Stability Analysis of the Viscous Polytropic Dark Energy Model in Einstein Cosmology *

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The viscous polytropic gas model as one model of dark energy is hot-spot and keystone to the modern cosmology. We study the evolution of the viscous polytropic dark energy model interacting with the dark matter in the Einstein cosmology. Setting the autonomous dynamical system for the interacting viscous polytropic dark energy with dark matter and using the phase space analysis method to investigate the dynamical evolution and its critical stability, we find that the viscosity property of the dark energy creates a benefit for the stable critical dynamical evolution of the interaction model between dark matter and dark energy in the flat Friedmann–Robertson–Walker universe and the viscosity of dark energy will soften the coincidence problem just like the interacting dark energy model.

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As we all know, two independent projects, the Supernova Cosmology Project and the High-Z Supernova Search Team, have simultaneously obtained the results suggesting an unexpected acceleration expansion of the universe.[1,2] Analysis of type-Ia supernovae indicates that the spatial expansion is accelerating. An explanation for the expansion of the Universe remains elusive. It is often attributed to 'dark energy', an unknown form of energy which is hypothesized to permeate space. Dark energy is the most accepted hypothesis to explain the acceleration expansion of the universe since the 1990s. The best current measurements indicate that dark energy contributes 68.3% of the total energy in the present-day observable universe. The mass energy of dark matter and ordinary (baryonic) matter contribute 26.8% and 4.9%, respectively, and other components such as neutrinos and photons contribute a very small amount. Dark energy can have such a profound effect on the universe, making up 68% of universal density. The two leading models are cosmological constant and quintessence. Both the models include the common characteristic that dark energy must have negative pressure.

Due to the fact that the nature of dark energy and the composition are still unknown, numerous speculations about the dark energy model and cosmic coincidence problem[3−7] are the most famous and difficult problems in cosmology. To alleviate this problem, a dynamical dark energy scenario has been proposed with time varying the equation-of-state (EoS) parameter ω. There are two different categories for the dynamical DE scenario which have been introduced to explain the accelerating expansion of universe: (i) the 'scalar fields' such as quintessence,[8,9] quintom,[10,11] phantom,[12,13] K-essence,[14−16] tachyon,[17−19] dilaton.[20] (ii) The 'interacting models' of dark energy such as Chaplygin gas models,[21−23] braneworld models,[24,25] holographic[26−28] and agegraphic models.[29] All those theoretical models describe dark energy. However, from the viewpoint of cosmic perturbations, it brings us to consider some correction dark energy models, such as the viscosity dark energy model and the interaction model. Thus the viscosity fluid dark energy model has become one of the active options[30] recently. We all know that the viscous dark energy model could represent our universe more exactly. Moreover, it contributes to entropy production in the universe. Numerous researchers have considered different viscous fluid cases, i.e., a five-dimensional space-time with viscous fluid,[31] a cosmological model with viscous fluid in higher-dimensional space-time,[32] a five-dimensional cosmological model with variable bulk viscosity,[33] the isotropic homogeneous spatially flat cosmological model with bulk viscous fluid[34] and the bulk viscous cosmological models with constant bulk viscosity coefficient.[35] In the recent work[36] the Friedmann–Robertson–Walker (FRW) bulk viscous cosmology and bulk viscous coefficient are considered in the flat space and then extend them to non-flat space.[37] In this Letter, we consider the polytropic gas dark energy model[38] and focus on its dynamical stability with the phase space method. The EoS of polytropic gas is given by $P = K\rho^{1+\frac{1}{n}} - 3\xi H$, where K is the polytropic constant, n is the polytropic index, and ξ is the viscous coefficient.

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According to Einstein's general relativity, the dynamics of the flat FRW universe[39] equations is

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho, \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + 3p, \tag{2}$$

where H is the Hubble parameter, i.e., a measure of the rate at which the universe is expanding, a is the scale factor, and G is a constant. The conservation relation of the cosmological total energy is as follows:

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{3}$$

where the total energy density is $\rho = \rho_{\rm E} + \rho_{\rm M}$ including dark energy and dark matter. In this work we set $c = 8\pi G = 1$ and consider the interaction term $Q = 3b_1 H\rho_{\rm E} + 3b_2 H\rho_{\rm M}$ between the dark energy and dark matter, thus the evolution equations are

$$\dot{\rho_{\rm M}} + 3H\rho_{\rm M} = Q, \tag{4}$$

$$\dot{\rho_{\rm E}} + 3H(\rho_{\rm E} + p) = -Q. \tag{5}$$

To analyze the dynamical system of the interacting viscous polytropic gas, we set

$$x = \frac{\rho_{\rm E}}{3H^2}, \tag{6}$$

$$y = \frac{p}{3H^2}, \tag{7}$$

$$z = \frac{\rho_{\rm M}}{3H^2}, \tag{8}$$

$$\frac{d}{dN} = \frac{1}{H}\frac{d}{dt}, \tag{9}$$

where a is the scale factor, and $N \equiv \ln a$ is the number of e-folding to present the cosmological time. To obtain the physically reasonable solutions, we have the following constraints $0 \le x \le 1$ and $0 \le z \le 1$. At the same time the variables x and z satisfy the following constraint

$$x + z = 1. \tag{10}$$

In terms of x and y, the EoS parameter of the viscous polytropic gas reads

$$\omega_{\rm c} = \frac{y}{x}. \tag{11}$$

The EoS parameter of the effective fluid reads

$$\omega_{\rm eff} = \frac{p}{\rho_{\rm E} + \rho_{\rm M}} = y. \tag{12}$$

Due to the fact that dark matter is considered as a pressureless fluid, it is not difficult to show that the deceleration parameter q reads

$$q = -1 - \frac{\dot{H}}{H^2} = \frac{1}{2}(1 + 3y). \tag{13}$$

Now we study late time attractors of the field equations for various interacting varying viscous polytropic gas cosmological scenarios. For simplicity, we combine an experience known from physical literature to impose some constraints on the model parameters. Then we consider a cosmological model with the interaction of the following form, which is involved into the darkness of the large universe

$$Q = 3Hb_1\rho_{\rm E} + 3Hb_2\rho_{\rm M}, \tag{14}$$

where b_1 and b_2 are coupling constants of interaction. To solve the coincidence problem, the interaction term should satisfy the condition $Q > 0$, which leads to the parameter $b_1 > -\frac{b_2}{3}$, which is the best range taken from the observational data.

Using those conditions (4), (5)and (14), we can construct the following autonomous dynamical system for the interacting viscous dark energy model in the Einstein cosmology,

$$\begin{aligned} x' &= f(x,y) = 3(-b_1 x(t) + b_2(x(t)-1) \\ &\quad + (x(t)-1)y(t), \end{aligned} \tag{15}$$

$$\begin{aligned} y' &= g(x,y) = -\big[3(n+1)(\xi + y(t))(b_1 x(t) \\ &\quad + b_2(-x(t)) + b_2 + x(t) + y(t))\big] \cdot [nx(t)]^{-1} \\ &\quad + \frac{1}{2}(-3)\xi(y(t)+1) + 3y(t)(y(t)+1). \end{aligned} \tag{16}$$

The dynamics is the general form $X' = F(X)$, where X is the column matrix constituted by the auxiliary variables. To analyze the stability of the dynamics system, we must linearly expand the dynamical system near the critical points by using the linearizing theory of the dynamical system. Thus we can acquire the stability properties of the dynamical system from the eigenvalues of linearizing matrix.

Firstly, we solve $x' = f(x,y) = 0$ and $y' = g(x,y) = 0$ to obtain the critical points: (I) $(x_{\rm c}^{\rm I}, y_{\rm c}^{\rm I}) = (\frac{-1+b_2}{-1-b_1+b_2}, -1)$, and (II) $(x_{\rm c}^{\rm II}, y_{\rm c}^{\rm II}) = (\frac{\xi(3n+2)-2b_2}{2b_1-2b_2+\xi(3n+2)}, -\frac{1}{2}\xi(2+3n))$.

For the critical point I, the solution is a late time scaling attractor, due to the fact that

$$r = \frac{\Omega_{\rm M}}{\Omega_{\rm E}} = \frac{a}{1-b}, \tag{17}$$

is a constant. Our analysis shows that this time scaling attractor describes the state of the universe where EoS of the effective fluid is $\omega_{\rm eff} = -1$. The deceleration parameter $q = -1$ and EoS of the polytropic gas exhibits a phantom behavior with

$$\omega_{\rm E} = \frac{b_1 - b_2 + 1}{b_2 - 1}. \tag{18}$$

To investigate the stability of the critical point II, we linearize the dynamical system near the critical

point as follows:

$$\delta x = 3(-b_1 + b_2 + y_c)\delta x + 3(-1 + x_c)\delta y, \tag{19}$$

$$\delta y = \frac{3(1+n)(b_2+y_c)(\xi+y_c)}{nx_c^2}\delta x - \Big[\frac{6(x_c(b_1 n + b_1 - 2b_1 y_c + 1) + (n+1)y_c)}{2nx_c} - \frac{3(-2b_2(n+1)(x_c-1) + \xi(n(x_c+2)+2))}{2nx_c}\Big]\delta y. \tag{20}$$

To analyze the stability of the dynamics system, we must linearly expand the dynamical system near the critical points using the linearizing theory of the dynamical system. From the above linearized equations, we can define the following linearizing matrix

$$M = \begin{pmatrix} \frac{\partial f}{\partial x}|_{x_c,y_c} & \frac{\partial f}{\partial y}|_{x_c,y_c} \\ \frac{\partial g}{\partial x}|_{x_c,y_c} & \frac{\partial g}{\partial y}|_{x_c,y_c} \end{pmatrix}.$$

For point Ⅱ, the solution is a late time scaling attractor, due to the fact that

$$r = \frac{\Omega_M}{\Omega_E} = \frac{2b_1}{\xi(3n+2) - 2b_2} \tag{21}$$

is a constant. Our analysis shows that the time scaling attractor describes the state of the universe where EoS of the effective fluid is

$$\omega_{\rm eff} = -\frac{1}{2}\xi(3n+2). \tag{22}$$

The deceleration parameter

$$q = \frac{1}{4}[2 - 3\xi(3n+2)], \tag{23}$$

and the EoS of the polytropic gas does exhibit a phantom behavior with

$$\omega_E = \frac{\xi(2+3n)[2b_1 - 2b_2 + \xi(2+3n)]}{4b_2 - 2\xi(2+3n)}. \tag{24}$$

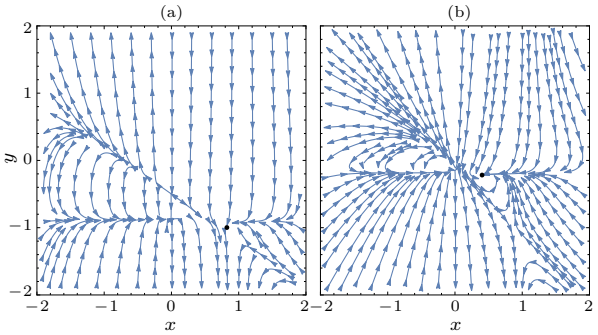

Fig. 1. The phase portraits for the viscous interacting polytropic gas model in the flat FRW universe by fixed parameters $n = 0.1$, $\xi = 0.87$, $b_1 = 0.5$, $b_2 = 0.2$ (a) and $n = 0.8$, $\xi = 0.1$ $b_1 = 0.1$ $b_2 = 0.15$ (b).

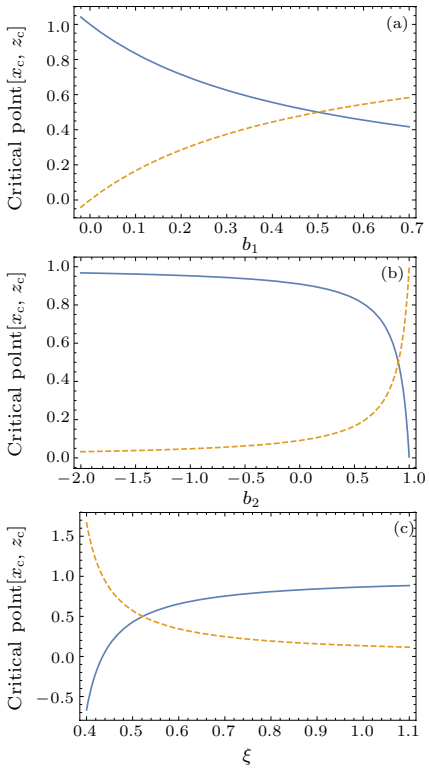

Fig. 2. Variable of critical point coordinates (x_c (solid) and z_c (dashed)) by fixed parameters $n = 0.1$, $b_2 = 0.5$, $\xi = 0.87$ (a); $n = 0.1$, $b_1 = 0.1$, $\xi = 0.87$ (b) and $n = 0.1$, $b_1 = 0.1$, $b_2 = 0.5$ (c).

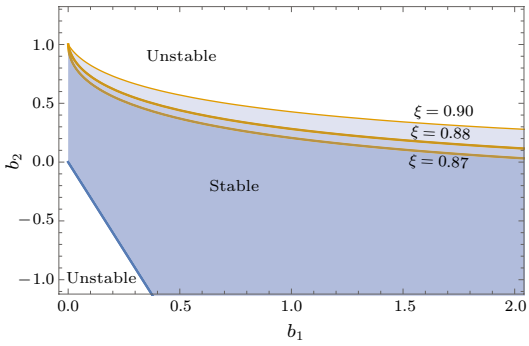

Fig. 3. The stability region of the critical points in interaction parameters space (b_1, b_2) for different values of viscosity coefficient ξ for the viscous polytropic gas model in the flat FRW universe by fixing the parameter $n = 0.1$ and $\xi = 0.87$, $\xi = 0.88$ and $\xi = 0.90$.

In Fig. 1 we have simulated the phase portraits of the autonomous dynamical system described by Eqs. (15) and (16) for the interacting viscous dark energy model in the Einstein cosmology. The point C is the critical point of the autonomous dynamical system by fixing the parameters $n = 0.1$, $\xi = 0.87$, $b_1 = 0.5$, $b_2 = 0.2$ and $n = 0.8$, $\xi = 0.1$, $b_1 = 0.1$, $b_2 = 0.15$. Figure 2 shows the critical point of the autonomous dynamical system changing with different physical parameters, such as interaction constant between dark matter and dark energy, viscous coefficient ξ. From these figures we can find that the universe is dominated by the dark matter when the interaction coeffi-

cients b_1 and b_2 become stronger. That is, if we take into account the interaction between the dark energy and the dark matter, it is good for dark matter formation in the universe. However, if we take into account the viscous fluid coefficient ξ, the situation is contrary. At the same time, both the critical points I and II show that their coordinates (x_c, y_c) in the phase space are not vanished in their stable region. These can alleviate the coincidence problem in the FRW universe. Thus we find that if we take into account the bulk viscosity of the dark energy, the viscosity will soften the coincidence problem just like the interacting dark energy model.

According to the dynamical theory, the critical point of the dynamical system is stable when the linearizing matrix M of the differential equations satisfies the conditions $\mathrm{Tr}M < 0$ and $\det M > 0$. In Fig. 3, we have shown the stability region of the critical points in the interaction parameter space (b_1, b_2) for different values of viscosity coefficient ξ. The shadow region is stable, and the rest region is unstable. In comparison of the regions for different viscosity coefficients ξ, we can see that the viscosity property of the dark energy creates a benefit for the stable critical dynamical evolution of the interaction model between dark matter and dark energy in the flat FRW universe.

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