Propagation of Surface Modes in a Warm Non-Magnetized Quantum Plasma System *

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(Received 2 June 2016)

The propagation of surface modes in warm non-magnetized quantum plasma is investigated. The surface modes are assumed to propagate on the plane between vacuum and warm quantum plasma. The quantum hydrodynamic model including quantum diffraction effect (the Bohm potential) and quantum statistical pressure is used to derive a new dispersion relation of surface modes. The new dispersion relation of surface modes is analyzed in some special interesting cases. It is shown that the dispersion relation can be reduced to the earlier results in some special cases. The results indicate that the quantum effects can facilitate the propagation of surface modes in such a semi-bounded plasma system. This work is helpful to understand the physical characteristics of the surface modes and the bounded quantum plasma.

PACS: *52.35.Bj*, *52.35.Fp*, *82.20.Xr* DOI: 10.1088/0256-307X/33/10/105201

The quantum plasma physics is an important branch of plasma physics, which can be used to describe some special physical phenomena in the degenerate electron gas in metals, dense astrophysical plasmas and next-generation optical and sensing technologies.[1–6] Recently, many studies involving quantum plasma are motivated by the rapid experimental progress and development of new materials, e.g., nanostructure materials,[7] the laboratory realization of ultra cold plasmas,[8] and quantum wells.[9] Quantum dispersive effects are significant for diagnostics of inertial fusion plasmas and intense laser–solid interaction.[10,11] Three well-known models are used to describe quantum plasma system, which are the Wigner–Poisson (WP) model (in the presence of magnetic fields, the so-called Wigner–Maxwell model), the Hartree model and the quantum hydrodynamic (QHD) model, respectively. The WP model describes the statistical behavior of quantum plasmas, whereas the Hartree model describes the hydrodynamic behavior.[12] As a fluid model, the QHD model has its advantage of mathematical efficiency and can be derived from the WP model and/or the Hartree model. It has been extensively used in the study of quantum plasmas transport, waves and instabilities. Dense plasmas can be described as a quantum fluid when the number density of electrons is so high that the electrons are degenerate. In the quantum regime, the equilibrium distribution function of the degenerate electrons follows the Fermi–Dirac statistics, as discussed by Haas.[1] The physical characteristics of quantum plasma have been extensively investigated including the quantum electron tunneling effects described by the Bohm potential term and the Fermi–Dirac quantum statistical pressure.[13] The influence of resistive effects on the quantum plasma system has been studied by Ren *et al.*[14]

Wave phenomenon is a common method to study the quantum plasma. Surface waves have attracted much attention due to their wide application in many areas. It is known that the surface waves can propagate along the interface between two different media, for example, the interface of a conducting medium or plasma with vacuum.[15] The spectrum of surface modes has been widely applied in many areas. Due to their special properties, the surface modes have already been the subject of much theoretical, experimental and numerical research.[16–19] In most of the relevant research, electromagnetic surface modes have been widely studied in the magnetized quantum plasma system. The dispersion properties of transverse electric (TE) surface waves propagating along the interface between a relativistic (nonrelativistic) quantum plasma system and vacuum are studied by using the QHD model, and it is found that the quantum effects play a crucial role to facilitate the propagation of TE surface waves.[20,21] The high frequency surface electromagnetic waves in a semi-bounded weakly ionized plasma were studied in Ref. [22]. They found that the high-frequency surface waves become strongly damped at a certain range of wavelength. The propagation of surface electromagnetic waves along a uniform magnetic field was studied in a quantum electron-hole semiconductor plasma.[23] A forward propagating mode was found by considering the effect of quantum tunneling. The propagation

*Supported by the National Natural Science Foundation of China under Grant No 11547137, and the Fundamental Research Funds for the Central Universities under Grant Nos JZ2015HGBZ0123 and JZ2016HGBZ0759.

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of electrostatic surface waves on the plane interface between vacuum and cold magnetized homogeneous plasma has already been investigated.[24] The results showed that surface waves are complementary to each other in the sense that when one wave has strong surface-wave characteristics, the other acts like a bulk wave. Moaied *et al.*[25] have studied quantum effects of nonlocal plasmons in Epsilon-near-zero properties of a thin gold film slab, and it was found that the spatial localization of the electron wave function was extended due to the quantum effects. In 2015, Mohamed *et al.*[26] have observed the TM surface modes in homogeneous magnetized quantum plasma, and the wave equation describing the excited field has been solved to obtain the dispersion relation for these modes. They found that the quantum effects play an important role on frequencies less and larger than plasma frequency such that the phase velocity of modes increases with the increasing quantum effects compared with the classical case. Furthermore, Zhu *et al.* investigated the propagation of surface plasmon polariton waves on the relativistic quantum plasma half-space.[27]

However, the physical properties of surface modes in bounded quantum plasma have not been investigated thoroughly. Thus in this work, we investigate the quantum effects on the propagation and the oscillation of surface modes since the dispersion properties of surface modes in semi-bounded warm quantum plasma can provide a useful tool for investigating the physical properties of quantum plasmas under certain circumstances.

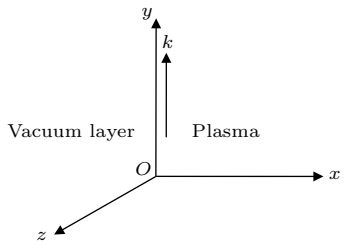

Fig. 1. Schematic description of the problem.

We suppose that the quantum plasma is composed of electrons and ions. The ions are assumed to be stationary and their quantum effects can be ignored since their inertia is too large. Meanwhile, the plasma is considered to fill the semi-infinite space, $x > 0$, which has been bounded with vacuum $x < 0$ (see Fig. 1). A pure surface mode propagates along the plasma–vacuum interface and decays away from it. Here we assume that the surface mode propagates in semi-infinite quantum plasma, due to the fact that this geometry gives the basic physical picture of the phenomenon. Also, we assume the vacuum–plasma interface to be sharp, an approximation is valid when the wavelengths of surface modes are larger than the thickness of the transition region (the region where the plasma density is from some finite value to zero and is of the order of the quantum Debye length).

By using the QHD model, the basic equations of surface mode are composed of the dynamic equation of an electron as follows:

$$m_{\rm e}n_{\rm e}\frac{d\boldsymbol{v}}{dt} = -en_{\rm e}\boldsymbol{E} - \nabla P + \boldsymbol{F}_{\rm Q}, \tag{1}$$

$$\frac{\partial n_{\rm e}}{\partial t} + \nabla\cdot(n_{\rm e}\boldsymbol{v}) = 0, \tag{2}$$

and the Maxwell equations

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}, \tag{3}$$

$$\nabla\times\boldsymbol{B} = \mu_0\Big(\boldsymbol{J} + \varepsilon_0\frac{\partial\boldsymbol{E}}{\partial t}\Big), \tag{4}$$

$$\nabla\cdot\boldsymbol{E} = -\frac{e}{\varepsilon_0}n_{\rm e}, \tag{5}$$

$$\nabla\cdot\boldsymbol{B} = 0. \tag{6}$$

By the way, SI units are used through the whole work. Here $d/dt = \partial/\partial t + (\boldsymbol{v}\cdot\nabla)$ is the hydrodynamic derivative, $m_{\rm e}$ is the electron mass, $n_{\rm e}$ is the electron number density, e is the magnitude of the electronic charge, $\boldsymbol{v}$ is the electron fluid velocity, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic field vectors, respectively, μ_0 is the magnetic permeability, ε_0 is the vacuum electric conductivity, $\boldsymbol{J} = -en_0\boldsymbol{v}$ is the current density due to free electrons, P is quantum statistical pressure, $\boldsymbol{F}_{\rm Q} = \frac{\hbar^2}{2m_{\rm e}}\nabla(\nabla^2\sqrt{n_{\rm e}}/\sqrt{n_{\rm e}}) \equiv -\nabla\varphi_{\rm B}$ is the quantum force due to electron tunneling through the Bohm potential, and $\varphi_{\rm B}$ represents the Bohm potential.

Now, the linear theory of surface modes propagation is considered, which means that all plasma parameters have the form $\phi = \phi_0 + \phi_1$, where ϕ_0 is the equilibrium part and ϕ_1 is the perturbation part owing to the wave field. Without any restrictions on generality, we assume that the oscillating quantities behave as the form

$$\phi_1 \propto \phi_1(x)\exp(ik_y\cdot y - i\omega t), \tag{7}$$

where k_y is the component of the wave vector along the y-axis, and ω is the wave frequency of the perturbations.

It should be pointed out that the quantum plasma is treated as a 2D Fermi gas here, thus the first order of quantum fermionic pressure in Eq. (1) can be written as [28]

$$P = 2T_{\rm Fe}n_1, \tag{8}$$

where $T_{\rm Fe} = \frac{1}{2}m_{\rm e}v_{\rm Fe}^2/k_{\rm B} = (\hbar^2/2m_{\rm e})(3\pi^2)^{2/3}n_{\rm e}^{2/3}$ is the Fermi temperature of degenerate electrons, $v_{\rm Fe}$ is the Fermi velocity, $k_{\rm B}$ is the Boltzmann constant. For the Maxwellian plasma, the Fermi temperature $T_{\rm Fe}$ will be replaced by the Maxwellian temperature $T_{\rm e}$,

and $n_1(\ll n_0)$ is the electron density perturbation. The first order quantum force $\boldsymbol{F}_{\rm Q}$ is[29]

$$\boldsymbol{F}_{\rm Q} = \frac{\hbar^2}{4m_{\rm e}}\nabla\nabla^2 n_1, \tag{9}$$

which is the quantum Bohm potential gradient (corresponding to the quantum corrections in the density fluctuation). By the way, we point out that the exact expression of the first order of the Bohm potential is

$$\frac{\hbar^2}{4m_{\rm e}}\nabla\Big[\nabla^2 n_1 - \frac{\nabla n_0\cdot\nabla n_1}{n_0} + \frac{(\nabla n_0)^2}{n_0^2}n_1 - \frac{\nabla^2 n_0}{n_0}n_1\Big]. \tag{10}$$

By taking account of the weak inhomogeneity approximation, the above formula can be simplified by keeping only the first term in the square brackets.

Plasma equilibrium is assumed $\boldsymbol{E}_0 = 0$ and $\boldsymbol{v}_0 = 0$, and the wave vector $\boldsymbol{k}$ is along the y-axis which is the plasma-vacuum interface. We consider the propagation of surface modes in a warm non-magnetized quantum plasma system, and the basic linearized equations in such a quantum plasma system can be obtained, which are expressed as follows:

$$m_{\rm e}n_0\frac{\partial \boldsymbol{v}_1}{\partial t} = -en_0\boldsymbol{E}_1 - 2T_{\rm Fe}\nabla n_1 + \frac{\hbar^2}{4m_{\rm e}}\nabla\nabla^2 n_1, \tag{11}$$

$$\frac{\partial n_1}{\partial t} + n_0\nabla\cdot\boldsymbol{v}_1 = 0, \tag{12}$$

$$\nabla\times\boldsymbol{E}_1 = -\frac{\partial \boldsymbol{B}_1}{\partial t}, \tag{13}$$

$$\nabla\times\boldsymbol{B}_1 = \mu_0\Big(\boldsymbol{J}_1 + \varepsilon_0\frac{\partial \boldsymbol{E}_1}{\partial t}\Big), \tag{14}$$

$$\nabla\cdot\boldsymbol{E}_1 = -\frac{e}{\varepsilon_0}n_1. \tag{15}$$

Using the linearized Eqs. (11) and (12), one can obtain the wave equation of the density perturbation as follows:

$$\omega^2 n_1 = \omega_{\rm pe}^2 n_1 - v_{\rm Fe}^2\frac{\partial^2 n_1}{\partial x^2} + v_{\rm Fe}^2 k_y^2 n_1 + \frac{\hbar^2}{4m_{\rm e}^2}\Big(\frac{\partial^4 n_1}{\partial x^4} - 2k_y^2\frac{\partial^2 n_1}{\partial x^2} + k_y^4 n_1\Big), \tag{16}$$

where $\omega_{\rm pe} = \sqrt{n_0e^2/\varepsilon_0 m_{\rm e}}$ and $v_{\rm Fe} = \sqrt{2k_{\rm B}T_{\rm Fe}/m_{\rm e}}$ are the electron plasma frequency and the electron Fermi velocity, respectively. The very slow nonlocal variations in Eq. (16) can be ignored (i.e., $k_y^{-2}\partial^4/\partial x^4 \ll \partial^2/\partial x^2 \ll k_y^2$). Then, Eq. (16) yields the following wave equation for the perturbed electron density

$$\frac{\partial^2 n_1}{\partial x^2} - \alpha_{\rm p}^2 n_1 = 0, \tag{17}$$

where

$$\alpha_{\rm p}^2 = \frac{(\omega_{\rm pe}^2 - \omega^2) + k_y^2\Big(v_{\rm Fe}^2 + \frac{\hbar^2k_y^2}{4m_{\rm e}^2}\Big)}{v_{\rm Fe}^2 + \frac{\hbar^2k_y^2}{2m_{\rm e}^2}}. \tag{18}$$

Equation (17) has the following finite solution inside the quantum plasma region ($x > 0$) as

$$n_1 = A\exp(-\alpha_{\rm p}x), \tag{19}$$

where A is the amplitude of the perturbed density.

We focus on the propagation and oscillation of surface modes. From the Maxwell equations, one obtains the wave equation of the magnetic field component for these surface modes

$$\frac{\partial^2 \boldsymbol{B}}{\partial x^2} - \beta_{\rm p}^2\boldsymbol{B} = 0, \tag{20}$$

where

$$\beta_{\rm p}^2 = k_y^2 + \frac{\omega_{\rm pe}^2 - \omega^2}{c^2}. \tag{21}$$

Equation (20) has a solution in the plasma region ($x > 0$) as

$$\boldsymbol{B}_{\rm p} = \boldsymbol{C}_{\rm p}\exp(-\beta_{\rm p}x), \tag{22}$$

and in the vacuum region ($x < 0$) in the form

$$\boldsymbol{B}_{\rm v} = \boldsymbol{C}_{\rm v}\exp(\beta_{\rm v}x), \tag{23}$$

where $\boldsymbol{C}_{\rm p}$ and $\boldsymbol{C}_{\rm v}$ are integration constants and $\beta_{\rm v}^2 = k_y^2 - \frac{\omega^2}{c^2}$ is the permeability of a vacuum. Now, it is possible to obtain the electric field of surface modes in plasma and vacuum region by using Eqs. (22) and (23) and the Maxwell equations. After some mathematical operations, the electric field of surface modes can be obtained,

$$\boldsymbol{E}_{\rm p} = \frac{eA\exp(-\alpha_{\rm p}x)}{\varepsilon_0(\omega^2 - \omega_{\rm pe}^2)}(ik_y\boldsymbol{e}_y - \alpha_{\rm p}\boldsymbol{e}_x)\cdot\Big[v_{\rm Fe}^2 - \frac{\hbar^2(\alpha_{\rm p}^2 - k_y^2)}{4m_{\rm e}^2}\Big] + \boldsymbol{R}_{\rm p}\exp(-\beta_{\rm p}x),\ x > 0, \tag{24}$$

$$\boldsymbol{E}_{\rm v} = \boldsymbol{R}_{\rm v}\exp(\beta_{\rm v}x),\ x < 0, \tag{25}$$

where $\boldsymbol{R}_{\rm p}$ and $\boldsymbol{R}_{\rm v}$ are constant. Considering the electromagnetic field components and some appropriate boundary conditions at $x = 0$, one can obtain the dispersion relation. Thus we use the boundary conditions, that is, (i) the tangential components of $\boldsymbol{E}$ and $\boldsymbol{B}$ are continuous at $x = 0$, (ii) the normal component of the displacement vector is continuous at $x = 0$, (iii) velocity components (along x-axis) vanish for electrons, i.e., $v_x = 0$ at $x = 0$. Thus

$$E_{{\rm p}y}|_{\rm surface} = E_{{\rm v}y}|_{\rm surface}, \tag{26}$$

$$D_{{\rm p}x}|_{\rm surface} = D_{{\rm v}x}|_{\rm surface}, \tag{27}$$

where $D_{{\rm p}x}$ and $D_{{\rm v}x}$ are the x-components of the electric displacement vector in the plasma and vacuum

regions, respectively. Substituting Eqs. (24) and (25) into Eqs. (26) and (27), one can derive

$$R_{py} = R_{vy} + \frac{ieAk_y}{\varepsilon_0(\omega_{pe}^2 - \omega^2)}\gamma^2, \tag{28}$$

$$R_{px} = R_{vx} - \frac{eA\alpha_p}{\varepsilon_0(\omega_{pe}^2 - \omega^2)}\gamma^2, \tag{29}$$

where

$$\gamma^2 = v_{Fe}^2 - \frac{\hbar^2(\alpha_p^2 - k_y^2)}{4m_e^2}. \tag{30}$$

By using the boundary conditions and the Maxwell equations again, one can obtain

$$R_{px} = \frac{1}{(\beta_v\beta_p + \beta_v^2)}\frac{eA}{\varepsilon_0(\omega^2 - \omega_{pe}^2)} \cdot [k_y^2\beta_p + \alpha_p\beta_v\beta_p - k_y^2\alpha_p + \alpha_p\beta_v^2]\gamma^2. \tag{31}$$

On the other hand, making use of Eqs. (11) and (24) at $x = 0$, we find

$$R_{px} = \frac{eA\alpha_p\gamma^2}{\varepsilon_0}\left[\frac{1}{\omega_{pe}^2} + \frac{1}{(\omega^2 - \omega_{pe}^2)}\right]. \tag{32}$$

Here we can derive the dispersion relation of surface modes by equalizing the right-hand-side of Eqs. (31) and (32). After some algebraic manipulations, the general dispersion relationship for surface modes in such a quantum plasma system is obtained as follows:

$$\alpha_p(\beta_v\beta_p + \beta_v^2)(\omega^2 - \omega_{pe}^2) = \omega_{pe}^2k_y^2(\beta_p - \alpha_p). \tag{33}$$

Here we restrict our study on surface modes of such a quantum plasma system, considering the electrostatic limit ($c \to \infty$) and overcritical dense plasma conditions, one can derive the solutions to Eq. (33) as follows:

$$\omega^2 = \frac{\omega_{pe}^2}{2}\left[1 + \frac{1}{\sqrt{\omega_{pe}^2 - \omega^2 + k_y^2v_{Fe}^2}}\sqrt{k_y^2v_{Fe}^2 + \frac{\hbar^2k_y^4}{4m_e^2}}\right]. \tag{34}$$

By ignoring the effect caused by the quantum Bohm potential, which means $\hbar \to 0$, Eq. (34) gives the solution of the dispersion relation as

$$\omega = \frac{1}{\sqrt{2}}\sqrt{(\omega_{pe}^2 + k_y^2v_{Fe}^2) \pm k_yv_{Fe}\sqrt{2\omega_{pe}^2 + k_y^2v_{Fe}^2}}. \tag{35}$$

If we also ignore the Fermi statistical pressure, that is, $v_{Fe} \to 0$, Eq. (35) will be simplified to

$$\omega = \frac{\omega_{pe}}{\sqrt{2}}. \tag{36}$$

This is just the surface plasmon polaritons (SPPs) on a cold plasma half space.[30]

To show the influence of the quantum effects due to the Bohm potential and the Fermi statistical pressure, we evaluate Eqs. (34) and (35) by considering some typical parameters in the intense laser produced solid density plasma,[31] where the corresponding physical parameters are $n_0 = 3 \times 10^{34}\,\mathrm{m}^{-3}$, $\omega_{pe} = 9.77 \times 10^{18}\,\mathrm{s}^{-1}$, $v_{Fe} = 1.11 \times 10^8\,\mathrm{m/s}$. We plot the frequency spectrum versus the wave number k_y.

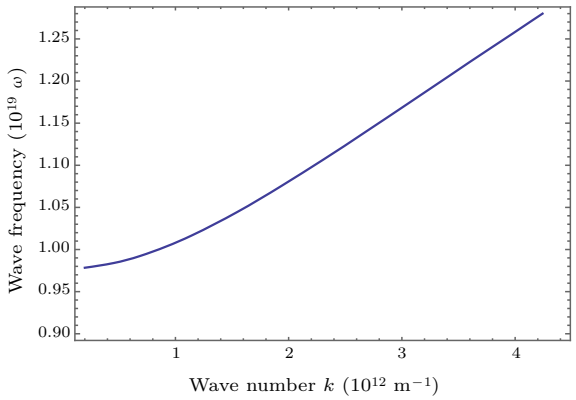

Fig. 2. Plot of frequency versus the wave number for the propagation of electrostatic surface mode on a quantum plasma–vacuum interface (contribution of the Bohm potential and quantum statistical pressure effect). The range of wave number k is from $2 \times 10^{11}\,\mathrm{m}^{-1}$ to $4.4 \times 10^{12}\,\mathrm{m}^{-1}$.

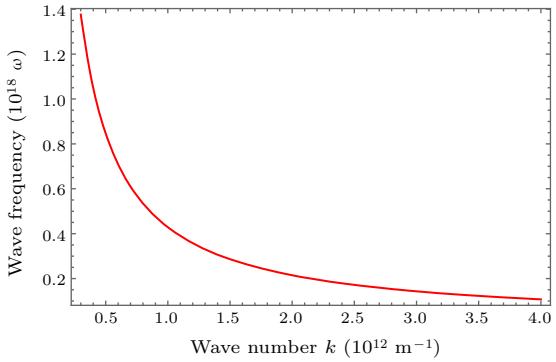

Fig. 3. Plot of frequency versus the wave number for the propagation of electrostatic surface mode on a quantum plasma–vacuum interface (only the contribution of quantum statistical pressure effect is considered). The wave number k is between $3 \times 10^{11}\,\mathrm{m}^{-1}$ and $4 \times 10^{12}\,\mathrm{m}^{-1}$.

Figure 2 is obtained from Eq. (34), and it describes the dispersion relation of surface modes on a quantum plasma–vacuum interface when the Bohm potential and quantum statistical pressure effect are simultaneously considered. The effect of the quantum Bohm potential correction on the dispersion relationship of electrostatic surface mode is apparent. From Fig. 2, the group velocity (the slope of the dispersion curve) increases with the wave number in a certain range, when considering both the Bohm potential effect and quantum statistical pressure effect.

Figure 3 is obtained from Eq. (35), which corresponds to the propagation of surface plasmon polaritons due to the quantum statistical pressure in a cer-

tain range of wave number. The regime of its propagation is due to the dynamical motion of the electrons, which introduces a restoring force that causes the propagation of oscillation.

In summary, we have studied the propagation of surface modes in the warm non-magnetized quantum plasma system, and a new general dispersion relation of surface modes is derived. The results show that quantum effects (including quantum diffraction effect and quantum statistical pressure) play an important role in the propagation of surface modes. If the quantum effects are ignored, there is only surface plasmon polariton oscillation without propagation as in the case of classical cold plasma. In Fig. 2, it is clear that the group velocity increases in a certain range of wave number when the quantum diffraction effect and the quantum statistical pressure both are considered. One important result in this work is the quantum effects on the dispersion equations of surface modes. Maybe, our work will provide a useful way to investigate the physical characteristic of quantum surface modes and physical properties of the bounded quantum non-magnetic plasma. The results may be significant for understanding the dispersion properties of new quantum surface modes in plasma, which may be observed experimentally in the near future.

We would like to thank Dr. Ren for valuable suggestions and comments.

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