$.

An overview of the proposed weak signal detection method for the SIMO system is depicted in Fig. 1. It can be roughly divided into five parts: normalization, matrix construction, eigenvalue calculation and transform, numerical KLD calculation, and decision.

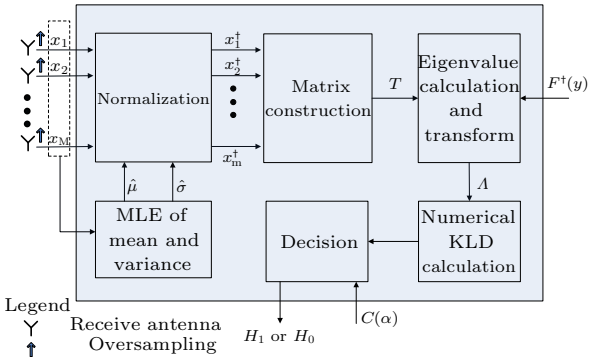

Fig. 1. Block scheme of the proposed signal detection method for the SIMO system.

Under the null hypothesis H_0, observation random vectors $\boldsymbol{x}_m$, $m = 1, \ldots, M$ are IID with the LQ-dimensional normal distribution. We can obtain the normalized vectors $\boldsymbol{x}_m^\dagger$, $m = 1, \ldots, M$ and the covariance matrix $\boldsymbol{C}_M$. Then, the framework of weak signal detection can use the normality test method proposed above.

Table 1. Detection probabilities of weak 8PSK signal for the SISO system ($P_{\mathrm{fa}} = 0.001$).

Method	f_{s} (GHz)	k	SNR −10 dB	SNR −8 dB	SNR −5 dB	SNR −3 dB
Chi2			**0.002**	**0.003**	**0.005**	**0.015**
			0.001	0.001	0.002	0.006
JB			**0**	**0.001**	**0.001**	**0.048**
			0	0	0.002	0.002
Lillie			**0**	**0.001**	**0.001**	**0.003**
			0	0	0.001	0.001
Dag	**2**		**0**	**0.001**	**0.001**	**0.037**
	0.4		0	0	0.001	0.002
Proposed method		100	**0.688**	**1**	**1**	**1**
			0.007	0.014	0.188	0.912
		50	**0.490**	**1**	**1**	**1**
			0.017	0.122	0.955	1
Method	N	k	SNR 0 dB	SNR 3 dB	SNR 5 dB	SNR 8 dB
Chi2			**0.874**	**1**	**1**	**1**
			0.052	0.978	1	1
JB			**0.988**	**1**	**1**	**1**
			0.078	0.998	1	1
Lillie			**0.522**	**1**	**1**	**1**
			0.010	0.853	1	1
Dag	**2**		**0.987**	**1**	**1**	**1**
	0.4		0.033	0.993	1	1
Proposed method		100	**1**	**1**	**1**	**1**
			1	1	1	1
		50	**1**	**1**	**1**	**1**
			1	1	1	1

For the SISO system, the received signal vector $\boldsymbol{x}$ can be split into M non-overlapping sub-vectors $\boldsymbol{x}_m$, $m = 1, \ldots, M$. The framework of the SIMO detection system can be exploited here to detect the signal.

Since a wireless communication system urgently needs the capacity of high speed data transmission, some high-order modulation methods have been widely studied and applied, such as the quadrature phase shift keying (QPSK), the 8 phase shift keying (8PSK), and the 64quadrature amplitude modulation (64QAM). Here we take the 8PSK signal as an example and assume the channel coefficient $h_m = 1$, $m = 1, \ldots, M$. The binary data that is conveyed by the transmitted signal is [111] $\otimes$

[10100010001001111001001000111101]. The symbol duration of the 8PSK signal is 0.45 μs. For QPSK and 64QAM signals, we have run the same simulations in the following, and the results show that our proposed method has almost the same superiorities.

For the SISO system, the compared normality tests are Pearson's χ^2 test, marked as Chi2; the Jarque-Bera test, marked as JB; the Lilliefors test, marked as Lillie; and the D'Agostino test, marked as Dag. For the SIMO system, the compared detection method is MME.

For the choice of the partition point number P, we perform a number of simulations with values ranging of 3–12 recommended by Dahiya *et al.*[13] and find that P almost has no effect on the detection performance. It should be noted that for a small sample size, it is not feasible to use a large number of subintervals, due to the possibility of zero observations in at least some of them. As a result, we suggest the choice of $P = 3$.

To compare the effects of the sample size on the detection performance, in the SISO system, the sample frequencies are set to 2 GHz and 0.4 GHz, respectively. At a false alarm rate of $P_{\rm fa} = 0.001$, Table 1 lists the detection probabilities of the weak 8PSK signals with different SNRs. It clearly validates that our proposed detection method outperforms other common normality tests. At a sample frequency of $f_{\rm s} = 2$ GHz (bold text), the detection probability is close to 1 until the SNR declines to −8 dB for the new method. In contrast, all compared normality tests fail to detect signals when the SNR is below 0 dB. For a small sample frequency of $f_{\rm s} = 0.4$ GHz (non-bold text), the new method still maintains its superiority, which can detect the weak 8PSK signals effectively above −3 dB. However, the SNR of effective detection needs more than 3 dB for the compared normality tests.

In regard to the choice of k for the SISO system, we run simulations with values 50 and 100. The results in Table 1 show that, for the sample frequency 2 GHz, a large value of k means a large detection probability. However, for the sample frequency 0.4 GHz, a large value of k means a small detection probability. Actually, this result is caused by different values of the sub-vector number M. In the simulation, for $f_{\rm s} = 2$ GHz, $k = 100$ corresponds to $M = 288$, while $k = 50$ corresponds to $M = 576$. For another sample frequency $f_{\rm s} = 0.4$ GHz, in the same time interval, $k = 100$ corresponds to $M = 57$, while $k = 50$ corresponds to $M = 115$. Thus we can obtain that a large value of M means a large detection probability with the same value of k. That is, the more samples there are, the better the detection performance during a fixed time interval.

For the SIMO system, the sample frequency is $f_{\rm s} = 20$ MHz in each receive antenna. Figure 2 depicts the detection probability curves with SNR for the 8PSK signal under different receive antenna numbers M and different false alarm rates $P_{\rm fa}$. It is obvious that a larger false alarm rate means a larger detection probability, which is validated in Fig. 2. In addition, we can also find that a larger value of the receive antenna number M equals better detection performance for the 8PSK signal under a specific false alarm rate in the SIMO system.

Figure 2 also shows that the detection probability maintains constant value 1 over −13 dB, which is superior to the MME method obviously, when the false alarm rate is no less than 0.001 and the receive antenna number M is no less than 500. For example, at $P_{\rm fa} = 0.1$ and $M = 1000$, for achieving 100% detection probability, SNR of the proposed method just needs to be greater than −15 dB, while that of the MME method should be greater than −5 dB. The 10 dB superiority of the proposed method is mainly because the MME method only uses the information of maximum and minimum eigenvalue, while the proposed method also utilizes the PDF information of eigenvalues to perform the detection.

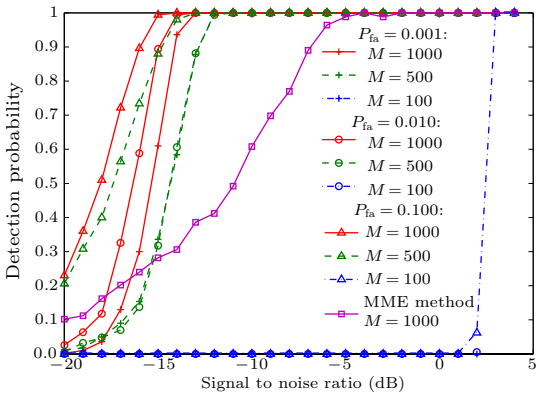

Fig. 2. Detection probability versus the SNR for the 8PSK signal detection in the SIMO system.

In summary, via reforming the white Gaussian noise sequence into a Wigner matrix and using its asymptotic spectral CDF as the inherent characteristic of the white Gaussian noise, we have proposed a new normality test method and have applied it to the weak signal detection in the SISO/SIMO system. Simulation results verify the superiority of the new detection method. Compared with the classical normality test method in the SISO system and the MME method in the SIMO system, our proposed method has 6 dB (with $P_{\rm fa} = 0.001$) and 10 dB (with $P_{\rm fa} = 0.1$) SNR improvements, respectively.

References

[1] Machiwal D and Jha M K 2012 *Hydrologic Time Series Analysis: Theory and Practice* (New Delhi: Springer) chap 3 p 42
[2] Tumminello M et al 2007 *Phys. Rev.* E **76** 031123
[3] Lu L et al 2013 *IEEE Trans. Wireless Commun.* **12** 2400
[4] Bai Z and Silverstein J W 2010 *Spectral Analysis of Large Dimensional Random Matrices* 2nd edn (New York: Springer) chap 2 p 15
[5] Chen J et al 2015 *IEEE Int. Symp. Inf. Theory* (Hong Kong China 14–19 June 2015) p 331
[6] Takami T and Fujisaki H 2007 *Phys. Rev.* E **75** 036219
[7] Yang S and Zhai H 2002 *Chin. Phys. Lett.* **19** 628
[8] Livan G et al 2011 *Phys. Rev.* E **84** 016113
[9] Conlon T et al 2007 *Physica* A **382** 565
[10] Wazir Z 2011 *Chin. Phys. Lett.* **28** 092503
[11] Couillet R and Debbah M 2011 *Random Matrix Methods For Wireless Communications* (New York: Cambridge University Press) chap 16 p 393
[12] Zeng Y and Liang Y 2009 *IEEE Trans. Commun.* **57** 1784
[13] Dahiya R C and Gurland J 1973 *J. Amn Stat. Assoc.* **68** 707