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Electromagnetic Resonance of Astigmatic Gaussian Beam to the High Frequency Gravitational Waves *

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The high frequency gravitational waves (around 10^8–10^{12} Hz) could interact with a specially designed electromagnetic resonance system. It is found that the power of transverse perturbative photon flux (PPF) of an electromagnetic resonance system can be improved significantly by virtue of an astigmatic Gaussian beam. Correspondingly the signal-to-noise ratio (SNR) would also be improved. When the eccentric ratio of waist satisfying $w_{0x} : w_{0y} > 1$, the peak value of signal photon flux could be raised by 2–4 times with typical systematic parameters, while the background photon flux would be depressed. Therefore, the ratio of transverse PPF to background photon flux (i.e., SNR) can be further improved 3–8 times with dimensionless amplitude of relic gravitational wave $h_t = 10^{-36}$.

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Recently LIGO collaboration has achieved the gravitational wave detection around 10^2–10^3 Hz.[1,2] As a supplement, the detection of gravitational waves in microwave frequency band (i.e., HFGWs) can provide a new frequency window to observe our universe and verify some novel cosmological models, such as braneworld scenarios with extra dimensions.[3,4] Furthermore on the earth the relic gravitational waves in high frequency band have an analogous plane waveform, thus they can cause some similar physical properties as electromagnetic (EM) waves. Therefore in the microwave frequency band the detection of HFGWs needs to adopt some different techniques, such as the electromagnetic resonance system designed by Li *et al.*[5–11] The electromagnetic resonance system could provide a possible way to capture the gravitational wave in the microwave band (10^8–10^{12} Hz), which is in laboratory scale and constituted by a static magnetic field on the y-axis, a standard Gaussian beam (GB) passing along the z-axis, a photon counter on the x-axis. Actually such a detection is an EM resonance process in curved spacetime, which can be divided into two crucial parts. (1) Firstly, once the HFGWs exist around our equipment and have a component propagating along the z-axis, the static magnetic field on the y-axis immersed in such perturbative spacetime would generate perturbative electromagnetic field on transverse (x and y) axis (the electro-dynamical equations in curved spacetime in Ref. [6]). (2) The electric component of such perturbative electromagnetic field ($E_y^{(1)}$) couples to the longitudinal magnetic component of GB ($B_z^{(0)}$) resulting in the transverse perturbative photon flux (PPF) on the x-axis. The PPF is considered to be our signal, which can be observed by the photon counter on the x-axis. Since the PPF contains some information of the HFGWs, we plan to realize the detection of HFGWs through the observation of such a PPF. In fact the dimensionless amplitude of gravitational waves in microwave band is very low, thus the power of the PPF is weak. Therefore based on its traditional structure, we have proposed a new approach to enhance the resonance of electromagnetic field to HFGWs by using an astigmatic GB.

Compared with a standard GB, the astigmatic GB has more flexible electromagnetic field, which is mainly manifested in the distribution of beam width $w(z)$. By adjusting the waist on the x and y axes, the PPF can be improved in different degrees. Furthermore, the methods of generating astigmatic GBs are convenient, and various kinds of astigmatic GBs can be achieved by using appropriate astigmatic lens. At present, more and more kinds of astigmatic GB appear, including collimated astigmatic GB, bottle beams, sin-Gaussian beams, cosine-squared Gaussian beams, elegant cosh Gaussian beams, J_0-correlated Schell-model beams and so on.[12–14] Here we just use the most general astigmatic GB-collimated astigmatic GB.

In this Letter, we focus on two points: (1) the relationship between the distribution of astigmatic GB's waist and the number of transverse PPF; and (2) the

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effect of eccentric ratio of waist on the ratio of the transverse PPF to the background transverse photon flux (BPF). Conservatively here we still focus on the relic gravitational waves (RGWs) in microwave band predicted by the inflation cosmological model. Certainly some other cosmological models also predict the existence of gravitational waves in the microwave band, and the strength of those gravitational waves are much stronger than RGWs. Thus the results in this work would be much better in those models. Therefore this work is meaningful for the future observation of high frequency gravitational waves.

Supposing we have an astigmatic GB with a fundamental frequency mode propagating along the z-axis, the amplitude can be expressed as[15]

$$\psi(x,y,z)=\phi\exp\Big(-\frac{x^2}{w_x^2(z)}-\frac{y^2}{w_y^2(z)}\Big), \tag{1}$$

where $w_x(z)=w_{0x}[1+(z\lambda/\pi w_{0x}^2)]^{1/2}$ and $w_y(z)=w_{0y}[1+(z\lambda/\pi w_{0y}^2)]^{1/2}$ (w_{0x} and w_{0y} are the GB waist on the x-axis and the y-axis, respectively). This can be deformed from a standard GB, which has the amplitude as

$$\psi_0(x_0,y_0,z)=\phi_0\exp\Big(-\frac{x_0^2+y_0^2}{w^2(z)}\Big). \tag{2}$$

According to energy conservation of EM field, i.e.,

$$\begin{aligned}&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi^2(x,y,z)dxdy\\&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi_0^2(x_0,y_0,z)dx_0dy_0,\end{aligned} \tag{3}$$

we can obtain

$$\phi=\sqrt{\frac{w^2(z)\phi_0^2}{w_x(z)w_y(z)}}. \tag{4}$$

Then we can assume the longitudinal magnetic component of GB in the astigmatic Gaussian form of

$$\begin{aligned}\tilde{B}_z^{(0)}=&\frac{\psi}{\sqrt{1+(z/\sqrt{f_xf_y})^2}}\\&\times e^{[i(k_ez-\omega_et-\arctan\frac{z}{\sqrt{f_xf_y}}+\frac{k_er^2}{2\sqrt{R_xR_y}}+\delta)]},\end{aligned} \tag{5}$$

where $\omega_{\rm e}$ is the angular frequency of GB, $k_{\rm e}=2\pi/\lambda_{\rm e}$ ($\lambda_{\rm e}$ is the wavelength of GB), δ is the phase difference between the GB and the resonant component of the HFGWs, $f_x=\pi W_{0x}^2/\lambda_{\rm e}$, $f_y=\pi W_{0y}^2/\lambda_{\rm e}$, $R_x=z+f_x^2/z$ and $R_y=z+f_y^2/z$ (the curvature radius of the wave fronts of the GB at z). Here the amplitude of magnetic field of GB ψ is in the astigmatic form as Eq. (1). Without loss of generality, we set $\tilde{B}_x^{(0)}=0$. Consequently, the other components of GB would be determined as follows:

$$\begin{aligned}\tilde{B}_y^{(0)}=&-\int\partial_z\tilde{B}_z^{(0)}dy,\\\tilde{E}_x^{(0)}=&\frac{ic^2}{\omega_{\rm e}}\Big(\frac{\partial\tilde{B}_z^{(0)}}{\partial y}-\frac{\partial\tilde{B}_y^{(0)}}{\partial z}\Big),\\\tilde{E}_y^{(0)}=&-\frac{ic^2}{\omega_{\rm e}}\frac{\partial\tilde{B}_z^{(0)}}{\partial x},\\\tilde{E}_z^{(0)}=&\frac{ic^2}{\omega_{\rm e}}\frac{\partial\tilde{B}_y^{(0)}}{\partial x}.\end{aligned} \tag{6}$$

Those support the background EM field for coupling to the electric component of the perturbative EM field, which is the product from interaction between static magnetic field $\hat{B}_y$ and the HFGW propagating along the z-axis with a resonant frequency (i.e., $\omega_{\rm g}=\omega_{\rm e}$).[6,9,16] Such an interaction can be explained by the electromagnetic equations in curved spacetime.[17] By solving the electromagnetic equations, the first-order perturbative electric component is given as[6]

$$\begin{aligned}E_y^{(1)}=&h_{\rm t}\hat{B}_y^{(0)}c\Big\{-\frac{1}{2}k_{\rm e}(z+l_1)\exp[i(k_{\rm e}z-\omega_{\rm e}t)]\\&+\frac{i}{4}\exp[i(k_{\rm e}z+\omega_{\rm e}t)]\Big\},\end{aligned} \tag{7}$$

where c denotes the speed of light in a vacuum, $h_{\rm t}$ is the dimensionless amplitude of HFGW, l_1 denotes the region of static magnetic field $\hat{B}_y^{(0)}$ on the z-axis ($-l_1\le z\le l_1$). Here we focus on the photon fluxes along the x-axis, which are composed of transverse background photon flux N_x^0 and PPF N_x^1. They can be derived from Eqs. (5)–(7) as follows:

$$N_x^0=\frac{1}{\hbar\mu_0\omega_{\rm e}}\int\int_{\Delta s}\langle\tilde{E}_y^{(0)}\tilde{B}_z^{(0)}-\tilde{E}_z^{(0)}\tilde{B}_y^{(0)}\rangle dydz, \tag{8}$$

$$N_x^1=\frac{1}{\hbar\mu_0\omega_{\rm e}}\int\int_{\Delta s}\langle\tilde{E}_y^{(1)}\tilde{B}_z^{(0)}\rangle dydz, \tag{9}$$

where Δs is the 'typical receiving surface' and generally equals to $3\times10^{-2}\,{\rm m}^2$. Note that the interaction between static magnetic field and the HFGW propagating along the z-axis will not generate longitudinal electric component (i.e., $E_z^{(1)}=0$). From Eq. (7) and (9), we can see that the effect of HFGW is contained in N_x^1. Thus N_x^1 is considered to be the signal of such a resonance system.

In this work, we investigate the electromagnetic effect of high frequency relic gravitational wave, which has the energy density[18,19]

$$\Omega_{\rm GW}(k)=\frac{\mathcal{P}_{\rm t}(k)}{12H_0^2}(\dot{\mathcal{T}}(\eta_0,k))^2, \tag{10}$$

where H_0 is the Hubble constant and the dot denotes the derivative with respect to cosmic time t, $\eta_0=1.41\times10^4\,{\rm Mpc}$ denotes the conformal time at

present epoch, $\mathcal{T}(\eta, k)$ represents the tensor transfer function, and $\mathcal{P}_{\rm t}(k)$ is the power spectrum of primordial tensor perturbations, which can be expressed as[20]

$$\mathcal{P}_{\rm t}(k) = r_{\rm t} A_{\rm s} \left(\frac{k}{k_{\rm p}}\right)^{n_{\rm t}}, \tag{11}$$

where $n_{\rm t}$ and $r_{\rm t}$ are the tensor spectral index and the tensor-to-scalar ratio, respectively, $k_{\rm p}$ is the pivot scale, $A_{\rm s}$ is the amplitude of power spectrum of primordial scalar perturbations, which can be fixed in the Planck 2015 cosmological results.[21] Then the amplitude of relic GW at present can be derived as

$$h_{\rm t}(\eta_0, k) = \sqrt{\mathcal{P}_{\rm t}(k)}\mathcal{T}(\eta_0, k). \tag{12}$$

Thus the amplitude of relic GW relies on the tensor spectral index $n_{\rm t}$ and the tensor-to-scalar ratio $r_{\rm t}$. Conservatively, here we choose $r_{\rm t} = 0.05$ and $n_{\rm t} = -r_{\rm t}/8$. Then $h_{\rm t} \sim 10^{-36}$ at GHz.[22]

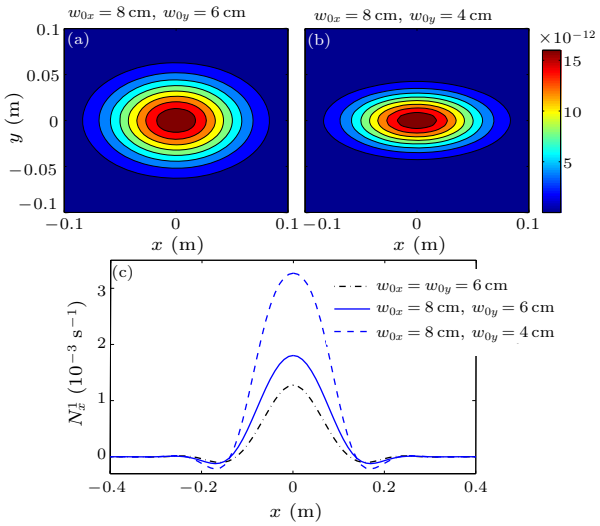

Fig. 1. (a) The intensity of astigmatic GB at $z = w_{0x}$ and $w_{0x} : w_{0y} = 4 : 3$. (b) The intensity of astigmatic GB at $z = w_{0x}$ and $w_{0x} : w_{0y} = 2 : 1$. (c) Comparison of the transverse PPF between standard GB and astigmatic GBs. Here we have made the intensity of two GBs satisfying the energy conservation of EM field, and the dimensionless amplitude of HFGW is $h_{\rm t} = 10^{-36}$, the frequency of HFGW is 3×10^9 Hz.

Then we adopt some typical parameters of the traditional GB resonance system of HFGW, such as the frequency of GB is $\nu_{\rm e} = 3 \times 10^9$ Hz, the power of GB $P = 10$ W, the static magnetic field $\hat{B}_y^{(0)} = 3$ T and $l_1 = 0.3$ m in Eq. (7). Consequently, we discuss the variations of signal (i.e., transverse PPF N_x^1) after astigmatic GB substituting for the standard GB. Figure 1 shows that our signal (PPF) can be enhanced through using astigmatic GB. We can find that with $w_{0x} = 0.08$ m, the maximum value of PPF (i.e., $N_x^1|_{x=0}$) can vary from 1.27×10^{-3} s^{-1} to 3.27×10^{-3} s^{-1} while w_{0y} varies from 0.08 m to 0.04 m. Meanwhile in the region of $(-0.14\,{\rm m}, 0.14\,{\rm m})$, the strength of PPF could be improved significantly.

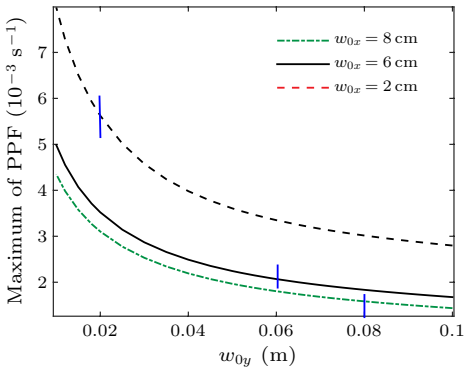

Fig. 2. The maximum values of transverse PPF vary with different eccentric ratios of waist. The curves correspond to $w_{0x} = 0.08$ m, 0.06 m and 0.02 m respectively, and the blue line denotes the position of $w_{0x} : w_{0y} = 1$ in each case. Here the dimensionless amplitude of HFGW is $h_{\rm t} = 10^{-36}$, and the frequency of HFGWs is 3×10^9 Hz.

Figure 2 illuminates the maximum value of transverse perturbation photon flux (N_x^1) varying with different eccentric ratios of waist. It can be found that given w_{0x}, the peak value of N_x^1 increases significantly with smaller w_{0y}. Furthermore under the condition of $w_{0x} > w_{0y}$, our signal strength would be stronger than that in the standard case (i.e., $w_{0x} = w_{0y} = w_0$), and vice versa. Such a property inspires us to adjust the eccentric ratio of waist to an appropriate value for generating much more transverse photon flux.

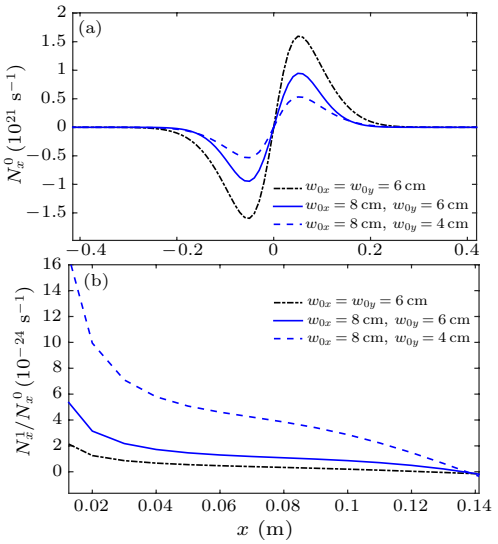

Fig. 3. The number of transverse background photon flux varying with different eccentric ratios of waist. (b) The ratio of the transverse PPF N_x^1 to the background transverse photon flux N_x^0 varying with different eccentric ratios of waist.

On the other hand, the ratio of the transverse PPF N_x^1 to the background transverse photon flux N_x^0 can be considered as the signal-to-noise ratio (SNR),

which is a very crucial index of such a resonance system. With the increase of the eccentric ratio of waist the background transverse photon flux $N_x^{(0)}$ can be depressed (Fig. 3(a)), thus N_x^1/N_x^0 increases in the region of $(-0.14\,\mathrm{m}, 0.14\,\mathrm{m})$ (Fig. 3(b)). Correspondingly the maximum value of N_x^1/N_x^0 varies from 2×10^{-24} to 1.6×10^{-23} while w_{0y} varies from 0.08 m to 0.04 m. In Ref. [22] with the same cosmological parameters the SNR at $x = 0.035\,\mathrm{m}$ is 10^{-28}. This means that we have improved the maximum value of SNR nearly four orders of magnitude.[22]

Compared with recent works about electromagnetic systems for detection of HFGWs,[22] with the same value of systematic parameters we have raised $N^1|_{x=0.035}$ from $10^{-3}\,\mathrm{s}^{-1}$ to $3.5\times10^{-3}\,\mathrm{s}^{-1}$. This is due to the fact that the transverse PPF (N_x^1) relies on $B_z^{(0)}$ directly (Eq. (9)), and we have focused the major power on the background longitudinal magnetic component $B_z^{(0)}$ (Eq. (5)) rather than on electric field $E_x^{(0)}$. Furthermore the transverse PPF N_x^1 can be enhanced when adjusting the eccentric ratio of waist to satisfy $w_{0x} : w_{0y} > 1$. Meanwhile transverse background photon flux N_x^0 could be reduced when $w_{0x} : w_{0y} > 1$. Therefore the signal-to-noise ratio N_x^1/N_x^0 can be raised greatly.

As mentioned above, we have chosen the most conservative dimensionless amplitude of HFGW at $3\times10^9\,\mathrm{Hz}$ as $h_\mathrm{t} = 10^{-36}$ corresponding to $n_\mathrm{t} = -6.25\times10^{-3}$. Actually according to some cosmological models, e.g., the extra dimension braneworld model,[3,4] inflation with axion potential[23] and the ekpyrotic model[24] and the latest Planck date, n_t could be larger than 0, which implies that the dimensionless amplitude of HFGW h_t can be much stronger, such as $h_\mathrm{t} = 10^{-28}$ with $n_\mathrm{t} = 0.61$. Then the signal N_x^1 and the signal-to-noise ratio N_x^1/N_x^0 would be further improved by 8 orders.

Even though the SNR of such an electromagnetic resonance system is still very low, the work which can possibly enhance the value of N_x^1/N_x^0 is thought to be meaningful. Moreover, since the transverse background photon flux N_x^0 has different physics properties from the transverse PPF N_x^1 (Figs. 1 and 3(a)), N_x^0 might be removed through some special materials.[25–28]

The astigmatic GB in this study is collimated. In fact, there are many other forms of astigmatic Gaussian beam, such as the rotating astigmatic Gaussian beam, and anisotropic twisted Gaussian beam. They are likely to generate some new resonance effects to high frequency gravitational waves. We plan to investigate those effects in the near future.

In addition to the EM resonance of HFGWs in the laboratory, the interaction of the HFGWs with the galactic-extragalactic magnetic fields might appear in our galaxies and galaxy clusters,[11] which would generate possibly detectable perturbed EM waves. Due to its unique properties, such an EM signal can be captured by some astronomical observations. Since the properties of the perturbed EM waves are strongly dependent on the characteristics of the HFGW's source, the astronomical approach of using galactic-extragalactic magnetic fields and their interaction with HFGWs can be applied to find some potential HFGW origins and to test various cosmological models.

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