Manipulating Backward Propagation of Acoustic Waves by a Periodical Structure *

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In the backward propagation of acoustic waves, the direction of phase velocity is anti-parallel to that of group velocity. We propose a scheme to manipulate the backward propagation using a periodical structure. The dynamic backward propagation process is further experimentally observed. It is demonstrated that the oblique incident plane wave moves backward when it travels through the periodical structure and the backward shift can be controlled within a certain range.

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Phononic crystals are artificial periodic composite materials consisting of periodically distributed individuals with high contrast of acoustic impedance, which can give rise to band gaps and band edge states due to the Bragg scattering as well as localized Mie resonance.[1−3] Due to the novel and unique modulation properties on the propagation of acoustic waves by phononic crystals, they have potential to be used in acoustic devices. Till now, a vast range of phenomena associated with wave propagation such as acoustic focusing,[4−7] cloaking,[8−11] negative refraction,[12−14] backward propagation,[15−18] and slow wave propagation[19] have led to applications of phononic crystals.

Backward propagation of acoustic waves in two-dimensional (2D) phononic crystals has been investigated.[15−18] It is found that backward waves can exist in a 2D triangular phononic crystal with a periodic lattice constant of 4.5 mm and a cylinder radius of 1.0 mm. Frequencies of the incident acoustic waves are localized to 60 and 73 kHz.[15,16] However, acoustic waves in frequency of several megahertz have been widely used in biology and non-destructive evaluation due to their high directivity. By using the scale model, it is difficult to fabricate a 2D phononic crystal to induce the backward propagation of acoustic waves in high frequency since the size of the phononic crystal becomes too small to be processed. Thus design of a feasible structure to manipulate the acoustic wave in high frequency is desirable. On the other hand, it has been proved that the backward wave exists in the elastic plate, in which the opposite sign of phase and group velocities is determined by the second branch of roots of the Rayleigh–Lamb frequency equation.[17] When the backward-wave motion is excited, the direction of the energy flux and group velocity is anti-parallel to that of the phase velocity, or that of motion of the phase fronts.[20,21] The backward propagation of the transmitted wave is due to the negative group velocity in the plate: the Lamb wave in a plate is formed by the reflection and interference of the incident wave from liquid. The acoustic energy in the plate propagates at the group velocity along the plate. The accumulation of the acoustic energy in the plate induces the plate to radiate acoustic energy into liquid. Thus the shift of the leaky wave is dependent on the group velocity. When the group velocity is negative, the leaky wave moves backward. However, the space shift of the backward wave in a plate is limited since the maximum negative group velocity in a plate with a fixed thickness is a definite value. For a high frequency acoustic wave, the thickness of the plate is small and the backward shift becomes negligible and its practical application will be limited.

To increase the backward shift of the acoustic waves, we propose a one-dimensional (1D) periodical structure (PS). In the structure, the shift of the backward wave is greatly increased and can be tuned by changing the number of layers or the frequency of the incident wave. The dynamic process of the acoustic wave propagation in the structure is experimentally observed by using the schlieren imaging method.[22] The effect of the frequency of the acoustic wave on the backward shift is also investigated. The experimental data are in good agreement with the simulation results.

The propagation of the elastic waves in a free infinite plate can be described by the Rayleigh–Lamb

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equation[23]

$$\frac{\omega^4}{v_T^4} = 4k^2q^2\Big(1 - \frac{p}{q}\frac{\tan(ph+\alpha)}{\tan(qh+\alpha)}\Big), \tag{1}$$

where $q^2 = \frac{\omega^2}{v_T^2} - k^2$, $p^2 = \frac{\omega^2}{v_L^2} - k^2$, $\alpha \in \{0, \frac{\pi}{2}\}$, $2h$ is the thickness of the plate, k, ω and α are the wave number, angular frequency and the incident angle of the acoustic wave, respectively, while $v_{\rm L}$ and $v_{\rm T}$ represent the longitudinal and shear wave velocities, respectively. The group velocity is given by

$$v_{\rm g} = \frac{d\omega}{dk} = \frac{df}{d(k/2\pi)}. \tag{2}$$

Figure 1 shows the group velocities of the first four Lamb modes (S0, S1, A0, A1) in a glass plate ($v_{\rm T}$ = 3673 m/s, $v_{\rm L}$ = 6202 m/s, and $2h$=1.1 mm). For the S1 mode, the group velocity remains negative in a rather wide frequency range (2.63–2.78 MHz) when the incident angle is 7°. A dotted line box in Fig. 1 shows the backward-wave region. We have proposed a 1D PS with two elements such as glass (density 2203 kg/m^3, Young's modulus 73.1 GPa, and Poisson's ratio 0.23) and water (density 998 kg/m^3 and sound speed 1482 m/s), as shown in Fig. 2(a). The band gap of the structure can be calculated by the transfer matrix and then the thickness of the water layer is determined. Figure 2(b) shows the frequency spectra of the structure in which the thickness of the water layer is 1 mm. There are five pass bands in the frequency range of 0–3.25 MHz. A broad pass band can be found between 2.60 and 3.20 MHz in which the backward-wave region (2.63–2.78 MHz) is included and the transmission coefficient is greater than 0.7 in the backward-wave region. Thus the acoustic wave shifts backward in every glass plate and resonances in the structure can allow transmission of plane waves from one side to the other.

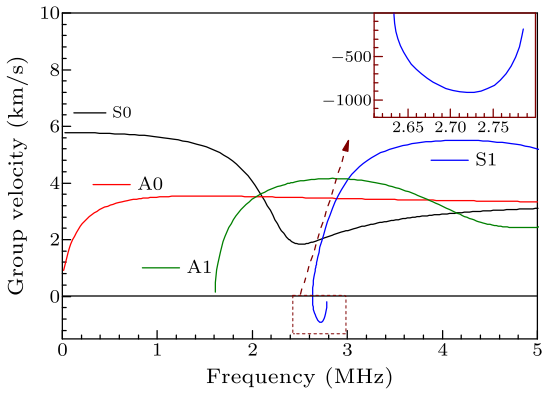

Fig. 1. The dispersion curves for a glass plate. The group velocities of the first two symmetrical (S) and antisymmetrical (A) Lamb wave modes are shown. Negative group velocity can be found in the mode S1.

The visualization of the acoustic fields is conducted by the schlieren method.[22] It is based on the acousto-optic effect to visualize the ultrasound field. Figure 3 shows the schematic diagram of the experimental setup. A wide beam of coherent laser light propagates through a transparent medium in which ultrasound propagates. The presence of ultrasound generates a light-diffraction grating which is caused by the spatially periodically varying refractive index in turn caused by traversing sound waves. Hence light is diffracted into several spatial orders. Zeroth-order field is eliminated by a spatial filter since it contains light that is not diffracted. The obtained light can be used to visualize ultrasound and is then recorded by an ICCD camera. A commercial ultrasonic transducer (Olympus, V204-RM) is used to emit plane waves. The diameter of the transducer is 6.4 mm. The transducer was driven by a signal generator (AFG3021, Tektronix) and a power amplifier (AG 1016, T&C Power conversion, Inc.). A high speed ICCD is used to record the acoustic field. The experimental details can be found in our previous work.[22]

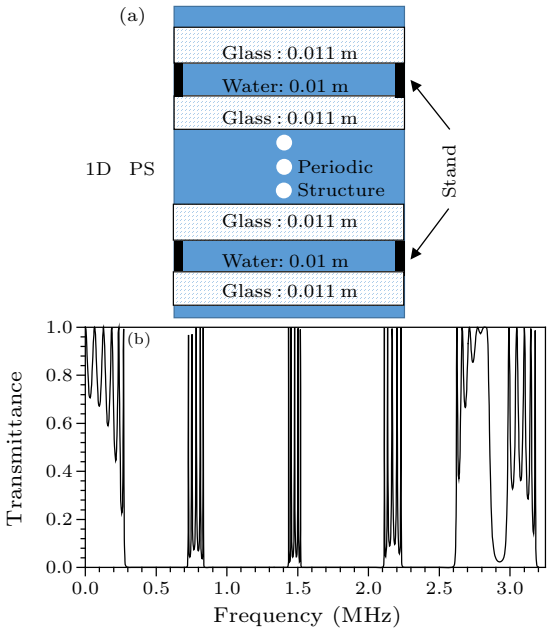

Fig. 2. (a) The geometry of the periodical structure and (b) the transmission coefficient of the structure as a function of frequency.

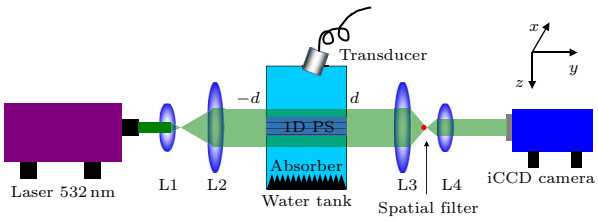

Fig. 3. Schematic illustration of the schlieren experimental apparatus.

Figure 4 shows a series of stroboscopic schlieren images of acoustic wave propagation. The dynamic process of acoustic wave propagation in the PS is re-

vealed. A long burst of the acoustic waves at frequency of 2.76 MHz is emitted from the transducer. The PS containing six layers of the water/glass structure has been used in all the experiments. The acoustic waves propagate from the upper side of the structure with an incident angle of 7° as shown in Fig. 4(a). It should be noted that the backward phenomenon can be observed in an arbitrary incident angle except normal incidence. The shift of the backward wave is dependent on the incident angle. With the increase of the incident angle, the backward shift also increases. However, the wave vector is inhomogeneous for an acoustic beam with finite width. The increase of the incident angle induces the dispersion of the transmitted wave and further broadens the transmission wave. Therefore, an incident angle of 7° is selected. After the waves pass through the first layer of the glass plate, the transmitted wave is shifted backward as observed in Fig. 4(b). Acoustic waves are multi-reflected and mutually interfered in each layer of the structure and the backward motion can be observed in each water layer. As shown in Figs. 4(c)–4(e), the transmitted waves in water are obviously shifted backward compared with the incident waves. From the experimental results, the backward shift between the incident and transmitted waves is 4.6 mm. However, the propagation directions of the incident and transmitted waves are the same. For comparison, we also show the simulation results for the backward propagation at $t = 108\,\mu s$ in Fig. 4(f). It is found that the propagation characteristics are approximately the same as the experimental results as shown in Fig. 4(e), and the backward shift is about 5 mm. The difference between the simulation and the experimental results may be due to the measurement error of the physical parameters of the glass, which causes the change of the group velocity in the calculation of the disperse curve and further influences the shift of the backward wave in simulation.

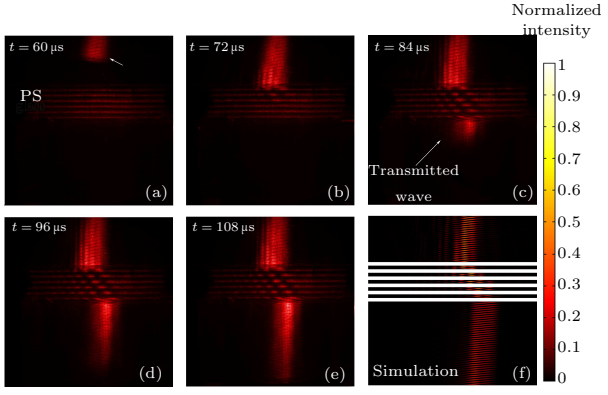

Fig. 4. The dynamic process of the ultrasonic wave ($f = 2.76$ MHz) propagating in the structure. Stroboscopic schlieren images at (a) 60 μs, (b) 72 μs, (c) 84 μs, (d) 96 μs, and (e) 108 μs. (f) Simulation result at 108 μs.

To obtain the different backward shifts of the transmitted waves with different frequencies, we adjust the frequency of the incident acoustic waves. Figure 5 shows the acoustic fields in the structure at various frequencies. It is observed that the shift of the backward wave depends on the frequency of the incident acoustic waves, i.e., the backward shifts are 2.2, 2.6, 3.3, 3.6, 4.0 and 4.6 mm at the respective frequencies of 2.66, 2.68, 2.70, 2.72, 2.74 and 2.76 MHz, respectively. Since the duration of the wave propagation through the plate is nearly a constant, the tendency of the backward shift is the same as that of the group velocity in the dispersion curve. As observed in Fig. 1, the absolute value of the negative group velocity increases due to the increase of the acoustic frequency from 2.66 to 2.76 MHz. Thus the shift of the backward wave can be tuned by changing the frequency of the acoustic waves. It is also noted that the transmitted waves are wider than the incident waves in the frequency range from 2.66 to 2.74 MHz, as shown in Figs. 5(a)–5(e). The broadening of the transmitted waves can be ascribed to the inhomogeneousity of the wave vector for the beam. As shown in Fig. 1, for the oblique acoustic incidence on the plate, the frequency on the disperse curve corresponds to a range rather than a point due to the Snell law. The broadening of the transmitted wave indicates the wave dispersion in the plate, and the width of the transmitted waves depends on the slope of the dispersion curve. With the increase of the frequency, the width of the transmitted wave decreases, and becomes nearly the same as that of the incident wave at 2.76 MHz.

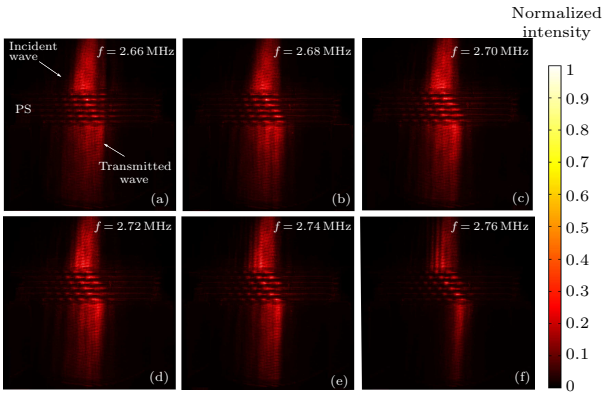

Fig. 5. Acoustic field distribution in the presence of the structure at (a) 2.66 MHz, (b) 2.68 MHz, (c) 2.70 MHz, (d) 2.72 MHz, (e) 2.74 MHz, and (f) 2.76 MHz.

In summary, we can manipulate the backward propagation of acoustic waves by a periodical structure. In the structure, the shift of the backward wave is greatly increased and can be tuned by changing the number of layers or the frequency of the incident wave. The dynamical process of the acoustic wave propagation is further experimentally observed in the structure using the schlieren imaging method.

References

[1] Liu Z Y, Zhang X X, Mao Y W, Zhu Y Y, Yang Z Y, Chan C T and Sheng P 2000 *Science* **289** 1734
[2] Cheng Y, Zhou C, Yuan B G, Wu D J, Wei Q and Liu X J 2015 *Nat. Mater.* **14** 1013
[3] Zhu X, Liang B, Kan W, Peng Y and Cheng J 2016 *Phys. Rev. Appl.* **5** 054015
[4] Christensen J and Javier García de Abajo F 2012 *Phys. Rev. Lett.* **108** 124301
[5] Cheng Y, Xu J Y and Liu X J 2010 *Appl. Phys. Lett.* **96** 071910
[6] Li J, Fok L, Yin X, Bartal G and Zhang X 2009 *Nat. Mater.* **8** 931
[7] Yuan B G, Cheng Y and Liu X J 2015 *Appl. Phys. Express* **8** 027301
[8] Cummer S and Schurig D 2007 *New J. Phys.* **9** 45
[9] Cheng Y, Yang F, Xu J Y and Liu X J 2008 *Appl. Phys. Lett.* **92** 151913
[10] Chen H and Chan C 2007 *Appl. Phys. Lett.* **91** 183518
[11] Zhu X, Liang B, Kan W, Zou X and Cheng J 2011 *Phys. Rev. Lett.* **106** 014301
[12] Xie Y, Popa B, Zigoneanu L and Cummer S 2013 *Phys. Rev. Lett.* **110** 175501
[13] Cheng Y, Xu J Y and Liu X J 2008 *Phys. Rev.* B **77** 045134
[14] Li J and Chan C 2004 *Phys. Rev.* E **70** 055602
[15] Lu M, Zhang C, Feng L, Zhao J, Chen Y, Mao Y, Zi J, Zhu Y, Zhu S and Ming N 2007 *Nat. Mater.* **6** 744
[16] Feng L, Liu X, Lu M, Chen Y, Chen Y, Mao Y, Zi J, Zhu Y, Zhu S and Ming N 2006 *Phys. Rev. Lett.* **96** 014301
[17] Li J, Liu Z and Qiu C 2006 *Phys. Rev.* B **73** 054302
[18] Ke M, Liu Z, Cheng Z, Li J, Peng P and Shi J 2007 *Solid State Commun.* **142** 177
[19] Zhu X, Li K, Zhang P, Zhu J, Zhang J, Tian C and Liu S 2016 *Nat. Commun.* **7** 11731
[20] Xie Y, Wang W, Chen H, Konneker A, Popa B and Cummer S 2014 *Nat. Commun.* **5** 5553
[21] Meitzler A 1965 *J. Acoust. Soc. Am.* **38** 835
[22] Jiang X, Qian M and Cheng Q 2013 *Chin. Phys. Lett.* **30** 084302
[23] Philippe F, Murray T and Prada C 2015 *Sci. Rep.* **5** 11112