CHIN. PHYS. LETT. Vol. 33, No. 10 (2016) 102101

First-Order Symmetry Energy Induced by Neutron–Proton Mass Difference *

Jian-Min Dong(董建敏)[1**], Wei Zuo(左维)[1], Jian-Zhong Gu(顾建中)[2]
[1]*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000*
[2]*China Institute of Atomic Energy, Beijing 102413*

(Received 8 June 2016)

The 1st-order symmetry energy coefficient of nuclear matter induced merely by the neutron–proton (n–p) mass difference is derived analytically, which turns out to be completely model-independent. Based on this result, the 1st-order symmetry energy $E_{\rm sym,1}^{\rm (npDM)}(A)$ of heavy nuclei such as ^{208}Pb induced by the n–p mass difference is investigated with the help of a local density approximation combined with the Skyrme energy density functionals. Although $E_{\rm sym,1}^{\rm (npDM)}(A)$ is small compared with the second-order symmetry energy, it cannot be dropped simply for an accurate estimation of nuclear masses as it is still larger than the rms deviation given by some accurate mass formulas. It is therefore suggested that one perhaps needs to distinguish the neutron mass from the proton one in the construction of nuclear density functionals.

PACS: 21.65.Ef, 21.65.Cd, 21.60.Jz, 23.60.+e DOI: 10.1088/0256-307X/33/10/102101

Much attention has been paid to the equation of state (EOS) of isospin asymmetric nuclear matter in nuclear physics and astrophysics with the development of radioactive beam facilities and astronomical observation facilities. The symmetry energy that measures the isospin dependence of the EOS is a quantity of critical importance in many important fields of nuclear physics. It involves the basic aspects of strong nuclear force,[1] heavy ion reactions,[2–10] stability of superheavy nuclei,[11] fusion cross sections[12] and structures, composition and thermal evolution of neutron stars.[13–15] Many theoretical and experimental efforts have been made to constrain its density dependence.

The energy per particle of nuclear matter with density $\rho = \rho_{\rm n} + \rho_{\rm p}$ and asymmetry $\beta = (\rho_{\rm n} - \rho_{\rm p})/\rho$ is usually written as

$$E(\rho,\beta) = E(\rho,0) + S_2(\rho)\beta^2 + \mathcal{O}(\beta^4), \qquad (1)$$

where $S_2(\rho)$ is the second-order symmetry energy coefficient, which is indeed a dominant quantity in determining the isospin dependence of the EOS. It has been widely investigated by employing various independent approaches, including heavy ion collisions, microscopic and phenomenological nuclear many-body theories and specific features of nuclear structure. One achieves more and more consistent results about $S_2(\rho)$ around the saturation density at present, while knowing almost nothing about its high density behavior. Recently, the high-order symmetry energy coefficients attracted some interest due to their non-negligible effects on some issues,[16–21] which deepens our understanding of symmetry energies. In Eq. (1), the exchange symmetry between neutrons and protons is assumed and thus only even-order terms with respect to β appear. However, this symmetry is just an approximation, and one of the reasons inducing the symmetry breaking is the n–p mass difference. In this Letter, we derive analytically the 1st-order symmetry energy coefficient of nuclear matter induced by the n–p mass difference.

The second-order symmetry energy coefficient of finite nuclei has received great interest[11,22–26] due to its importance for constraining the density dependence of the nuclear matter symmetry energy coefficient,[25–27] and can be calculated with nuclear energy density functionals.[27–29] In this work, we calculate the 1st-order symmetry energy (coefficient) of finite nuclei in the framework of density functional to achieve a further understanding on symmetry energies. The Skyrme interaction as a very powerful tool in low energy nuclear physics[30–32] is employed. It is stressed that a distinction between the symmetry energy and the symmetry energy coefficient is made in this work.

We calculate in detail the first-order term in β caused by the neutron–proton mass difference, defined as

$$\begin{aligned} S_1^{\rm (npDM)}(\rho) &= \frac{\partial E(\rho,\beta)}{\partial\beta}\Big|_{\rm (npDM),\beta=0} \\ &= \frac{1}{\rho}\frac{\partial \mathcal{H}_{\rm T}(\rho,\beta)}{\partial\beta}\Big|_{\rm (npDM),\beta=0}, \end{aligned} \qquad (2)$$

where $\mathcal{H}_{\rm T}$ is the kinetic energy density for asymmetric nuclear matter. Usually, $\mathcal{H}_{\rm T}$ is given by

$$\mathcal{H}_{\rm T}(\rho,\beta) = \frac{3\hbar^2}{10m}(3\pi^2)^{2/3}(\rho_{\rm n}^{5/3} + \rho_{\rm p}^{5/3}). \qquad (3)$$

If we distinguish the neutron mass from the proton one, the kinetic energy density should be expressed as

$$\mathcal{H}_{\rm T}(\rho,\beta) = \frac{3\hbar^2}{10m_{\rm n}}(3\pi^2)^{2/3}\rho_{\rm n}^{5/3} + \frac{3\hbar^2}{10m_{\rm p}}(3\pi^2)^{2/3}\rho_{\rm p}^{5/3}, \qquad (4)$$

*Supported by the National Natural Science Foundation of China under Grant Nos 11405223, 11175219, 11275271 and 11435014, the National Basic Research Program of China under Grant No 2013CB834405, the Knowledge Innovation Project of Chinese Academy of Sciences under Grant No KJCX2-EW-N01, the Funds for Creative Research Groups of China under Grant No 11321064, the Youth Innovation Promotion Association of Chinese Academy of Sciences, and the K. C. Wong Education Foundation.
**Corresponding author. Email: dongjm07@impcas.ac.cn
© 2016 Chinese Physical Society and IOP Publishing Ltd

which does not rely on specific models. Comparing Eq. (3) with Eq. (4), we obtain the β-dependent mass $m = m(\beta)$ appearing in Eq. (3),

$$m(\beta) = m_{\rm n}m_{\rm p}\frac{(1+\beta)^{5/3}+(1-\beta)^{5/3}}{m_{\rm p}(1+\beta)^{5/3}+m_{\rm n}(1-\beta)^{5/3}}. \quad (5)$$

Therefore, the kinetic energy density $\mathcal{H}_{\rm T}$ of Eq. (3) is rewritten as

$$\mathcal{H}_{\rm T}(\rho,\beta) = \frac{C}{m(\beta)}, \quad (6)$$

$$C = \frac{3\hbar^2}{20}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{5/3}[(1+\beta)^{5/3}+(1-\beta)^{5/3}], \quad (7)$$

and accordingly the first-order symmetry energy density is given by

$$\mathcal{H}_{\rm sym,1}^{\rm (npDM)}(\rho) = -\frac{\hbar^2}{2m^*}\left(\frac{3\pi^2}{2}\right)^{2/3}\left(\frac{m_{\rm n}-m_{\rm p}}{m_{\rm n}+m_{\rm p}}\right)\rho^{5/3}\beta, \quad (8)$$

where m^* is the reduced mass $m^* = 2m_{\rm n}m_{\rm p}/(m_{\rm n}+m_{\rm p})$. The nucleonic masses $m_n = 939.56533$ and $m_p = 938.27199$[33] are used in the following. The first-order symmetry energy coefficient is given analytically by

$$S_1^{\rm (npDM)}(\rho) = -\frac{\hbar^2}{2m^*}\left(\frac{3\pi^2}{2}\right)^{2/3}\left(\frac{m_{\rm n}-m_{\rm p}}{m_{\rm n}+m_{\rm p}}\right)\rho^{2/3}, \quad (9)$$

which is completely free from model-independence since it is unrelated to the interaction. The density dependence of the first-order symmetry energy coefficients in nuclear matter is shown in Fig. 1. Here $S_1^{\rm (npDM)}(\rho)$ in absolute value increases with the increasing density. It is indeed much less than the second-order one, and is often reasonably ignored in the investigation of density-dependent symmetry energy coefficient.

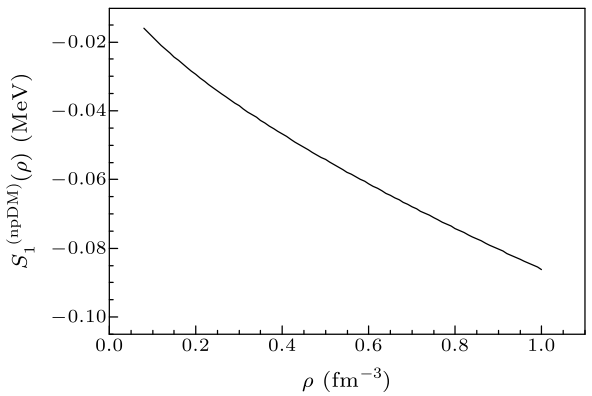

Fig. 1. The first-order symmetry energy coefficient as a function of the nuclear matter density.

Once the density distribution in a nucleus is achieved within the Skyrme–Hartree–Fock approach after subtracting the Coulomb interaction, the first-order symmetry energy $E_{\rm sym,1}^{\rm (npDM)}(A)$ of ^{208}Pb is calculated with a local density approximation

$$E_{\rm sym,1}^{\rm (npDM)}(A) = a_{\rm sym,1}^{\rm (npDM)}(A)IA = \int_0^\infty 4\pi\mathcal{H}_{\rm sym,1}^{\rm (npDM)}r^2dr, \quad (10)$$

just like the calculation of the second-order symmetry energy of finite nuclei in Ref. [27] $I = (N-Z)/A$ is the isospin asymmetry of nuclei. Here $g_1(r) = 4\pi\mathcal{H}_{\rm sym,1}^{\rm (npDM)}r^2$ is the radial density of the first-order symmetry energy, and $a_{\rm sym,1}^{\rm (npDM)}(A)$ is the corresponding first-order symmetry energy coefficient.

Table 1. The first-order symmetry energy coefficient as a function of nuclear matter density.

Force	$a_{\rm sym,1}^{\rm (npDM)}(A)$	$E_{\rm sym,1}^{\rm (npDM)}(A)$
SLy4[35]	−0.018	−0.81
LNS1[36]	−0.019	−0.84
LNS5[36]	−0.019	−0.82
HFB-17[37]	−0.019	−0.82
SLy5[35]	−0.018	−0.81
SkI2[38]	−0.018	−0.80
SkI5[38]	−0.017	−0.74
SkP[39]	−0.019	−0.82
SkM*[40]	−0.018	−0.81
SGII[41]	−0.019	−0.83
T6[42]	−0.019	−0.82
BSk8[43]	−0.019	−0.84
MSk1[44]	−0.019	−0.82
v090[45]	−0.019	−0.85
MSkA[46]	−0.018	−0.80
T4[42]	−0.017	−0.76
SK255[47]	−0.017	−0.76
SK272[47]	−0.017	−0.76
SIV[48]	−0.018	−0.78
SkMP[49]	−0.018	−0.79
MSL0[50]	−0.018	−0.79
SKa[51]	−0.018	−0.77
SkSC15[52]	−0.019	−0.86
SIII[48]	−0.018	−0.81

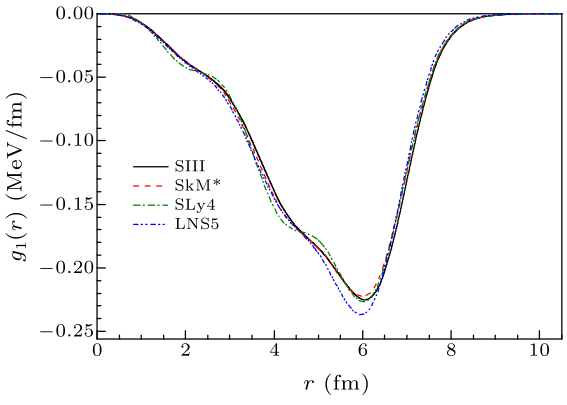

Fig. 2. (Color Online) The radial first-order symmetry energy density $g_1(r) = 4\pi\mathcal{H}_{1,\rm sym}r^2$ as a function of r in ^{208}Pb for different Skyrme interactions.

The calculated $E_{\rm sym,1}^{\rm (npDM)}(A)$ and $a_{\rm sym,1}^{\rm (npDM)}(A)$ for ^{208}Pb are presented in Table 1, which are found to be very weakly model-dependent, and $a_{\rm sym,1}^{\rm (npDM)}(A) \simeq -0.018$ MeV and $E_{\rm sym,1}^{\rm (npDM)}(A) \simeq -0.8$ MeV. The weak

model dependence merely results from the slightly different nucleon density profiles in ^{208}Pb obtained with different Skyrme interactions. The radial density distribution $g_1(r) = 4\pi\mathcal{H}_{\rm sym,1}^{\rm (npDM)} r^2$ is presented in Fig. 2. Similar to the situation of the second-order symmetry energy, the contribution to the first-order symmetry energy mainly stems from the nuclear surface region due to the large volume of the spherical shell and large isospin asymmetry in this region. Here $a_{\rm sym,1}^{\rm (npDM)}(A)$ is smaller than that of nuclear matter at the saturation density due to the very low density near the nuclear surface. The third-order symmetry energy caused by the proton–neutron mass difference $E_{\rm sym,3}^{\rm (npDM)}(A)$ and its coefficient are also derived. For ^{208}Pb, $E_{\rm sym,3}^{\rm (npDM)}(A) = 0.02$ MeV is much less than the first-order one. For the nucleus with a smaller isospin asymmetry, $E_{\rm sym,3}^{\rm (npDM)}(A)$ is even less. Incidentally, the second-order symmetry energy induced by the proton–neutron mass difference is zero due to

$$\left.\frac{\partial^2\mathcal{H}_{\rm T}(\rho,\beta)}{\partial\beta^2}\right|_{\rm (npDM),\beta=0} = Cm^{-3}(\beta)\left[2\left(\frac{\partial m(\beta)}{\partial\beta}\right)^2 - m(\beta)\frac{\partial^2 m(\beta)}{\partial\beta^2}\right]_{\beta=0} = 0. \quad (11)$$

The calculated $E_{\rm sym,1}^{\rm (npDM)}(A)$'s for other widely-investigated nuclei are shown in Fig. 3. For light nuclei, they are not so large. However, for superheavy nuclei, such as ^{298}114, $E_{\rm sym,1}^{\rm (npDM)}(A)$ can be as large as about -1.2 MeV. For neutron-rich nuclei, such as ^{184}Ce, due to their large neutron excess, $E_{\rm sym,1}^{\rm (npDM)}(A)$ can be also as large as those in the superheavy region. Although the first-order symmetry energy is much less than the second-order one, it is non-negligible for an accurate estimation of nuclear masses. Currently, there are a few successful nuclear mass formulas with a very high accuracy, for example, the new mass table FRDM-2011a with the rms deviation $\sigma = 0.5700$ MeV by Möller *et al.*,[53] the improved nuclear mass formula containing a unified prescription for the shell and pairing corrections with $\sigma = 0.493$ MeV by Zhang *et al.*[54] and the WS4 model with $\sigma = 0.298$ MeV by Wang *et al.*[55] The first-order symmetry energy $E_{\rm sym,1}^{\rm (npDM)}(A)$ can be much larger than the rms deviations in absolute value given by the above-mentioned formulas. Consequently, as one of the main conclusions of this work, $E_{\rm sym,1}^{\rm (npDM)}(A)$ cannot simply be ignored, in particular for the neutron-rich nuclei which have large isospin asymmetries $I = (N-Z)/A$. In other words, the neutron–proton mass difference should be taken into account in high-precision mass estimations for neutron-rich nuclei. In addition, since some relativistic and non-relativistic nuclear energy density functionals also predict nuclear masses with high accuracies, such as the HFB-17–HFB-21[37] and PC-PK1,[56] it may be necessary to distinguish the neutron mass from the proton one in those nuclear energy density functionals, just like the interactions PK1, PK1R and PKDD[57] in relativistic energy density functional theory. Therefore, we suggest taking into account the neutron–proton mass difference in building new effective nucleon–nucleon interactions in the future.

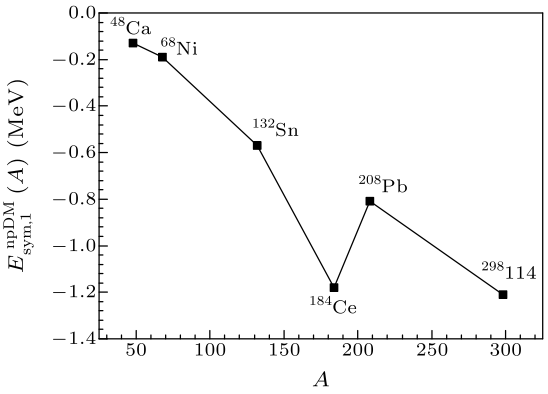

Fig. 3. The calculated $E_{\rm sym,1}^{\rm (npDM)}(A)$ for some nuclei in different mass regions with SLy4 interaction.

In summary, the first-order symmetry energy coefficient of nuclear matter induced only by the n–p mass difference is obtained without introducing any model-dependence. We then study the first-order symmetry energies and their coefficients of heavy nuclei with the help of local density approximation where the nucleonic density distribution is computed in the framework of the Skyrme energy density functionals taking the nucleus ^{208}Pb as an example. The first-order symmetry energy $E_{\rm sym,1}^{\rm (npDM)}(A)$ merely originating from the n–p mass difference is equal to -0.8 MeV, and originates dominantly from the nuclear surface region. Although the first-order symmetry energy is much less than the second-order one, it should be taken into account in nuclear mass formulas due to the fact that it is distinct compared with the uncertainties of the current macroscopic–microscopic mass formulas. The non-negligible first-order symmetry energy here also suggests that it may be necessary to distinguish the neutron mass from the proton one in the construction of nuclear energy density functionals.

References

[1] Xu C and Ren Z Z 2012 *Chin. Phys. Lett.* **29** 122102
[2] Baran V, Colonna M, Greco V and Toro M D 2005 *Phys. Rep.* **410** 335
[3] Li B A, Chen L W and Ko C M 2008 *Phys. Rep.* **464** 113
[4] Lattimer J M and Prakash M 2007 *Phys. Rep.* **442** 109
[5] Kumar S, Ma Y G, Zhang G Q and Zhou C L 2011 *Phys. Rev.* C **84** 044620
[6] Feng Z Q 2011 *Phys. Rev.* C **83** 067604
[7] Yong G C 2011 *Phys. Rev.* C **84** 014607
[8] Ma C W, Wang F, Ma Y G and Chan J 2011 *Phys. Rev.* C **83** 064620
[9] Ma C W, Wei H L, Wang S S, Ma Y G, Wada R and Zhang Y L 2015 *Phys. Lett.* B **742** 19 and reference therein
[10] Ma C W, Pu J, Wang S S and Wei H L 2012 *Chin. Phys. Lett.* **29** 062101
[11] Dong J, Zuo W and Scheid W 2011 *Phys. Rev. Lett.* **107** 012501
[12] Rizzo C, Baran V, Colonna M, Corsi A and Toro M D 2011 *Phys. Rev.* C **83** 014604
[13] Horowitz C J and Piekarewicz J 2001 *Phys. Rev. Lett.* **86**

5647
[14] Lattimer J M and Prakash M 2000 *Phys. Rep.* **333** 121
[15] Cavagnoli R, Menezes D P and Providencia C 2011 *Phys. Rev.* C **84** 065810
[16] Cai B J and Chen L W 2012 *Phys. Rev.* C **85** 024302
[17] Xu J, Chen L W, Li B A and Ma H R 2009 *Phys. Rev.* C **79** 035802
[18] Seif W M and Basu D N 2014 *Phys. Rev.* C **89** 028801
[19] Moustakidis C C 2012 *Phys. Rev.* C **86** 015801
[20] Jiang H, Bao M, Chen L W, Zhao Y M and Arima A 2014 *Phys. Rev.* C **90** 064303
[21] Jiang H, Wang N, Chen L W, Zhao Y M and Arima A 2015 *Phys. Rev.* C **91** 054302
[22] Centelles M, Roca-Maza X, Viñas X and Warda M 2009 *Phys. Rev. Lett.* **102** 122502
[23] De J N and Samaddar S K 2012 *Phys. Rev.* C **85** 024310
[24] De J N, Samaddar S K and Agrawal B K 2012 *Phys. Lett.* B **716** 361
[25] Dong J, Zhang H, Wang L and Zuo W 2013 *Phys. Rev.* C **88** 014302
[26] Fan X, Dong J and Zuo W 2014 *Phys. Rev.* C **89** 017305
[27] Dong J, Zuo W and Gu J 2015 *Phys. Rev.* C **91** 034315
[28] Dong J, Zuo W and Gu J 2013 *Phys. Rev.* C **87** 014303
[29] Zhang Z W, Bao S S, Hu J N and Shen H 2014 *Phys. Rev.* C **90** 054302
[30] Bennaceur K and Dobaczewski J 2005 *Comput. Phys. Commun.* **168** 96
[31] Stoitsov M V, Dobaczewski J and Nazarewicz W R 2005 *Comput. Phys. Commun.* **167** 43
[32] Stone J R and Reinhard G P 2007 *Prog. Part. Nucl. Phys.* **58** 587 and references therein
[33] Nakamura K et al (Particle Data Group) 2010 *J. Phys.* G **37** 075021
[34] Chabanat E, Bonche P, Haensel P, Meyer J and Schaeffer R 1997 *Nucl. Phys.* A **627** 710
[35] Chabanat E, Bonche P, Haensel P, Meyer J and Schaeffer R 1998 *Nucl. Phys.* A **635** 231
[36] Gambacurta D, Li L, Colò G, Lombardo U, Giai N V and Zuo W 2011 *Phys. Rev.* C **84** 024301
[37] Goriely S, Chamel N and Pearson J M 2009 *Phys. Rev. Lett.* **102** 152503
Goriely S, Chamel N and Pearson J M 2010 *Phys. Rev.* C **82** 035804
[38] Reinhard P G and Flocard H 1995 *Nucl. Phys.* A **584** 467
[39] Dobaczewski J, Flocard H and Treiner J 1984 *Nucl. Phys.* A **422** 103
[40] Bartel J, Quentin P, Brack M, Guet C and Hatkansson H B 1982 *Nucl. Phys.* A **386** 79
[41] Nguyen V G and Sagawa H 1981 *Phys. Lett.* B **106** 379
[42] Tondeur F, Brack M, Farine M and Pearson J M 1984 *Nucl. Phys.* A **420** 297
[43] Samyn M, Goriely S, Bender M and Pearson J M 2004 *Phys. Rev.* C **70** 044309
[44] Tondeur F, Goriely S, Pearson J M and Onsi M 2000 *Phys. Rev.* C **62** 024308
[45] Pearson J M and Goriely S 2001 *Phys. Rev.* C **64** 027301
[46] Sharma M M, Lalazissis G, Kö nig J and Ring P 1995 *Phys. Rev. Lett.* **74** 3744
[47] Agrawal B K, Shlomo S and Au V K 2003 *Phys. Rev.* C **68** 031304
[48] Beiner M, Flocard H, Giai N V and Quentin P 1975 *Nucl. Phys.* A **238** 29
[49] Bennour L, Heenen P H, Bonche P, Dobaczewski J and Flocard H 1989 *Phys. Rev.* C **40** 2834
[50] Chen L W, Ko C M, Li B A and Xu J 2010 *Phys. Rev.* C **82** 024321
[51] Kohler H S 1976 *Nucl. Phys.* A **258** 301
[52] Pearson J M and Nayak R C 2000 *Nucl. Phys.* A **668** 163
[53] Möller P, Myers W D, Sagawa H and Yoshida S 2012 *Phys. Rev. Lett.* **108** 052501
[54] Zhang H, Dong J, Ma N, Royer G, Li J and Zhang H 2014 *Nucl. Phys.* A **929** 38
[55] Wang N, Liu M, Wu X and Meng J 2014 *Phys. Lett.* B **734** 215
[56] Zhao P W, Li Z P, Yao J M and Meng J 2010 *Phys. Rev.* C **82** 054319
[57] Long W H, Meng J, Giai N V and Zhou S G 2004 *Phys. Rev.* C **69** 034319