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On Time-Fractional Cylindrical Nonlinear Equation *

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Properties of cylindrical electron acoustic solitons are studied in vortex plasmas. The modified cylindrical Korteweg–de Vries (KdV) equation is acquired and converted into the time fractional cylindrical modified KdV equation by Agrawal's analysis. Via the Adomian decomposition method, a cylindrical soliton solution to the equation is obtained. The cylindrical time fractional effect on the wave properties is investigated. Further, the increase of the fractional order of time α and hot to trapped electrons temperature β are minimized in both solitary width and amplitude. These influences on the structures of the soliton may be an alternative to the use of higher order perturbation analysis.

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Space observations confirmed the electron acoustic (EA) propagation in the magnetosphere.[1−7] According to fundamentals of kinetic theory, the BGK modes investigated the electron trapping and phase-space holes.[8−10] Lu *et al.*[9] performed electrostatic particle simulations for generation of electrostatic solitary mechanism in the auroral region of the Earth. They recorded that, for large EA amplitude, part of the beam and hot electrons were trapped. Moreover, Wu *et al.* discussed the effects of perpendicular thermal velocities and electron hole evolutions on transverse instability by simulation in weak magnetized plasmas.[11] They obtained that the transverse instability resorted to stability with perpendicular thermal velocity. El-Taibany *et al.* derived the modified form fifth-order nonlinear equation to explicate the soliton noises spotted by the spacecraft in Earth's magnetosphere.[12] More specifically, a comparative investigation on excited electrostatic waves in electron beam-plasmas have been discussed.[13] It was reported that the EA wave is one of the unstable modes excited in such a plasma. Sabry *et al.* examined the nonextensive effectiveness on spherical and cylindrical electron acoustic wave packets.[14] They found that the prolonged spherical envelope soliton is affected by the nonextensive parameter rather than the cylindrical one.

The investigation of wave propagation in nonplanar geometry is a realistic modality in laboratories and space plasma. Accordingly, many investigations debated plasma properties in a nonplanar geometry.[15−17] In the last few years, research works have been investigating nonplanar electron-acoustic solitons.[18−21] Pakzad introduced nonextensive hot electrons to discuss cylindrical electron acoustic solitons using numerical simulations.[19] It was noted that cylindrical solitary amplitude is less than the spherical ones. Shuchy *et al.* derived the Gardner equation for critical cylindrical and spherical electron-acoustic solitons in nonthermal plasmas.[20] Spherical properties of electron-acoustic solitons in superthermal hot electrons have been reported.[21] In physics interactions in plasmas, hot electrons having different high-energy tails caused by trapping in a wave potential will not follow the equilibrium Boltzmann velocity distribution.[22−25] In other words, nonlinear fractional equations and its applications received one of the master attentions in fluid mechanics flow, viscoelasticity, optics and physics of space plasmas.[26−29]

A lot of research has discussed the studies on plasma physics by using the fractional nonlinear evolution equations.[29−33] Guo *et al.* reported the wave progress in ion pair plasma via the Gardner equation with a time fraction term. They used the method of variational iteration to investigate the nonthermal electrons effect on the produced wave.[34] Later, Demiray *et al.* for the first time studied the cylindrical electron-acoustic mode using the Shamel vortex case via analytical and spectral scheme.[35] In this Letter, the cylindrical electron-acoustic model with vortex hot electrons is studied. The modified cylindrical Korteweg–de Vries (mcKdV) equation is derived and Agrawal's analysis[36] is applied to obtain the time-fractional mcKdV (TFmcKdV) equation solved by means of the Adomian-decomposition method.[37,38]

Our fluid model has three unmagnetized homogenous collisionless plasmas with a pressure term having ions in a stationary state, two electrons, one is

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a mobile cold electron fluid and the other is trapped vortex-like distributed hot electrons. The system and all normalization of physical quantities in small but finite amplitude are given in[35]

$$\frac{\partial}{\partial t}n_{\rm c}(r,t)+\frac{\partial}{\partial r}[n_{\rm c}(r,t)u_{\rm c}(r,t)] + \frac{1}{r}(n_{\rm c}(r,t)^{u}_{\rm c}(r,t))=0, \tag{1a}$$

$$\frac{\partial}{\partial t}u_{\rm c}(r,t)+u_{\rm c}(r,t)\frac{\partial}{\partial r}u_{\rm c}(r,t)-\delta\frac{\partial}{\partial r}\phi(r,t)=0, \tag{1b}$$

$$\frac{\partial^2}{\partial r^2}\phi(r,t)+\frac{1}{r}\frac{\partial}{\partial r}\phi(r,t) =\frac{1}{\delta}n_{\rm c}(r,t)+n_{\rm h}(r,t)-(1+\frac{1}{\delta}), \tag{1c}$$

where $n_{\rm c,h}(r,t)$ is the density of cold (trapped hot) electron, $\phi(r,t)$ is the electric potential, $u_{\rm c}(r,t)$ is the cold electron fluid velocity, and $\delta=n_{\rm h0}/n_{\rm c0}$ is the electron density ratio, and $\theta=T_{\rm h}/T_{\rm c}\gg 1$ denotes the ratio of the electron temperature ratio. The normalizations are defined as

$$\begin{aligned}&\phi(r,t)\to k_{\rm B}T_{\rm h}/e,\\ &n_{\rm c,h}\to n_{\rm c0}\ \ \text{and}\ \ n_{\rm h0},\\ &u_{\rm c}(r,t)\to\sqrt{k_{\rm B}T_{\rm h}/(m\delta)},\\ &t\to\omega_\pi^{-1},(\omega_\pi=\sqrt{4\pi n_{\rm c0}e^2/m}),\\ &r\to\lambda_{\rm D}=\sqrt{k_{\rm B}T_{\rm h}/(4\pi n_{\rm h0}e^2)},\end{aligned}$$

where e is the electron charge, m is the electron mass, $\lambda_{\rm D}$ is the Debye length of hot electron, ω_π^{-1} is the inverse of plasma frequency, and $k_{\rm B}$ is the Boltzmann constant. The hot density of electrons $n_{\rm h}(r,t)$ can be given by a trapping-vortex-like distribution[17–19]

$$n_{\rm h}(r,t)=1+\phi(r,t)-\frac{4}{3}b\phi^{3/2}(r,t) +\frac{1}{2}\phi^2(r,t)+\ldots, \tag{2a}$$

$$b=(1-\beta)/\pi^{1/2},\quad \beta=T_{\rm h}/T_{\rm ht}, \tag{2b}$$

where b is a parameter including the temperature of resonant (both free with temperature $T_{\rm h}$ and trapped with temperature $T_{\rm ht}$) hot electrons. The third term $-\frac{4}{3}b\phi^{3/2}$ denotes the contribution of electrons resonant to electron density,[24]

$$\tau=\varepsilon^{3/2}t,\quad \xi=\varepsilon^{1/2}(r-\lambda t), \tag{3}$$

where λ and ε are the normalized phase velocity and a smallness perturbation amplitude. We expand all physical quantities in Eq. (1) in ε,

$$n_{\rm c}(\xi,\tau)=1+\varepsilon^2n_1(\xi,\tau)+\varepsilon^3n_2(\xi,\tau) +\varepsilon^4n_3(\xi,\tau)+\ldots, \tag{4a}$$

$$u_{\rm c}(\xi,\tau)=\varepsilon^2u_1(\xi,\tau)+\varepsilon^3u_2(\xi,\tau) +\varepsilon^4u_3(\xi,\tau)+\ldots, \tag{4b}$$

$$\phi(\xi,\tau)=\varepsilon^2\phi_1(\xi,\tau)+\varepsilon^3\phi_2(\xi,\tau) +\varepsilon^4\phi_3(\xi,\tau)+\ldots. \tag{4c}$$

Substituting Eqs (2)–(4) into Eq. (1) and eliminating the second order perturbation quantities $n_2(\xi,\tau)$, $u_2(\xi,\tau)$ and $\phi_2(\xi,\tau)$, we can obtain the mcKdV equation as follows:[33]

$$\frac{\partial\phi_1(\xi,\tau)}{\partial\tau}+\frac{\phi_1(\xi,\tau)}{2\tau}+\frac{(1-\beta)}{\sqrt{\pi}}\phi_1^{\frac{1}{2}}(\xi,\tau)\frac{\partial\phi_1(\xi,\tau)}{\partial\xi} +\frac{1}{2}\frac{\partial^3\phi_1(\xi,\tau)}{\partial\xi^3}=0. \tag{5}$$

For simplicity $\phi_1(\xi,\tau)=\Phi(\xi,\tau)$, the mcKdV equation in (1+1)-dimension reads

$$\frac{\partial\Phi(\xi,\tau)}{\partial\tau}+\frac{\Phi(\xi,\tau)}{2\tau}+\frac{(1-\beta)}{\sqrt{\pi}}\Phi^{\frac{1}{2}}(\xi,\tau)\frac{\partial\Phi(\xi,\tau)}{\partial\xi} +\frac{1}{2}\frac{\partial^3\Phi(\xi,\tau)}{\partial\xi^3}=0. \tag{6}$$

The TFmcKdV equation can be subedited in the following way. Using a potential function $\Psi(\xi,\tau)$ where $\Phi(\xi,\tau)=\Psi_\xi(\xi,\tau)$ denotes potential equation (6) as

$$\frac{\partial^2\Psi(\xi,\tau)}{\partial\xi\partial\tau}+\frac{\Phi(\xi,\tau)}{2\tau}+\frac{2}{3}\frac{(1-\beta)}{\sqrt{\pi}}\frac{\partial}{\partial\xi}\Big(\frac{\partial\Psi(\xi,\tau)}{\partial\xi}\Big)^{\frac{3}{2}} +\frac{1}{2}\frac{\partial^4\Psi(\xi,\tau)}{\partial\xi^4}=0. \tag{7}$$

Using the semi-inverse method, the functional of potential Eq. (7) reads

$$J(\Psi)=\int_R d\tau\,\Psi(\xi,\tau)\Big[c_1\frac{\partial^2\Psi(\xi,\tau)}{\partial\xi\partial\tau}+c_2\frac{\Phi(\xi,\tau)}{2\tau} +c_3\frac{2}{3}\frac{(1-\beta)}{\sqrt{\pi}}\frac{\partial}{\partial\xi}\Big(\frac{\partial\Psi(\xi,\tau)}{\partial\xi}\Big)^{\frac{3}{2}} +c_4\frac{1}{2}\frac{\partial^4\Psi(\xi,\tau)}{\partial\xi^4}\Big], \tag{8}$$

where c_1, c_2, c_3 and c_4 are constants to be determined.

Integrating Eq. (8) by parts where, and taking the variation with respect to $\Psi(\xi,\tau)$ after integration, $\Psi_\xi|_R=\Psi_\tau|_T=0$, assuming that $\Phi(\xi,\tau)$ is a fixed, one can obtain

$$c_1=\frac{1}{2},\ c_2=1,\ c_3=\frac{2}{5},\ c_4=\frac{1}{2}. \tag{9}$$

Lagrangian of Eq. (7) reads

$$\begin{aligned}&L(\Psi,\Psi_\tau,\Psi_\xi,\Psi_{\xi\xi})\\ &=-\frac{1}{2}\Psi_\tau(\xi,\tau)\Psi_\xi(\xi,\tau)+\frac{\Phi(\xi,\tau)}{2\tau}\Psi(\xi,\tau) -\frac{4}{15}\frac{(1-\beta)}{\sqrt{\pi}}\Big(\frac{\partial\Psi(\xi,\tau)}{\partial\xi}\Big)^{\frac{5}{2}}+\frac{1}{4}\Big(\frac{\partial^2\Psi(\xi,\tau)}{\partial\xi^2}\Big)^2,\end{aligned} \tag{10}$$

where $\Psi_\tau(\xi,\tau) = \frac{\partial\Psi(\xi,\tau)}{\partial\tau}$, $\Psi_\xi(\xi,\tau) = \frac{\partial\Psi(\xi,\tau)}{\partial\xi}$, and $\Psi_{\xi\xi}(\xi,\tau) = \frac{\partial^2\Psi(\xi,\tau)}{\partial\xi^2}$.

Similarly, Lagrangian in the time-fractional range can be given as

$$\begin{aligned} &F(\Psi, {}_0D_\tau^\alpha\Psi, \Psi_\xi, \Psi_{\xi\xi}) \\ &= -\frac{1}{2}{}_0D_\tau^\alpha\Psi(\xi,\tau)\Psi_\xi(\xi,\tau) + \frac{\Phi(\xi,\tau)}{2\tau^\alpha}\Psi(\xi,\tau) \\ &\quad - \frac{4}{15}\frac{(1-\beta)}{\sqrt{\pi}}\Big(\frac{\partial\Psi(\xi,\tau)}{\partial\xi}\Big)^{\frac{5}{2}} + \frac{1}{4}\Big(\frac{\partial^2\Psi(\xi,\tau)}{\partial\xi^2}\Big)^2, \\ &\quad 0<\alpha\le 1, \end{aligned} \tag{11}$$

where ${}_0D_t^\alpha\Psi(\xi,\tau)$ is the left Riemann–Liouville fractional derivative definition[37]

$$\begin{aligned} {}_aD_t^\alpha f(t) &= \frac{1}{\Gamma(k-\alpha)}\frac{d^k}{dt^k}[\textstyle\int_a^t f(\tau)], \\ &\quad k-1\le\alpha\le k,\ t\in[a,b]. \end{aligned} \tag{12}$$

The time-fractional potential functional form is given as

$$J(\Psi) = \textstyle\int_R d\,\tau F(\Psi, {}_0D_\tau^\alpha\Psi, \Psi_\xi, \Psi_{\xi\xi}). \tag{13}$$

Following Agrawal's method,[22] taking variation of Eq. (12) to $\Psi(\xi,\tau)$ gives

$$\begin{aligned} &\delta J(\Psi) = \textstyle\int_R d\tau\Big[\Big(\frac{\partial F}{\partial {}_0D_\tau^\alpha\Psi}\Big)\delta {}_0D_\tau^\alpha\Psi + \Big(\frac{\partial F}{\partial\Psi_\xi}\Big)\delta\Psi_\xi \\ &+ \Big(\frac{\partial F}{\partial\Psi_{\xi\xi}}\Big)\delta\Psi_{\xi\xi} + \Big(\frac{\partial F}{\partial\Psi}\Big)\delta\Psi\Big], \end{aligned} \tag{14}$$

$$\begin{aligned} &\int_a^b dt f(t){}_aD_t^\alpha g(t) = \int_a^b dt\, g(t){}_tD_b^\alpha f(t), f(t), \\ &g(t)\in[a,b]. \end{aligned} \tag{15}$$

The right Riemann–Liouville fractional derivative ${}_tD_b^\alpha$ is defined by

$$\begin{aligned} {}_tD_b^\alpha f(t) &= \frac{(-1)^k}{\Gamma(k-\alpha)}\frac{d^k}{dt^k}[\textstyle\int_t^b f(\tau)], \\ &\quad k-1\le\alpha\le k,\ t\in[a,b]. \end{aligned} \tag{16}$$

By integrating

$$\begin{aligned} \delta J(\Psi) = \textstyle\int_R d\tau\Big[&{}_\tau D_{T_0}^\alpha\Big(\frac{\partial F}{\partial {}_0D_\tau^\alpha\Psi}\Big) - \frac{\partial}{\partial\xi}\Big(\frac{\partial F}{\partial\Psi_\xi}\Big) \\ &+ \frac{\partial^2}{\partial\xi^2}\Big(\frac{\partial F}{\partial\Psi_{\xi\xi}}\Big) + \Big(\frac{\partial F}{\partial\Psi}\Big)\Big]\delta\Psi, \end{aligned} \tag{17}$$

the optimizing functional $J(\Psi)$, i.e., $\delta J(\Psi)=0$, gives the Euler–Lagrange equation for the time-fractional of the potential equation in the form

$$\begin{aligned} &{}_\tau D_{T_0}^\alpha\Big(\frac{\partial F}{\partial {}_0D_\tau^\alpha\Psi}\Big) - \frac{\partial}{\partial\xi}\Big(\frac{\partial F}{\partial\Psi_\xi}\Big) \\ &+ \frac{\partial^2}{\partial\xi^2}\Big(\frac{\partial F}{\partial\Psi_{\xi\xi}}\Big) + \Big(\frac{\partial F}{\partial\Psi}\Big) = 0. \end{aligned} \tag{18}$$

Using Eq. (11), we obtain

$$\begin{aligned} &-\frac{1}{2}[{}_\tau D_{T_0}^\alpha\Psi_\xi(\xi,\tau)] + \frac{1}{2}[{}_\tau D_{T_0}^\alpha\Psi_\xi(\xi,\tau)] + \frac{\Phi(\xi,\tau)}{2\tau^\alpha} \\ &+ \frac{2}{3}\frac{(1-\beta)}{\sqrt{\pi}}\frac{\partial}{\partial\xi}\Big(\frac{\partial\Psi(\xi,\tau)}{\partial\xi}\Big)^{\frac{3}{2}} + \frac{1}{2}\frac{\partial^4\Psi(\xi,\tau)}{\partial\xi^4} = 0. \end{aligned} \tag{19}$$

Substituting for the potential function $\Psi_\xi(\xi,\tau) = \Phi(\xi,\tau)$, the TFmcKdV equation is

$$\begin{aligned} &-\frac{1}{2}[{}_\tau D_{T_0}^\alpha\Psi_\xi(\xi,\tau)] + \frac{1}{2}[{}_\tau D_{T_0}^\alpha\Psi_\xi(\xi,\tau)] + \frac{\Phi(\xi,\tau)}{2\tau^\alpha} \\ &+ \frac{(1-\beta)}{\sqrt{\pi}}\Phi^{\frac{1}{2}}(\xi,\tau)\frac{\partial\Phi(\xi,\tau)}{\partial\xi} + \frac{1}{2}\frac{\partial^3\Phi(\xi,\tau)}{\partial\xi^3} = 0, \end{aligned} \tag{20}$$

where ${}_0D_\tau^\alpha\Phi(\xi,\tau)$ and ${}_\tau D_{T_0}^\alpha\Phi(\xi,\tau)$ are the left and right Rieman–Liouville fractional derivatives and defined by Eqs. (12) and (16).

The TFmcKdV Eq. (20) can be rewritten as

$$\begin{aligned} &{}_0^RD_\tau^\alpha\Phi(\xi,\tau) + \frac{\Phi(\xi,\tau)}{2\tau^\alpha} + \frac{(1-\beta)}{\sqrt{\pi}}\Phi^{\frac{1}{2}}(\xi,\tau)\frac{\partial\Phi(\xi,\tau)}{\partial\xi} \\ &+ \frac{1}{2}\frac{\partial^3\Phi(\xi,\tau)}{\partial\xi^3} = 0,\ 0\le\alpha\le 1,\ t\in[0,T_0]. \end{aligned} \tag{21}$$

The Riesz fractional derivative ${}_0^RD_\tau^\alpha$ is given by[39]

$$\begin{aligned} &{}_0^RD_\tau^\alpha\Psi(\xi,\tau) = -\frac{1}{2}[{}_\tau D_{T_0}^\alpha\Psi(\xi,\tau)] + \frac{1}{2}[{}_\tau D_{T_0}^\alpha\Psi(\xi,\tau)] \\ &= \frac{1}{2}\frac{1}{\Gamma(k-\alpha)}\frac{d^k}{dt^k}[\int_a^t dt|\tau-t|^{k-\alpha-1}\Psi(\xi,t)], \\ &\quad k-1\le\alpha\le k,\quad t\in[a,b]. \end{aligned} \tag{22}$$

The Adomian decomposition technique decomposes the solution to a nonlinear equation $\Phi(\xi,\tau)$ into infinite series as

$$\Phi(\xi,\tau) = \sum_{n=0}^{\infty}\Phi_n(\xi,\tau), \tag{23a}$$

and the nonlinear term $F(\Phi(\xi,\tau))$ is represented by the Adomian series

$$F(\Phi(\xi,\tau)) = \sum_{n=0}^{\infty}A_n, \tag{23b}$$

where A_n are the Adomian polynomials given by

$$A_0 = F(\Phi_0(\xi,\tau)), \tag{24a}$$

$$A_1 = \Phi_1\frac{\partial F(\Phi_0)}{\partial\Phi_0}, \tag{24b}$$

$$A_2 = \Phi_2\frac{\partial F(\Phi_0)}{\partial\Phi_0} + \frac{1}{2}\Phi_1^2\frac{\partial^2F(\Phi_0)}{\partial\Phi_0^2}, \tag{24c}$$

and other polynomials can be generated in the same manner. Applying the operator ${}_0^RD_\tau^{-\alpha}$ on both sides

of Eq. (16) yields

$$\Phi(\xi,\tau) = \Phi(\xi,0) - {}_0^R D_\tau^{-\alpha}\Big[\frac{\Phi(\xi,\tau)}{2\tau} + A\Phi^{1/2}(\xi,\tau)\frac{\partial\Phi(\xi,\tau)}{\partial\xi} + B\frac{\partial^3\Phi(\xi,\tau)}{\partial\xi^3}\Big],$$
$$A = \frac{(1-\beta)}{\sqrt{\pi}}, \quad B = \frac{1}{2}, \tag{25}$$

where ${}_0^R D_\tau^{-\alpha}$ is the Riemann–Liouville fractional integral, which is defined by

$${}_0^R D_\tau^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(\tau)}{(t-\tau)^{(1-\alpha)}}d\tau, \ 0 \le \alpha < 1. \tag{26}$$

Substituting Eq. (18) into Eq. (20) leads to

$$\sum_{n=0}^{\infty}\Phi_n(\xi,\tau) = \Phi(\xi,0) - {}_0^R D_\tau^{-\alpha}\Big[\frac{1}{2\tau}\sum_{n=0}^{\infty}\Phi_n(\xi,\tau) + A\sum_{n=0}^{\infty}A_n + B\frac{\partial^3\sum_{n=0}^{\infty}\Phi_n(\xi,\tau)}{\partial\xi^3}\Big]. \tag{27}$$

The components $\Phi_n(\xi,\tau)$ of the solution $\Phi(\xi,\tau)$ can be computed by applying the following recursive relation

$$\Phi_0(\xi,\tau) = \Phi(\xi,0), \tag{28a}$$
$$\Phi_{k+1}(\xi,\tau) = -{}_0^R D_\tau^{-\alpha}\Big[\frac{1}{2\tau}\Phi_k(\xi,\tau) + AA_k + B\frac{\partial^3\Phi_k(\xi,\tau)}{\partial\xi^3}\Big], \ k \ge 1. \tag{28b}$$

The initial condition will be taken as

$$\Phi(\xi,0) = \phi_m \sec h^4(c\xi), \tag{29}$$

where ϕ_m and c are constants, and their values depend on the physical parameters of the system $\phi_m = (\frac{15v}{8A})^2$ and $c = \sqrt{\frac{v}{16B}}$, with ν being the traveling velocity.

The zero order of ADM solution can be taken from the initial value of state variable, which is taken as the solution to the regular equation at time equal to zero, in this case as

$$\Phi_0(\xi,\tau) = \Phi(\xi,0) = \phi_m \mathrm{sech}^4(c\xi). \tag{30}$$

Substituting with this zero order in the recursive relation Eq. (23b) leads to the first recursive, in the same manner, higher recursive order can be given by using the Maple package. Substituting from these different recursive orders into Eq. (23a) represents the solution of the TFMCKdV equation.

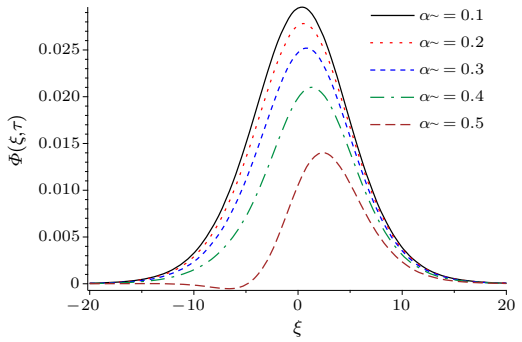

Fig. 1. The electrostatic potential $\Phi(\xi,\tau)$ against the position ξ at $\tau = 5$, $\nu = 0.1$, and $\beta = -0.5$ for different values of the fractional parameter α.

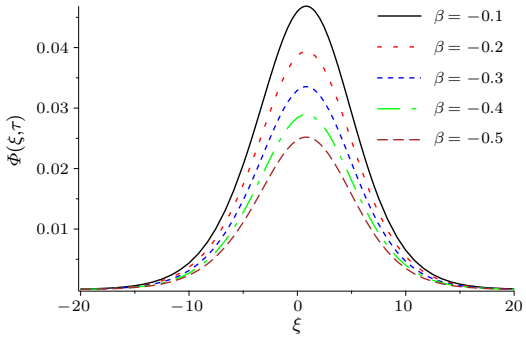

Fig. 2. The electrostatic potential $\Phi(\xi,\tau)$ against the position ξ at $\tau = 5$, $\nu = 0.1$, and the fractional parameter $\alpha = 0.3$ for different values of β.

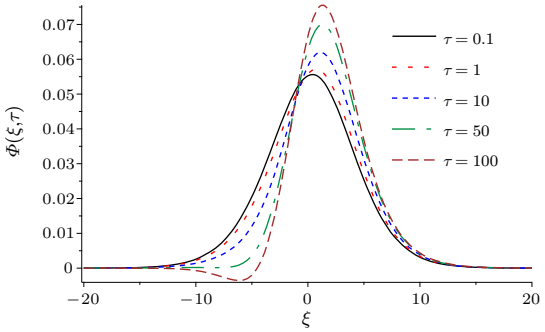

Fig. 3. The electrostatic potential $\Phi(\xi,\tau)$ against the position ξ at $\beta = -0.5$, $\nu = 0.15$, and the fractional parameter $\alpha = 0.3$ for different values of τ.

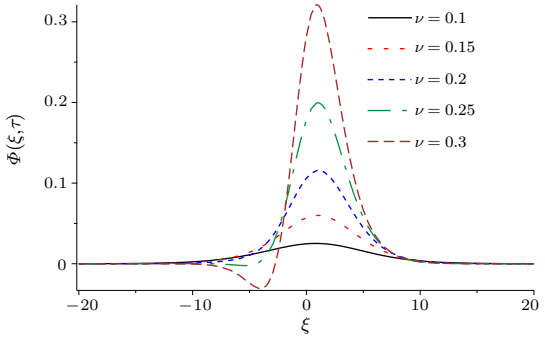

Fig. 4. The electrostatic potential $\Phi(\xi,\tau)$ against the position ξ at $\beta = -0.5$, $\tau = 5$, and the fractional parameter $\alpha = 0.3$ for different values of ν.

Three components of cylindrical collisionless unmagnetized plasma having stationary state ions, fluid of cold electrons and vortex-like distributed hot electrons have been inspected. The mcKdV equation is gained via RPT. The TFmcKdV equation has been derived by employing a variational method represented by Agrawal.[27] The derived TFmcKdV equation is solved via ADM.[37,38] The fourth order of ADM approximation is used. Numerical calculations are used to study the importance of fractional order of time α and other parameters of plasma in solitary wave features. Clearly, the effect of α on the soliton profile is shown in Fig. 1. It is noted that α reduces soliton width and amplitude. This means that the increase of α increases the Riesz fractional derivative order that affects the higher recursive order solutions and reduces the soliton amplitude and width. Effects of hot-to-trapped-electron temperature β and time τ on the solitary properties are shown in Figs. 2 and 3. It is illustrated that β decreases both the amplitude and width while τ increases both width and amplitude. The reason for β effect is that the parameter β which refers to shore of resonant hot electrons to electron density affects directly the nonlinear coefficient that controls the solitary wave amplitude.

Finally, the speed v will increase the soliton amplitude and distort the soliton width. The above calculations show that the fractional time order in cylindrical coordinates can be used to modify the solitary shape and to verify the experimental data for the definite set of plasma parameters. Further, the fractional order of the mcKdV equation modulates the shape of the wave progress in plasmas without needing to study higher orders of the mcKdV equation.[12,40]

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