Theoretical Analysis of the Frequency Jump in E-fishbone Experiments *

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It is identified that barely passing electrons are the drive of the e-fishbones, rather than the barely trapped electrons at low frequency. The frequency jump in e-fishbone experiments is reproduced and analyzed. It is found that the e-fishbone frequency increases with the hot electron energy, which is consistent with the experiments. The growth rate of the mode ($m = 2$, $n = 2$) is greater than that of the mode ($m = 1$, $n = 1$).

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Fishbone instability excited by energetic electrons during electron cyclotron resonant heating (ECRH) is investigated on HL-2A. With high-power ECRH, periodic frequency jump phenomena are observed by soft x-ray arrays. Soft x-ray tomography shows that the poloidal and toroidal mode numbers are 1/1 and 2/2 with the frequency jump.

Fishbone instability has been experimentally studied in many tokamaks.[1–3] It was first observed in the PDX tokamak during nearly perpendicular neutral injection (NBI),[1] and then an instability excited by circulating energetic ions was also observed during tangential beam injection into the PBX tokamak.[4] The phenomena were explained by Chen *et al.*[5] and Betti *et al.*[6] successfully.

In the JET tokamak, during high-power ion cyclotron resonant heating (ICRH), a crash of a monster sawtooth abruptly changed the type of observed ion fishbone (i-fishbone) mode from a low frequency to a high frequency.[7] An instability driven by electron cyclotron current drive (ECCD) together with NBI was first found in DIII-D,[8] and then it was observed during electron cyclotron resonant heating (ECRH) only in HL-1M.[9,10] Experiments on HL-1M showed that fishbone instabilities can be excited by energetic electrons purely with off-axis ECRH and enhanced by lower hybrid current drive (LHCD).[9] The fishbone mode was also observed with LHCD only on the FTU and Tore Supra tokamaks.[13,14] Theoretical explanation of the electron fishbone first appears in the work by Wang *et al.*[11] The following theoretical explanations are in the articles by Wang *et al.*[12] and Zonca *et al.*[13]

An e-fishbone frequency jump has been observed on Tore Supra,[15] which is important for the redistribution of energetic electrons and energetic particle losses. New results about e-fishbone frequency jump phenomena with high-power ECRH on HL-2A are reported. In this Letter we present a theoretical base of the frequency jump in the e-fishbones experiment.

It is assumed that the modes are fluted near the $q = 1$ surface[5,6,9] and are driven by the barely circulating electrons which could be resonant with the modes. The dispersion relation given by Chen *et al.*[5] is used in the analysis. The MHD potential energy given by Wesson *et al.*[16] is employed. For the barely passing particles we calculate the kinetic contribution.

The plasma consists of two components. One is a relatively cold MHD part. The other is a hot particle component treated with the gyro-kinetics theory.

The Hamiltonian of a charged particle is given by

$$H = \frac{1}{2M}\Big[(P_R - eA_R)^2 + (P_z - eA_z)^2 + \frac{1}{R^2}(P_\varphi + e\psi)^2\Big] + e\Phi, \tag{1}$$

where A_R and A_z are the vector potential components of the magnetic field. ψ is the flux of the magnetic field, Φ is the electrical potential, M is the mass of the electron, e is the charge, and P_R, P_φ and P_z are the canonical momenta conjugate to R, φ and z, respectively.

$$P_R = Mv_R + \frac{e}{c}A_R, \tag{2}$$

$$P_\varphi = RMv_\varphi + \frac{e}{c}RA_\varphi, \tag{3}$$

$$P_z = Mv_z + \frac{e}{c}A_z, \tag{4}$$

$$\psi = -RA_\varphi, \tag{5}$$

where A_R, A_φ and A_z are the components of the magnetic vector potential.

The magnetic field can be expressed as

$$B = \nabla\varphi \times \nabla\Psi + I\nabla\varphi, \tag{6}$$

where Ψ is related to the poloidal flux of the magnetic field, I is related to the poloidal current, and R is the major radius. Then, in tokamaks we have

$$A_R = 0,\ A_z = -I\ln\frac{R}{R_0},\ A_\varphi = -\frac{\Psi}{R}. \tag{7}$$

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To derive the dispersion relation, a set of canonical variables is employed, which is rigorously derived from canonical transformation.[17]

We form a new invariant

$$\Pi = \frac{1}{2\pi}\oint P_x dX, \tag{8}$$

where P_x is a new momentum in the transformation conjugate to new coordinate X. New momenta Π, P_α, P_ζ are conjugate to η, α, ζ and $\frac{d\eta}{dt} = \omega_{\rm b}$, $\frac{d\zeta}{dt} = \omega_\zeta$, $P_\zeta = -\frac{e}{c}\Psi_0 = \Psi^*$, which are actually the position variable of the banana center.[18] The bounce frequency and the precession frequency are obtained from the Hamiltonian equation

$$\omega_{\rm b} = \frac{d\eta}{dt} = \frac{\partial H}{\partial \Pi} = \left(\frac{\partial \Pi}{\partial H}\right)^{-1}, \tag{9}$$

$$\omega_\zeta = \frac{d\zeta}{dt} = \frac{\partial H}{\partial \Psi^*} = -\left(\frac{\partial \Pi}{\partial \Psi^*}\right)\Big/\left(\frac{\partial \Pi}{\partial H}\right). \tag{10}$$

According to Eqs. (9) and (10), for the circulating particles, the bounce frequency and the precession frequency are obtained as follows:[12,19,20]

$$\Pi_{\rm c} = \frac{\Omega_0 M_0 r^2}{2} + \frac{2qR_0 M_0 v_\varphi \sigma}{\pi} E(k^2), \tag{11}$$

$$\omega_{\rm bc} = \frac{\pi v_\varphi \sigma}{2qR_0 K(k^2)}, \tag{12}$$

$$\omega_{\zeta{\rm c}} = q\omega_{\rm bc} + \omega_{\zeta 0}, \tag{13}$$

$$\omega_{\zeta 0} = \frac{u_\varphi^2}{2\Omega_{\rm p} r R_0}\left[\frac{E(k^2)}{K(k^2)} - \left(1 - \frac{k^2}{2}\right)\right] + \frac{v_\varphi^2 s}{\Omega_{\rm p} R_0^2}\frac{E(k^2)}{K(k^2)} = \frac{v_\varphi^2}{2\Omega_{\rm p} r R_0} G, \tag{14}$$

where q is a safety factor, σ represents the direction of the circulating particle, G is the normalized precession velocity of the circulating particle, and $E(k^2)$ and $K(k^2)$ are the complete elliptic integral of the second kind and the complete elliptic integral of the first kind. For the barely passing electrons, $q\omega_{\rm bc}$ could be larger than $\omega_{\zeta 0}$.

In the new coordinates η, α, ζ, in which Π and P_α are constants of motion, the gyrokinetic equation for the non-relativistic scenario is easily derived,[11]

$$\frac{\partial f_1}{\partial t} + \dot{\zeta}\frac{\partial f_1}{\partial \zeta} + \dot{\eta}\frac{\partial f_1}{\partial \eta} = -\frac{i}{T}(\omega - \omega^*) F_{\rm m} \delta H_1, \tag{15}$$

where $F_{\rm m}$ is Maxwellian with temperature T, f_1 the non-adiabatic part of the distribution function, $\dot{\zeta} = \omega_\zeta$, and $\delta H_1 = -\omega_\zeta \Omega_{\rm p} R_0 \xi$ is the perturbed Hamiltonian by the radial MHD displacement

$$\xi = \xi_0 e^{i(n\zeta - m\eta - \omega t)}. \tag{16}$$

It is indicated that only the change of guiding center position can induce the change of particle kinetic energy. Here ω^* is the electron diamagnetic drift frequency, m and n are the poloidal and toroidal mode numbers, respectively.

Equation (15) is derived in a complete set of canonical variables for both the trapped particles and circulating particles. For the circulating electrons we can sum Eq. (15) over σ. The solution of Eq. (15) is given as

$$f_1 = \frac{\omega - \omega^*}{\omega - n\omega_\zeta + m\omega_{\rm b}} \cdot \frac{F_{\rm m}\omega_\zeta \Omega_{\rm p} R_0 \xi}{T}, \tag{17}$$

Using Eqs. (16) and (17), we can calculate the kinetic energy

$$\delta W_k = \frac{1}{2}\int d\tau f_1 \delta H_1. \tag{18}$$

The coordinates P_α and Π are converted to P_α and v_φ, the Jacobian is

$$J = \frac{2qM_0 R_0}{\pi} E(k^2). \tag{19}$$

Using Eqs. (16)–(18), we obtain

$$\delta W_k = \frac{(2\pi)^3}{2}\int d\Psi^* dv_\varphi dP_\alpha J f_1 \delta H_1. \tag{20}$$

The magnetic moment is

$$P_\alpha = \frac{1}{2}\frac{v_{\rm t}^2}{\Omega_{\rm c}}, \tag{21}$$

where $\Omega_{\rm c}$ is the gyro-frequency at the guiding center $R_{\rm c}$, $v_{\rm t}$ is the thermal velocity, and T is the thermodynamic temperature.

We set some parameters as follows:

$$\omega_{b_0} = \frac{\pi v_{\rm t}}{2qR_0 K(k^2)}, \tag{22}$$

$$a = \frac{\omega}{n\omega_{b_0}\left(q - \frac{m}{n}\right)}, \tag{23}$$

$$x = \frac{v_\varphi}{v_{\rm t}}, \tag{24}$$

$$y = \frac{v_\perp}{v_{\rm t}}, \tag{25}$$

where v_φ is the toroidal velocity, and $v_\perp$ is the perpendicular velocity. The Maxwell distribution can be written as

$$F_{\rm m} = N\left(\frac{M_0}{2\pi kT}\right)^{3/2} e^{-(x^2+y^2)}, \tag{26}$$

where N is the density of the particles, and M_0 is the mass of the particles.

Thus we obtain

$$\delta H_1 = -q\frac{\omega_{\rm b}}{\omega_{b_0}}\omega_{b_0}\Omega_{\rm p} R_0 \xi = q\omega_{b_0}\Omega_{\rm p} R_0 \xi x, \tag{27}$$

$$\delta W_k = \frac{(2\pi)^3}{2}\left(\frac{M_0}{2\pi kT}\right)^{3/2}\iiint \Omega_{\rm p} R_0 dr dy 2dx \cdot \frac{(v_{\rm t})^3 E(k^2)}{2\Omega}\cdot\frac{2qR_0}{\pi}\cdot\frac{\omega * \omega_{b_0}(q\Omega_{\rm p} R_0 \xi)^2}{nT(q - \frac{m}{n})} \cdot [(x+a) + \frac{a^2}{x-a}]e^{-(x^2+y^2)}, \tag{28}$$

where

$$\omega^* = \frac{NT}{\Omega_{\rm p} R_0 M_0 L_n},$$
$$L_n = \left(-B_{\rm p} R_0 \frac{\partial}{\partial \Psi}(\ln(f_{\rm m}))\right)^{-1}.$$

For the barely passing particles and the flute modes, we fix k and q, then we obtain

$$\delta W_k = 4\sqrt{\pi} \int r dr \frac{E(k^2)\omega_{b_0}\Omega_{\rm p}}{L_n} \cdot \frac{N(R_0 q\xi)^2}{n(q-\frac{m}{n})}[1+aZ(a)]\sqrt{\pi}a, \quad (29)$$

and

$$\delta \hat{W}_k = 4\pi\beta_{\rm h}\frac{a(1+aZ(a))}{q-\frac{m}{n}}, \quad (30)$$

where $Z(a)$ is the dispersion function, and

$$\beta_{\rm h} = \frac{2}{\varepsilon}\int z^2 dz \frac{NTr_a}{L_n B^2 \rho_{\rm L}} \cdot \frac{E(k^2)}{K(k^2)}, \quad (31)$$

where $z = \frac{r}{r_a}$, r_a is the minor radius at the resonant surface, $\rho_{\rm L}$ is the Larmor radius, and ε is the inverse aspect ratio.

The dispersion relation is given[5,21]

$$-i\frac{\hat{\omega}}{\omega_{\rm A}} + \delta\hat{W}_k + \delta\hat{W}_{\rm f} = 0. \quad (32)$$

The MHD potential energy $\delta W_{\rm f}$ in Ref. [16] is

$$\delta W_{\rm f} = \frac{\pi^2 B_\varphi^2}{\mu_0 R}\int_0^{r_a}\left\{\left[\left(r\frac{d\xi}{dr}\right)^2 + (m^2-1)\xi^2\right] \cdot \left(\frac{n}{m}-\frac{1}{q}\right)^2 r dr\right\} + \left[\frac{2}{q_a}\left(\frac{n}{m}-\frac{1}{q_a}\right) + \left(\frac{n}{m}-\frac{1}{q_a}\right)^2\right]a^2\xi_a^2, \quad (33)$$

where μ_0 is the vacuum permeability, and B_φ is the poloidal magnetic field.

If $q_a = 1$ and $\xi = r^m$, we can obtain

$$\delta\hat{W}_{\rm f} = 2\pi^2\frac{2m^2-1}{m+1}\left(\frac{n}{m}-\frac{1}{q}\right)^2. \quad (34)$$

Substituting Eqs. (30) and (32) into Eq. (34), we obtain the dispersion relation

$$-i\frac{\omega}{\omega_{\rm A}} + 4\pi\beta_{\rm h}\frac{a(1+aZ(a))}{n\left(q-\frac{m}{n}\right)} + 2\pi^2\frac{2m^2-1}{m+1}\left(\frac{n}{m}-\frac{1}{q}\right)^2 = 0, \quad (35)$$

where $\omega_{\rm A} = \frac{V_{\rm A}}{R_0 q}$, and $V_{\rm A}$ is the Alfven velocity.

HL-2A is a medium-sized tokamak (major radius $R_0 = 1.65$ m and minor radius $a = 0.40$ m) with a circular cross section,[22] which performs with the plasma current $I_{\rm p} = 100$–350 kA, the line-averaged electron density $n_{\rm e} = 0.2$–4.0×10^{19} cm^{-3} and the toroidal magnetic field $B_{\rm t} = 1.0$–2.5 T. In the process of calculation the k used in the complete elliptic integral is near to 1. The ECRH system consists of four 68 GHz/500 kW/1 s and two 68 GHz/500 kW/1.5 s gyrotrons provide ECRH power (PECRH) up to 3.0 MW. The ECRH power is injected into the plasma from the low-field side (LFS) with O-mode or X-mode in the perpendicular direction.[23,24] The line-averaged electron density is measured by a hydrogen cyanide (HCN) interferometer. The evolution of the e-fishbone behavior is detected by a 100-channel soft x-ray multi-camera system and a Mirnov coil system, which is composed of 10 toroidal Mirnov coils installed in the LFS and 18 poloidal Mirnov coils.

Solutions to the dispersion relation are obtained by using a non-linear complex equation solver of a mathematical calculation software.

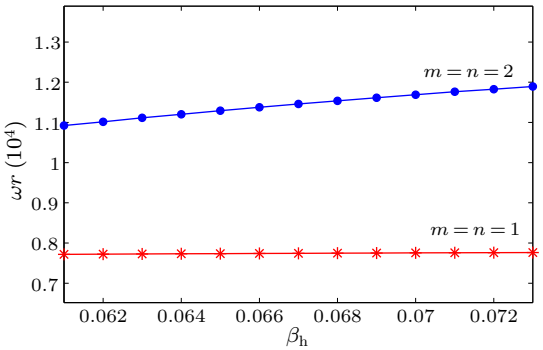

Fig. 1. The real part of the frequency $\omega_{\rm r}$ versus $\beta_{\rm h}$.

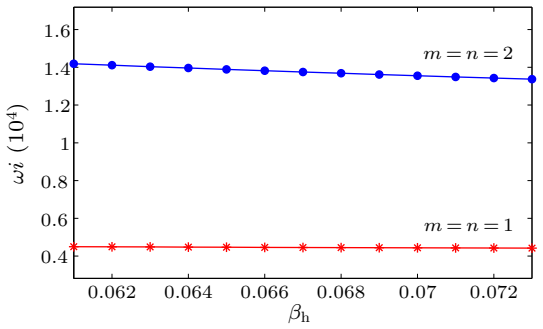

Fig. 2. The imaginary part of the frequency $\omega_{\rm i}$ versus $\beta_{\rm h}$.

We calculate two cases: the first case is m=n=1, and the second case is m=n=2. The calculated results are shown in Figs. 1 and 2. Figure 1 shows the real component of frequency $\omega_{\rm r}$ versus $\beta_{\rm h}$. Figure 2 exhibits the imaginary component of frequency $\omega_{\rm i}$ versus $\beta_{\rm h}$. In Fig. 1, if m and n are equal to 2, the real frequency $\omega_{\rm r}$ increases with $\beta_{\rm h}$, which is consistent with the experiments. The real frequency $\omega_{\rm r}$ of the (1/1) mode also increases with $\beta_{\rm h}$, while the degree of increase is smaller than that of the (2/2) mode. There are differences between the (2/2) mode and the (1/1) mode since the topologies of the modes (2/2) and (1/1) are different.

Figure 3 confirms the existence of the two modes,

which is in agreement with the theory. The e-fishbone frequency increases with the hot electron energy, which is compatible with the variation of data observed in the experiments.[25] Figure 3 describes the frequency jump phenomena with 1.56 MW ECRH power. The data in Fig. 3 come from the HL-2A experiments. With the soft x-ray tomography the modes in Fig. 3 are identified as (2/2) and (1/1) modes.

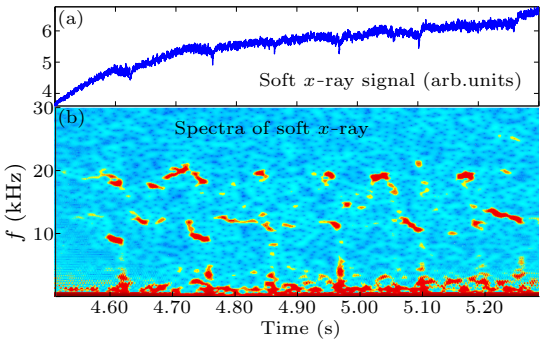

Fig. 3. Fishbone jump phenomena appearing during 1.56 MW on-axis ECRH on shot 18146, (a) soft x-ray signal, and (b) spectra of the soft x-ray.

It is found that the $(2, 2)$ mode and the $(1, 1)$ mode exist with periodical frequency jump phenomena in the nonlinear process during the long time. In the next step we will carry out the further work to explain the periodical frequency jump phenomena in the nonlinear process.

In summary, a set of canonical variables are employed to give a clear picture of the particle dynamics. Looking at a series of data, it is found that the calculated frequency jumps between 7 and 13 kHz are consistent with the experimental observation[25] in the vicinity of the $q = 1$ surface. It is also found for the first time that the increase of the frequency also increases with the energy of the particles.

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