# Determination of the Minimum Wave Speed for the Modelling of Ventilated Cavitation *

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*Ventilated cavitation plays an important role on the drag reduction of underwater vehicles and surface ships. For the modelling of ventilated cavitation, the minimum speed of the pressure wave is a crucial parameter for the closure of the pressure-density coupling relationship. In this study, the minimum wave speed is determined based on a theoretical model coupling the wave equation and the bubble interface motion equation. The influences of several paramount parameters (e.g., frequency, bubble radius and void fraction) on the minimum wave speed are quantitatively demonstrated and discussed. Compared with the minimum wave speed in the traditional cavitation, values for the ventilated cavitation are much higher. The physical mechanisms for the above difference are briefly discussed with the suggestions on the usage of the present findings.*

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Modelling of ventilated cavitation is of great importance for the design of fast-speed underwater vehicles (e.g., torpedoes) and surface ships.[1–5] Different from traditional cavitation (which is induced by the low local pressure with vapor bubbles inside), the ventilated cavitation mainly consists of artificially injected non-condensable gases. The remarkable differences in nature of these two kinds of cavitation lead to the significant discrepancy of the dynamic behaviors and associated physical models.

For the modelling of ventilated cavitation, the homogeneous mixture theory is widely adopted[2–4] assuming that every phase is well mixed with each other. For example, Yu *et al.*[3] simulated the ventilated cavitation using the level set method with the smoothed Heaviside function (corresponding to Eq. (8) in Ref. [3]) regulating the interfaces between different phases. For a rigorous modelling of the ventilated cavitation, it is important to determine the interface between the liquids and bubbles physically. For that purpose, the coupling relationship between pressure and density in the bubbly mixture is essential.[6–12] Based on the barotropic state law, Delannoy *et al.*[6] proposed the following pressure-density coupling relationship

$$p = p_{\rm g} + \frac{1}{2}(\rho_{\rm l} - \rho_{\rm g})c_{\rm min}^2 \arcsin(1-2\alpha), \qquad (1)$$

where $p$ is the pressure in the mixture, $p_{\rm g}$ is the characteristic pressure relating to bubbles (at the reference temperature), $\rho_{\rm l}$ and $\rho_{\rm g}$ are the densities of the pure liquid and the pure gas, respectively, $c_{\rm min}$ is the minimum wave speed, which is inversely proportional to the square of the slope of the curve of the barotropic state law, and $\alpha$ is the void fraction of bubbles. In Eq. (1), $c_{\rm min}$ is a crucial parameter for modelling the ventilated cavitation. Figure 1 demonstrates the importance of the values of $c_{\rm min}$ on the barotropic model (e.g., in terms of transition regions represented by $\Delta P$ and $\Delta P'$, respectively, for different values of $c_{\rm min}$). In Fig. 1, $\rho_{\rm l}$ and $\rho_{\rm g}$ are the density of pure liquid and pure gas, respectively. Specifically, from the physical viewpoints of cavitation dynamics, $c_{\rm min}$ is strongly related to the thickness and the internal structure of sheet cavitation.[7] By varying the parameter $c_{\rm min}$, the thickness and shape of the attached cavity can be controlled.

For the traditional cavitation (with vapor bubbles inside), the experimental and theoretical studies suggested that the value of $c_{\rm min}$ is of the order of several meters per second.[7–12] Jakobsen[8] proposed a theoretical model to determine the value of $c_{\rm min}$ as 3.3 m/s at the normal temperature. Coutier-Delgosha *et al.*[10] numerically obtained the value of $c_{\rm min}$ as 1.5 m/s, which agrees well with the experimental value (i.e., 1–2 m/s).[11] Shamsborhan *et al.*[7] experimentally determined the value of $c_{\rm min}$ for high void fraction cavitating flow, taking into account the effects of the Mach number and flow compressibility. Unfortunately, all the above studies focused on the traditional cavitating flows and could not be applied into the modelling of the ventilated cavitation.

In this study, the value of the minimum wave speed ($c_{\rm min}$) for the ventilated cavitation is determined based on a theoretical model coupling the wave equation and the bubble interface motion equation. The influences of several paramount parameters (e.g., frequency, bubble radius, and void fraction) on the

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minimum wave speed are quantitatively demonstrated and discussed within a wide parameter zone. The determined values of $c_{\min}$ are compared with those of traditional cavitation (with vapor bubbles inside) and the physical mechanisms involved are briefly discussed.

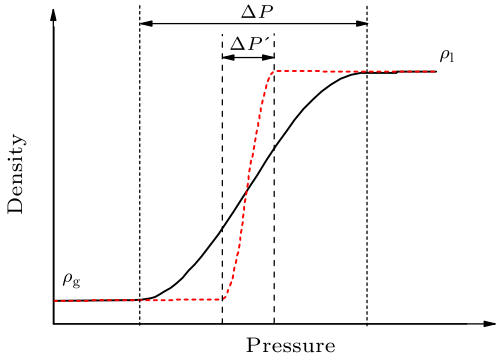


**Fig. 1.** The basic principles of the cavitation models based on the barotropic state law in terms of the pressure-density coupling relationship. Here $\rho_{\rm l}$ and $\rho_{\rm g}$ are the densities of pure liquid and pure gas, respectively, and $\Delta P$ and $\Delta P'$ denote the range of transition of the law for the bubbly mixture cases with high and low values of $c_{\min}$, respectively (referring to the solid and dashed lines).

The theoretical model is briefly introduced with the definition of the nomenclature. A framework presented by Zhang *et al.*[13] is employed in the following. In their work,[13] the effects of non-uniform pressure outside bubbles and liquid compressibility are both considered. Moreover, Zhang's model[13] has also been validated by using the experimental data (referring to Fig. 3 of Ref. [13]). For the wave propagations in gas bubbly mixture, the wave equation can be modified as[13]

$$\frac{1}{c_{\rm l}^2}\frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 4\pi\rho_{\rm l} n \int_0^\infty R_0^2 \ddot{R} f(R_0) dR_0, \quad (2)$$

where $c_{\rm l}$ is the speed of wave in the pure liquid, $P$ is the pressure in the mixture, $t$ is the time, $n$ is the total number of ventilated gas bubbles per unit volume, $R_0$ is the non-condensable gas bubble radius, $R$ is the instantaneous bubble radius, overdot denotes the time derivative, and $f(R_0)$ is the distribution function of bubble sizes with

$$\int_0^\infty f(R_0) dR_0 = 1. \quad (3)$$

The void fraction ($\alpha$) is defined as

$$\alpha = n\frac{4\pi}{3}\int_0^\infty R_0^3 f(R_0) dR_0. \quad (4)$$

For the closure of the model, the equation of bubble interface motion should be employed to couple with Eq. (2). The details of the above coupling dynamics could be found in Ref. [13]. As a result, the following dispersion relation of the wave propagation could be obtained[13]

$$\frac{1}{c_m^2} = \frac{1}{c_{\rm l}^2} + 4\pi n \int_0^\infty \frac{R_0 f(R_0) dR_0}{\psi}, \quad (5)$$

where $c_m$ is the speed of acoustic wave in the bubbly mixture. In Eq. (5), $\psi$ can be given as[13]

$$\psi = \frac{kR_0}{\sin(kR_0)}\Big\{ - \frac{\omega^2}{1+(\omega R_0/c_{\rm l})^2} + \omega_0^2 + i\omega\Big[\frac{4(\mu_{\rm l}+\mu_{\rm th})}{\rho_{\rm l} R_0^2} + \frac{R_0}{c_{\rm l}}\frac{\omega^2}{1+(\omega R_0/c_{\rm l})^2}\Big]\Big\}, \quad (6)$$

where $k = \omega/c_{\rm l}$ is the wave number, $\omega$ and $\omega_0$ are the angular frequency and the natural frequency of the acoustic wave, respectively, and $\mu_{\rm l}$ and $\mu_{\rm th}$ are the viscosity of the liquid and the effective thermal viscosity, respectively. Then, the phase speed in the mixture ($V$) can be determined as[13]

$$V = \{\mathrm{Re}(1/c_m)\}^{-1}. \quad (7)$$

The value of $c_{\min}$ can be determined by varying the parameters (e.g., frequency[7]).

Several demonstrating examples for the determination of the values of $c_{\min}$ are given and discussed. During the modelling processes, the physical properties listed in Table 1 are employed. For simplicity, air is selected as the ventilated gas with the water as the liquid. According to the practical engineering cases,[7] the value of $c_{\min}$ within the frequency range between 0 and 10 kHz is given in the following examples. In our discussions, mono-disperse gas bubbles are employed to avoid much complex discussion of the bubble size distribution ($f(R_0)$ in Eq. (2)).

**Table 1.** Physical parameters employed in the present simulation. Here $D_{\rm g,p}$ and $D_{\rm g,v}$ refer to the thermal diffusivity of the gas at the constant pressure and volume, respectively.

| Parameters | Values |
|---|---|
| Ambient temperature | $T_\infty = 288.15\,\mathrm{K}$ |
| Ambient pressure | $P_\infty = 1.01\times10^5\,\mathrm{Pa}$ |
| Molecular weight of the gas | $M_{\rm g} = 28.88\,\mathrm{g/mol}$ |
| Ratio of specific heats of the gas | $\gamma = 1.40$ |
| Surface tension coefficient | $\sigma = 0.0728\,\mathrm{N/m}$ |
| Density of gas | $\rho_{\rm gas} = 1.205\,\mathrm{kg/m^3}$ |
| Density of liquid | $\rho_{\rm l} = 998.207\,\mathrm{kg/m^3}$ |
| Speed of sound in liquid | $c_{\rm l} = 1486\,\mathrm{m/s}$ |
| Ideal gas constant | $R_{\rm g} = 8.314\,\mathrm{J{\cdot}mol^{-1}K^{-1}}$ |
| Viscosity of liquid | $\mu_{\rm l} = 1\times10^{-3}\,\mathrm{Pa{\cdot}S}$ |
| Gas thermal diffusivity | $D_{\rm g,p} = 2.08\times10^{-5}\,\mathrm{m^2/s}$ |
| | $D_{\rm g,v} = 2.9\times10^{-5}\,\mathrm{m^2/s}$ |

Figure 2 indicates the process of determining the value of $c_{\min}$. For fixed void fraction (e.g., $\alpha = 1\%$ in Fig. 2), the wave speed varies strongly with the frequency, demonstrating a lowest point ($c_{\min}$, corresponding to 32.90 m/s in Fig. 2) marked as the solid

point in Fig. 2. The variations of the wave speed versus the bubble radius show a quite similar trend, which is not shown here. In fact, the wave speed depends on a non-dimensional parameter ($fR_0/c_l$, representing the ratio between bubble radius and the wavelength in the liquid) as shown in our previous work (referring to Fig. 4 of Zhang *et al.*[13]). This non-dimensional parameter consists of both the frequency and bubble radius. The importance of the parameter $fR_0/c_l$ on the bubble dynamics was discussed by Zhang *et al.*[14]

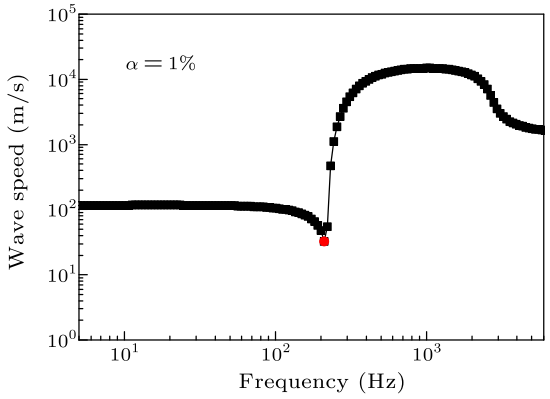


**Fig. 2.** The variations of the wave speed versus the frequency with the fixed void fraction. The minimum wave speed ($C_{\min}$) is marked as a solid red point. Here $\alpha = 1\%$.

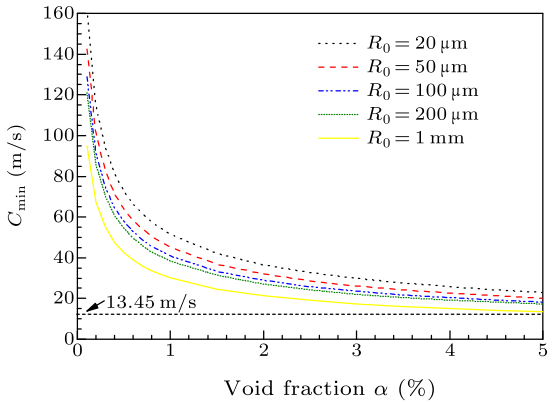


**Fig. 3.** The variations of the minimum wave speed ($C_{\min}$) versus the void fraction. The bubble radii vary from 20 µm to 1 mm with the void fraction up to 5%. The marked value (13.45 m/s) is the minimum value of $C_{\min}$ shown in our simulation.

Figure 3 shows the predictions of the minimum wave speed ($c_{\min}$) versus void fraction with bubble radii in the range of [20 µm, 1 mm]. Based on Fig. 3, $c_{\min}$ decreases with the increase of void fraction. For fixed void fraction, $c_{\min}$ decreases slightly with the increase of the bubble radii. In Table 2, the minimum values of $c_{\min}$ are summarized in terms of different bubble radii. In the parameter zone investigated here (with frequency in [0 Hz, 10 kHz] and bubble radii in [20 µm, 1 mm]), all values of $c_{\min}$ are above 13.45 m/s. These values of $c_{\min}$ for the ventilated cavitation (with gas bubbles inside) are much higher than $c_{\min}$ for the traditional cavitation (with vapor bubbles inside), which usually drop in the region of 1–3 m/s for most cases as discussed above.

**Table 2.** The minimum values of $C_{\min}$ corresponding to different bubble radii with void fraction up to 5% as shown in Fig. 3.

| $R_0$ | $C_{\min}$ (m/s) |
|---|---|
| 20 µm | 23.02 |
| 50 µm | 20.20 |
| 100 µm | 18.27 |
| 200 µm | 17.16 |
| 1 mm | 13.45 |

The physical mechanisms for the ventilated cavitation with larger values of $c_{\min}$ can be explained as follows. For the vapor bubbles in the traditional cavitation, the oscillations are violent due to the phase change, leading to a significant alternation of the wave speed. However, for the gas bubbles in the ventilated cavitation, the oscillations of the bubble interface are less prominent. Take the compression process (i.e., the shrinkage of the gas bubbles) as an example. During the above process, a limited amount of the gases could move in or out of the bubbles through the interface (termed as 'rectified diffusion'[15,16]) and the numbers of gas molecules inside the bubbles can be treated as a constant. Hence, during the process of compression, the pressure inside the bubbles will increase according to the ideal gas law, leading to a drop of the pressure difference across the bubble interface. Hence, there exists a limited amplitude for the oscillations of gas bubbles, which is quite different from the behavior of the vapor bubbles.

Owing to the occurrence of the minimum wave speed being very close to the resonance frequency region, it is difficult to measure the accurate value of $C_{\min}$ in the experiments. Compared with the previous work (e.g., $C_{\min} = 35$ m/s in Fig. 10 of Ref. [17]), the value of $C_{\min}$ obtained by us is quite similar with the same void fraction 1% (corresponding to 32.90 m/s in Fig. 2). In comparison with the value of $C_{\min}$ for the traditional cavitation (with vapor bubbles inside), the value of $C_{\min}$ for the ventilated cavitation is much larger (14 m/s versus 1–3 m/s). The influential parameters (e.g., frequency, bubble radii, and void fraction) on the determination of $C_{\min}$ are also identified and discussed within a wide range of parameters. The void fraction has strong influences on the values of $C_{\min}$ while the effects of the bubble radii are limited. This work provides a fundamental parameter for a rigorous modelling of the ventilated cavitation. In the future work, a detailed analysis of the above findings on the cavitation model should be discussed in terms of cavitation dynamics (e.g., sheet structure). For more complex cases with a mixture of bubbles (e.g., with both the gas and vapor bubbles inside cavitation zones),

researchers are referred to a recent work.[18]

## References


[1] Brennen C E 2005 *Fundamentals of Multiphase Flows* (Cambridge: Cambridge University Press)
[2] Ceccio S L 2010 *Annu. Rev. Fluid Mech.* **42** 183
[3] Yu A, Luo X and Ji B 2015 *Sci. Bull.* **60** 1833
[4] Ji B, Luo X W, Zhang Y, Ran H J, Xu H Y and Wu Y L 2010 *Chin. Phys. Lett.* **27** 096401
[5] Wang Y W, Huang C G, Du T Z, Wu X Q, Fang X, Liang N G and Wei Y P 2012 *Chin. Phys. Lett.* **29** 014601
[6] Delannoy Y and Kueny J L 1990 *ASME Cavitation Multiphase Flow Forum* (Toronto Canada 4–7 Jun 1990) **98** p 153
[7] Shamsborhan H, Coutier-Delgosha O, Caignaert G and Nour F A 2010 *Exp. Fluids* **49** 1359
[8] Jakobsen J K 1964 *J. Basic. Eng. -T. ASME* **86** 291
[9] Testud P, Moussou P, Hirschberg A and Aurégan Y 2007 *J. Fluids Struct.* **23** 163
[10] Coutier-Delgosha O, Reboud J L and Delannoy Y 2003 *Int. J. Numer. Meth. Fl.* **42** 527
[11] Stutz B and Reboud J L 2000 *Exp. Fluids* **29** 545
[12] Goncalvés E and Patella R F 2009 *Comput. Fluids* **38** 1682
[13] Zhang Y N and Du X 2015 *Ultrason. Sonochem.* **26** 119
[14] Zhang Y N and Li S C 2010 *J. Acoust. Soc. Am.* **128** EL306
[15] Zhang Y N and Li S C 2014 *J. Heat Trans. T. ASME* **136** 042001
[16] Zhang Y N and Li S C 2014 *Int. J. Heat Mass Tran.* **69** 106
[17] Commander K W and Prosperetti A 1989 *J. Acoust. Soc. Am.* **85** 732
[18] Prosperetti A 2015 *Interface Focus* **5** 20150024