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Coexistence of Prolate and Oblate Shapes in ^{98}Sr Nuclei Using IBM2 *

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The properties of the low-lying states and the shape coexistence in ^{98}Sr are investigated within the framework of the proton–neutron interacting boson model (IBM2). By considering the relative energy of the d proton boson to be different from that of the neutron boson, it is found that the calculated energy levels and the $B(E2)$ transition strengths agree with the experimental data perfectly. Particularly, the second 0^+ state, which is associated with the shape coexistence phenomenon and has the lowest energy $E(0_2^+)$ among all known even–even nuclei, is reproduced very well. The behavior of the calculated quadrupole shape invariants is consistent with the experimental results.

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A very rapid onset of deformation is observed around neutron number $N = 60$ for neutron-rich even–even isotopes of Sr.[1] The energies of the 2_1^+ states, the $B(E2; 0_1^+ \to 2_1^+)$ values, as well as the two-neutron separation energies and the mean-square charge radii change drastically from $N = 58$ to $N = 60$ in Sr isotopes.[2–4] This phenomenon is interpreted as the intrusion of the deformed 0^+ state. The presence of low-lying excited 0^+ states in even–even nuclei has been systematically discussed in the context of shape coexistence.[5] In particular, the second 0^+ state in ^{98}Sr has lowest energy $E(0_2^+)$ among all known even–even nuclei at 215 keV.[6]

From the theoretical point of view, the very low-lying 0_2^+ state in ^{98}Sr has been considered as evidence for shape coexistence, which means that the deformed and spherical states are slightly mixed. The low-lying 0_2^+ state is explained as asymmetric shape, in which state there exists an axial asymmetry.[7] A simple band-mixing calculation supports the picture of structure of the low-lying levels in ^{98}Sr as mixing between a nearly spherical S band (the main component of 0_2^+ and 2_2^+ states) and a prolate deformed D band (the main component of the yrast band). Another band-mixing calculation, which is deduced without assuming a rigid rotor[6,7] for the energy levels of the unmixed D band, shows that ^{98}Sr has a highly deformed ($\beta \sim 0.4$) yrast band and a moderately deformed ($|\beta| \sim 0.2$) low-lying band.[8] The configuration mixing between the first two 0^+ states in ^{98}Sr has been re-analyzed by introducing the intrinsic $E2$ matrix element between them in the two-state-mixing model.[9] The calculation shows that the shape coexistence phenomenon in ^{98}Sr exhibits a strongly deformed ground state and a weakly deformed excited 0^+ state. Further calculation predicts that the deformed ground state band has a robust $\beta_{\rm D}$ of 0.38, while the S band has a $\beta_{\rm S}$ value between -0.22 and -0.24 for ^{98}Sr in a simple two-state mixing model.[6] Meanwhile, the projected shell model has been used to explain the nature of the onset of deformation at $N = 60$ in Sr isotopes.[10] The self-consistent mean-field models with the Gogny force[4] or with the Skyrme force[11] find that the competing prolate minimum and oblate minimum occur simultaneously in ^{98}Sr. Furthermore, the calculation of the covariant density functional theory also indicates that the coexistence of prolate and oblate shapes takes place in ^{98}Sr nuclei.[12]

Very recently, the electric monopole transition strength $\rho^2(E0)$ for ^{98}Sr was measured in a β-decay spectroscopy experiment.[6] At the same time, the detail of shape coexistence in ^{98}Sr was investigated with the low-energies Coulomb excitation experiments.[13] A complete set of matrix elements was obtained for the ground and 0_2^+, 2_2^+ states, including the diagonal matrix elements. The experimental results indicate that there is shape coexistence of highly deformed and spherical configurations, and a very small mixing between the coexisting states in ^{98}Sr. Meanwhile, the extensive set of matrix elements for ^{98}Sr are important benchmarks for theoretical calculations of shape coexisting structure of the ^{98}Sr nuclei.[1]

The interacting boson model (IBM)[14] has also been used to describe the even–even nuclear shape and shape coexistence.[15–18] Within the two configurations mixing, the shape coexistence and shape transition in the Mo,[19] and Ge[20] isotopes have been discussed in the IBM framework. Furthermore, the IBM Hamiltonian can be derived by mapping the potential energy surface of the mean-field model with the Gogny-D1S energy density function.[21] This approach was extended to handle shape coexistence in the neutron-rich Kr isotopes,[16] ^{98}Mo nuclei,[22] and the neutron-deficient Pb,[23] Hg,[24,25] and Pt[26] isotopes. On the other hand, in the proton–neutron interacting boson model (IBM2),[14] which distinguishes the proton-pairs from neutron-pairs, one can introduce the relative energy of the d proton boson to be different from that of the neutron bosons.[27] As a result, the properties of the low-lying states for Mo iso-

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topes can be reproduced very well without introducing the mixing of two different configurations. More recently, this method was extended to study the shape coexistence in ^{98,100}Mo isotopes.[28,29] The calculated results correctly described the strong mixing between the coexisting states in ^{98,100}Mo isotopes. In particular, the calculated values of the quadrupole shape invariants for ^{98}Mo are in good agreement with the experimental results both quantitatively and qualitatively. However, a detailed discussion on a very small mixing between the coexisting states in ^{98}Sr with this method is lacking. In this Letter, we investigate the properties of the low-lying states and shape coexistence of ^{98}Sr within the framework of the proton–neutron interacting boson model. By considering the relative energy of the d proton boson to be different from that of the neutron boson, we give the theoretical predictions of the low-lying levels and compare the $B(E2)$ transition strengths with the experimental data. The quadrupole deformation parameters of the ground state and second 0^+ states for ^{98}Sr will also be calculated. Then we attempt to give a clear picture of the shape coexistence in ^{98}Sr.

The major assumption of the IBM is the introduction of $L = 0^+$ (s) and $L = 2^+$ (d) bosons, which reflect the collective S and D pairs of valence nucleons.[14] Since the IBM is closely linked to group theory, a symmetry-dictated analysis of shape coexistence is also possible. In fact, nuclei are composed of both protons and neutrons. To describe them in the framework of the IBM, the IBM2 should be implemented, which distinguishes the proton-pairs (proton-bosons) from neutron-pairs (neutron-bosons). In the IBM2 model, the physical dominant interactions contained in the Hamiltonian[14] is expressed as

$$\begin{aligned}\hat{H} =& \varepsilon_{d\pi}\hat{n}_{d\pi} + \varepsilon_{d\nu}\hat{n}_{d\nu} + \kappa\hat{Q}_\pi \cdot \hat{Q}_\nu \\ &+ \omega_\nu \hat{L}_\nu \cdot \hat{L}_\nu + \hat{M}_{\pi\nu}, \end{aligned} \tag{1}$$

where $\hat{n}_{d\rho} = d^\dagger_\rho \cdot \tilde{d}_\rho$, $(\rho = \pi, \nu)$ and $\hat{Q}_\rho = (s^\dagger_\rho \tilde{d}_\rho + d^\dagger_\rho s_\rho)^{(2)} + \chi_\rho (d^\dagger_\rho \tilde{d}_\rho)^{(2)}$ represent the d-boson number operator and quadrupole operator for the proton ($\rho = \pi$) and neutron ($\rho = nu$), respectively. The parameter χ_ρ, which appears in the quadrupole operator, determines the type of deformation, and L_ν is the neutron angular momentum operator with $\hat{L}_\nu = \sqrt{10}[d^\dagger_\nu \cdot \tilde{d}_\nu]^{(1)}$. The last term on the right-hand side of Eq. (1) represents the Majorana interaction, whose strength is given by $\hat{M}_{\pi\nu} = \lambda_2(s^\dagger_\pi d^\dagger_\nu - s^\dagger_\nu d^\dagger_\pi)^{(2)} \cdot (s_\pi \tilde{d}_\nu - s_\nu \tilde{d}_\pi)^{(2)} + \sum_{k=1,3} \lambda_k (d^\dagger_\pi d^\dagger_\nu)^{(k)} \cdot (\tilde{d}_\pi \tilde{d}_\nu)^{(k)}$, $\varepsilon_{d\rho}$ is the energy of d-boson, and κ is the strength of quadrupole–quadrupole interaction between neutron-bosons and proton-bosons. The parameter ω_ν is the strength of dipole neutron–neutron interaction we consider explicitly in this calculation similar to Refs. [30,31]. The parameters λ_k ($k = 1$, 2 and 3) are strengths of the Majorana interaction. For more details in this model see Ref. [14].

Here ^{98}Sr is composed of $Z = 38$ protons and $N = 60$ neutrons, which locates between the proton shell closures $Z = 28$ and $Z = 50$, and between neutron shell closures $N = 50$ and $N = 82$. For the description of ^{98}Sr, our investigation is based on the validity of the $Z = 40$ subshell closure, and we consider the $^{90}_{40}$Zr$_{50}$ as a doubly magic core the same as in Refs. [19,22,29]. In this case, two valence proton holes form $N_\pi = 1$ hole-like proton-boson from the $Z = 40$ shell, and ten valence neutrons form $N_\nu = 5$ particle-like neutron-bosons from the $N = 50$ shell for ^{98}Sr. In this calculation we have taken the relative energy of the d proton boson to be different from that of the neutron boson, i.e., $\varepsilon_{d\pi} \neq \varepsilon_{d\nu}$ the same as in Refs. [27–30]. To fix the free parameters in the Hamiltonian Eq. (1), a parameter scan was performed to optimize the calculation to reproduce the experimental energies, especially the order of the energy level and transition strengths in ^{98}Sr. By a least-square fit to the energies of the experimental data, the best fit IBM2 parameters used here are $\varepsilon_{d\pi} = 1.100$ MeV, $\varepsilon_{d\nu} = 0.226$ MeV, $\kappa = -0.360$ MeV, $\chi_\pi = 0.300$, $\chi_\nu = -\sqrt{7}/2$, $\omega_\nu = 0.0094$ MeV, $\lambda_1 = 1.000$ MeV, $\lambda_2 = -0.400$ MeV, and $\lambda_3 = 1.000$ MeV for ^{98}Sr. The diagonalization of the IBM Hamiltonian is performed numerically by the NPBOS code.[32]

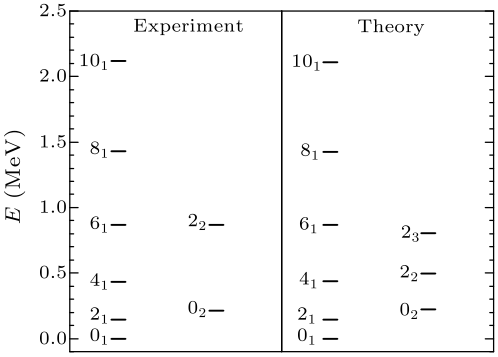

Fig. 1. Comparison of experimental and calculated energy levels for positive-parity low-spin states of ^{98}Sr. The experimental data are taken from Refs. [6,13].

The calculated results for ^{98}Sr are given in Fig. 1 in comparison with the experimental levels available, which fall into two groups: the ground-state band and the non-yrast state band, that is, low-spin positive-parity levels. The experimental data are taken from Refs. [6,13]. From Fig. 1, we know that the experimental ground-state band has been reproduced quite precisely by our calculation, even for the high-spin state 10^+_1. In IBM2, the characteristic ratio $R_{4/2} = E(4^+_1)/E(2^+_1)$ has the limit of 2.00 for a spherical vibration (the $U_{\pi\nu}(5)$ dynamical symmetry), 2.50 for a triaxial rotor (the $SU^*_{\pi\nu}(3)$ symmetry), and 3.33 for an axial rotor (the $SU_{\pi\nu}(3)$ symmetry).[33] For ^{98}Sr, the experimental value $R_{4/2}$ is 3.01, and the calculated value is 3.02. Both the experimental and the theoretical characteristic ratios predict that ^{98}Sr is a nearly axial rotor. More importantly, the second 0^+ state is associated with the shape coexistence phenomenon.

Figure 1 indicates that the result of the calculated energy level for this 0_2^+ state reproduces the experimental data very well, although ^{98}Sr has the lowest energy $E(0_2^+)$ among all known even–even nuclei at 215 keV. This indicates that two different values $\varepsilon_{d\pi} \neq \varepsilon_{d\nu}$ chosen for the proton and neutron d bosons energy are sufficient to account for this shape coexistence. However, the calculated energy level of the second 2^+ state is significantly lower than the corresponding experimental data, which is due to the fact that the calculated main n_d boson number for 2_2^+ state is different from that in the observed result. Figure 1 shows the observed 2_2^+ state energy level lies close to the 8_1^+ state energy level. This indicates that the observed 2_2^+ state contains much more pronounced $n_d = 3$ components in the case of the $SU(3)$–$U(5)$ coexistence in IBM. While the calculated 2_2^+ state energy level lies far from the 8_1^+ state, and is close to the 4_1^+ state. This implies that the calculated 2_2^+ state contains much more pronounced $n_d = 2$ components.

In addition to the energy spectrum, other important properties are the $E2$ transition strengths and their branching ratios for nuclear structure. The calculated $B(E2)$ values and $B(E2)$ ratios give us a better insight into the nature of the shape coexistence for nuclei, due to the fact that $B(E2)$ values and $B(E2)$ ratios are more sensitive to the shape parameters than the energy levels. In IBM2, the $B(E2)$ transition strengths are given in terms of the reduced matrix elements as

$$B(E2; J \to J') = \frac{1}{2J+1} |\langle J' \| \hat{T}^{(E2)} \| J \rangle|^2, \qquad (2)$$

where J and J' are the angular momenta for the initial and final states, respectively. The $E2$ transition operator is explicitly given by $\hat{T}^{(E2)} = e_\pi \hat{Q}_\pi + e_\nu \hat{Q}_\nu$, where the term Q_ρ is the same as in Eq. (1), and for consistency we choose the same value for χ_ρ as in the Hamiltonian. Here $e_\pi(e_\nu)$ is the effective charge of proton (neutron) bosons in units of eb. In principle, the values of e_π and e_ν could be different from each other. For simplicity, we take $e_\pi = e_\nu$, which does not affect our conclusion. By exactly fitting to the experimental data $B(E2, 2_1^+ \to 0_1^+) = 0.259 \mathrm{e}^2\mathrm{b}^2$[13] in ^{98}Sr, we obtain $e_\pi = e_\nu = 0.136$ eb. The calculated results are listed in Table 1 with the comparison with the corresponding known experimental $E2$ transition strengths.

Table 1 shows that the calculated results are in good agreement with the experimental data. In particular, the intraband $E2$ transition strengths of the ground-state band describe the experimental results very well. In IBM2, the value of the key sensitive quantities $R_1 = B(E2, 4_1^+ \to 2_1^+)/B(E2, 2_1^+ \to 0_1^+) = 2$ corresponds to the $U_{\pi\nu}(5)$ symmetric limit (spherical vibration), and $R_1 = 10/7$ corresponds to the $SU_{\pi\nu}(3)$ symmetric limit (axial rotor).[33] For ^{98}Sr, the experimental value of R_1 is 1.33, while the theoretic value of R_1 is 1.41. Both the calculated value and experimental value are much lower than the $U_{\pi\nu}(5)$ symmetric limit, while they are rather close to the $SU_{\pi\nu}(3)$ symmetric limit. This indicates that the ground-state band has an almost axial rotor feature, which is consistent with the characteristic ratio $R_{4/2}$. The calculated value of the $E2$ transition from 0_2^+ to 2_1^+ state reproduces the experimental data very satisfactorily, although the calculated value is slightly less than the experimental data. Indeed, the $B(E2, 0_2^+ \to 2_1^+)$ transition strength is a sensitive observable to distinguish $U_{\pi\nu}(5)$ and $SU_{\pi\nu}(3)$ limit in IBM2. In the $U_{\pi\nu}(5)$ limit, the 0_2^+ state is a member of the two-phonon triplet with the number of d boson $n_d = 2$ and the $\triangle n_d = 1$ selection rule only allows a decay to the 2_1^+ state. For the transition in the $SU_{\pi\nu}(3)$ limit, the selection rule is $\triangle(\lambda, \mu) = (0, 0)$, only intraband transition is allowed in this case.[33] Thus transition between the first excited 0^+ state and first 2^+ state is forbidden in the $SU_{\pi\nu}(3)$ limit. Therefore, both the experimental and the theoretical $B(E2, 0_2^+ \to 2_1^+)$ transition strengths suggest that the 0_2^+ state is situated between the $U_{\pi\nu}(5)$ and $SU_{\pi\nu}(3)$ limits in ^{98}Sr, but closer to the $U_{\pi\nu}(5)$ limit. Furthermore, the strong $B(E2, 0_2^+ \to 2_1^+)$ transition, relative to the $2_1^+ \to 0_1^+$ transition, presents a stringent test of configuration mixing. For ^{98}Sr, the observed $B(E2, 0_2^+ \to 2_1^+)$ value is smaller than the $B(E2, 2_1^+ \to 0_1^+)$ value, which reflects a relatively weak mixing of the coexisting structure in ^{98}Sr.[6,34] The slight deviation between the calculated value and the experimental data for $B(E2, 0_2^+ \to 2_1^+)$ may be due to the effect of weak configuration mixing. Meanwhile, the observed $B(E2)$ values from 2_2^+ state to the ground-state band are small, which also suggest weak mixing between these configurations. The calculated results for these $B(E2)$ transition strengths are in good agreement with experiment qualitatively and quantitatively. However, the largest discrepancy is found in the decay from 2_2^+ state to 0_2^+ state, similar to the calculated energy level for 2_2^+ state. The experimental $B(E2, 2_2^+ \to 0_2^+)$ value of ^{98}Sr is about a factor 4 smaller than the theoretical prediction. This is due to the fact that the calculated 2_2^+ state contains more $n_d = 2$ component than its actual case.

Table 1. Comparison of experimental and calculated energy levels for positive-parity low-spin states of ^{98}Sr. The experimental data are taken from Refs. [6,9,13].

$J_i^\pi \to J_f^\pi$	Expt.	Calc.
$2_1^+ \to 0_1^+$	0.259	0.2580
$4_1^+ \to 2_1^+$	0.345	0.3670
$6_1^+ \to 4_1^+$	0.460	0.3579
$8_1^+ \to 6_1^+$	0.340	0.2875
$0_2^+ \to 2_1^+$	0.150	0.1163
$2_2^+ \to 0_2^+$	0.027	0.1319
$2_2^+ \to 0_1^+$	0.002	0.0014
$2_2^+ \to 2_1^+$	0.002	0.0034
$2_2^+ \to 4_1^+$	0.012	0.0311

To further confirm the shape coexistence in ^{98}Sr, we calculate the theoretical values for the quadrupole shape invariants: the quadrupole overall deformation

parameter Q^2 and the triaxiality parameter $\cos 3\delta$. They correspond to[35]

$$Q_i^2 = \Sigma_j \langle 0_i^+ \| E2 \| 2_j^+ \rangle \langle 2_j^+ \| E2 \| 0_i^+ \rangle, \quad (3)$$

$$Q_i^3 \cos 3\delta_i = -\sqrt{\frac{7}{10}} \Sigma_{j,k} \langle 0_i^+ \| E2 \| 2_j^+ \rangle \cdot \langle 2_j^+ \| E2 \| 2_k^+ \rangle \langle 2_k^+ \| E2 \| 0_i^+ \rangle, \quad (4)$$

where $E2$ also denotes the $E2$ transition operator. The signs of the matrix elements have been chosen as the same in Ref. [36]. The calculated results of the quadrupole deformation for the first and second 0^+ states in ^{98}Sr are $1.320\,\mathrm{e}^2\mathrm{b}^2$ and $0.727\,\mathrm{e}^2\mathrm{b}^2$, respectively. The calculated Q^2 is close to the experimental data for the ground state within the experimental uncertainty,[13] which shows that the ground state is a highly deformed one. The calculated Q^2 for the second 0_2^+ state fairly describes the characteristic feature of the weakly deformed 0_2^+ state. Subsequently, our calculation indicates that there exists a shape coexistence between highly deformed prolate and spherical configurations in ^{98}Sr, although the calculated value of quadrupole deformation for second 0^+ state is slightly larger than the experimental data. For the triaxiality parameter, $\cos 3\delta = 1$ corresponds to the prolate shape, $\cos 3\delta = -1$ corresponds to the oblate shape, and $\cos 3\delta$=0 corresponds to the triaxiality shape. The calculated triaxiality parameters for $0_{1,2}^+$ states are 0.720 and 0.717, respectively. They are both close to their upper limit of the experimental value 0.46(13), and this suggests that there exists a mostly prolate shape with a certain triaxiality for $0_{1,2}^+$ states in ^{98}Sr. Recalling the best fitted parameters, we adopt $\chi_\nu = -\sqrt{7}/2$ and $\chi_\pi = 0.30$ to describe the character of the shape for ^{98}Sr. Here $\chi_\pi + \chi_\nu \approx -1$ results in the axial symmetric feature, and this behavior is embedded in ^{98}Sr structure. On the other hand, the parameters χ_ν and χ_π imply that a neutron fluid with a pure prolate deformed and a proton fluid with some oblate deformation simultaneously exist in the equilibrium structure of ^{98}Sr. Due to their symmetry axes orthogonal to each other, this brings about a certain triaxiality for the 0^+ states in ^{98}Sr.[37] Combining the two effects, we can conclude that ^{98}Sr has a prolate deformation with a certain triaxiality shape, which is consistent with the experimental results.[13] However, the present value of $\cos 3\delta$ for 0_2^+ state is still larger than the experimental data, due to the fact that the theoretical prediction of the 2_2^+ coupling to 0_2^+ state is larger than the experimental data.

In conclusion, the properties of the low-lying states and shape coexistence in ^{98}Sr have been investigated within the framework of the proton–neutron interacting boson model. By considering the relative energy of the d proton-boson to be different from that of the neutron-boson, it is found that our calculated results agree with the experimental data very well. Particularly, the second 0^+ state has been reproduced quite well in ^{98}Sr, which is associated with the shape coexistence phenomenon and has the lowest energy $E(0_2^+)$ among all known even–even nuclei. The behavior of the calculated quadrupole shape invariants is consistent with the experimental results. Our calculation indicates that choosing relative energy of the d proton boson to be different from that of the neutron boson, i.e., $\varepsilon_{d\pi} \neq \varepsilon_{d\nu}$ could correctly reproduce the shape coexistence not only for strong mixing between the coexisting configurations but also for weak mixing. However, the calculated 2_2^+ energy level is lower than the experimental data, and the property of $B(E2, 2_2^+ \to 0_2^+)$ transition could not be reproduced well. More investigations are needed for these aspects.

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