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Modulation of Void Motion Behavior in a Magnetized Dusty Plasma *

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Based on fluid equations, we show a time-dependent self-consistent nonlinear model for void formation in magnetized dusty plasmas. The cylindrical configuration is applied to better illustrate the effects of the static magnetic field, considering the azimuthal motion of the dusts. The nonlinear evolution of the dust void and the rotation of the dust particles are then investigated numerically. The results show that, similar to the unmagnetized one-dimensional model, the radial ion drag plays a crucial role in the evolution of the void. Moreover, the dust rotation is driven by the azimuthal ion drag force exerting on the dust. As the azimuthal component of ion velocity increases linearly with the strength of the magnetic field, the azimuthal component of dust velocity increases synchronously. Moreover, the angular velocity gradients of the dust rotation show a sheared dust flow around the void.

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Dusty plasmas are formed when submicron- and micron-sized dust grains are immersed and charged in fully or partially ionized low-temperature gases. Dust and dusty plasmas are ubiquitous in space, such as in interstellar media, in planetary rings, in cometary tails, in supernova remnants, and in the lower ionosphere of the earth.[1]

The massive and charged dusts have a great influence on the collective behaviors of dusty plasmas, in which the dusts interact with electrons, neutral gas molecules and ions. Dust particles usually acquire a negative charge due to collection of background plasma electrons whose thermal speed is much higher than that of ions. The damping force resulting from collisions with neutral gas molecules can cause a deceleration of dust particles. Moreover, the ion drag force due to the momentum transfer from flowing ions to charged grains often determines the static and dynamical properties of dusts, which is responsible for a rich variety of phenomena occurring in the system, including dust voids, mach cones, dusty plasma crystal and dust lattice waves, etc.[2–4] The external plasma parameters influence the motion of dusts by affecting the ion flow, and ultimately have an impact on the whole system. For instance, the rotation of dust structures in the presence of a magnetic field is due to the drag force of the azimuthal ion flow under the magnetic field.[5–9] The ion drag force is still a subject of intensive research activity since a complete self-consistent model for its role in those collective behaviors in dusty plasmas is few.

A dust void is a dust-free region in the central part of the discharge where the dusty plasma has been generated. Since the void has been observed in many dusty plasma experiments, in particular under microgravity conditions, extensive research about its evolution has been carried out both experimentally and theoretically. Avinash *et al.* proposed a time-dependent model for void formation process in a dusty plasma, which is initiated by an instability caused by the ion drag, and then reaches a saturation state with a void by ion-neutral collisions.[10] In recent years, dust voids in magnetized dusty plasmas have also been found, as well as a slow rotation of the dust cloud around the void in the azimuthal direction in a plane perpendicular to the magnetic field. Huang *et al.* investigated the structure and dynamics of dust rotation under a magnetic field at different gas pressures in the sheath of a radio-frequency discharge and found the formation of a void in the dust cloud at a sufficiently high angular velocity.[11–13] Pilch *et al.* reported the formation of a torus-shaped dust cloud with a void in the central region, which are confined in a magnetized anodic plasma. The particles confined in the torus are rotating in the $E \times B$ direction of the ions.[14,15] Several models have been proposed to explain these phenomena. Reichstein *et al.* proposed a model to describe the torus confinement of dust particles by means of a force balance,[16] then developed a one-dimensional N-particle model with strong radial confinement to

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account for inertia and many-body effects,[17] and further extended it to a three-dimensional one to describe dust streaming in a toroidal trap with weak radial confinement by means of molecular dynamics (MD) simulations.[18–20] They found that the essential force for particle motion is the azimuthal component of ion drag force, which determines the particle velocity as well as the onset of particle rotation, and that friction with neutral gas has a crucial influence on the rotation characteristics. Kaw *et al.* gave an explanation of the rigid rotation of dust in a static magnetic field by using fluid equations.[6] Land *et al.* used a 2D fluid model to solve the transportation of dust particles in the presence of a homogeneous magnetic field.[21] Jovanović *et al.* made a theoretical prediction of a dust void in association with a double layer in a magnetoplasma and concluded that the ion drag force on charged dust grains does not play an important role in the creation of dust voids.[22] Bai *et al.* observed a rotating-void behavior in complex plasmas investigated in a cylindrical system with an axial symmetry,[23] Farokhi *et al.* further observed the void behavior in magnetized dusty plasmas.[24,25]

In this Letter, we propose a time-dependent self-consistent nonlinear model for void formation in a magnetized dusty plasma in the form of a cylindrical symmetry, and study in detail the motion behavior of the dusts rotating in the static magnetic field. We find that the dusts around the void rotate around the axis of the field due to the azimuthal ion drag force, also a sheared rotation of dust particles is observed. The azimuthal ion drag force results from the azimuthal ion flow in the presence of the magnetic field. These results are in agreement with existing experiments. In particular, we find that the radial ion drag force on charged dust grains still plays an important role in the creation of the dust void in the presence of the magnetic field, which contradicts the conclusion in Ref. [22].

For void formation in magnetized dusty plasmas, we extend the nonlinear time-dependent model proposed by Avinash *et al.* in Ref. [10] to be in an external magnetic field. Moreover, we adopt cylindrical coordinates rather than Cartesian coordinates. In the three-dimensional cylindrical coordinates $\boldsymbol{R} = \{\rho, \varphi, z\}$, we assume a uniform distribution in the z-direction, which means $\partial/\partial z = 0$, and an approximation of axial symmetry.

For electrons, the inertial terms can be ignored due to the slow timescale of dusts. Thus their momentum equation is

$$en_{\rm e}\boldsymbol{E} + T_{\rm e}\nabla n_{\rm e} = 0, \tag{1}$$

where $-e$, $n_{\rm e}$ and $T_{\rm e}$ are the electron charge, density and temperature, respectively, and $\boldsymbol{E}$ is the electric field.

Also for ions, the inertial terms can be safely ignored on the slow dust timescale. In addition, when we focus on the relatively extremely slow timescale of dusts, we can assume that the ion density adjusts quickly and attains a quasi-steady constant.[10] This assumption implicitly contains an ionization mechanism and is consistent with the condition of Avinash's model. The external magnetic field has an influence on the ion momentum equation. The ion density and velocity then satisfy approximately

$$n_{\rm i} = \text{const}, \tag{2}$$

$$e\boldsymbol{E} - \nu_{\rm in} m_{\rm i}\boldsymbol{u}_{\rm i} + e\boldsymbol{u}_{\rm i} \times \boldsymbol{B} = 0, \tag{3}$$

where $m_{\rm i}$, $n_{\rm i}$, and $\boldsymbol{u}_{\rm i}$ are the ion mass, density and fluid velocity, respectively, $\nu_{\rm in}$ is the ion-neutral collision frequency, and $\boldsymbol{B}$ is the magnetic field.

The dust continuity and momentum equations can be written as

$$\frac{\partial n_{\rm d}}{\partial t} + \nabla \cdot (n_{\rm d}\boldsymbol{u}_{\rm d}) = D\nabla \cdot (\nabla n_{\rm d}), \tag{4}$$

$$m_{\rm d}\frac{\partial \boldsymbol{u}_{\rm d}}{\partial t} = -Ze\boldsymbol{E} + \boldsymbol{F}_{\rm d} - \nu_{\rm dn} m_{\rm d}\boldsymbol{u}_{\rm d} - \frac{T_{\rm d}}{n_{\rm d}}\nabla n_{\rm d}, \tag{5}$$

where the dusts are unmagnetized and the convective nonlinearity in the dust equation has been ignored, $n_{\rm d}$, $-Ze$, $m_{\rm d}$, $\boldsymbol{u}_{\rm d}$, and $T_{\rm d}$ are the number density, charge, mass, fluid velocity and temperature of dust particles, respectively, D is a particle diffusion coefficient for dust, $\nu_{\rm dn}$ is the dust-neutral collision frequency, and $\boldsymbol{F}_{\rm d}$ is the ion drag force on the dusts which is approximated to

$$\boldsymbol{F}_{\rm d} = \frac{m_{\rm d}\nu_{\rm di}\boldsymbol{u}_{\rm i}}{b + \left(\frac{\boldsymbol{u}_{\rm i}}{u_{\rm thi}}\right)^3}.$$

Finally the self-consistent Poisson's equation is

$$\nabla \cdot \boldsymbol{E} = 4\pi e(n_{\rm i} - n_{\rm e} - Zn_{\rm d}). \tag{6}$$

We apply a static uniform magnetic field which has only the axial component, $\boldsymbol{B} = \{0, B_z\}$. The electric field is thereby a function of $\boldsymbol{\rho}$. Our model Eqs. (1)–(6) can be written as

$$\frac{\partial n_{\rm e}}{\partial \rho} = -\frac{n_{\rm e}E}{\tau_{\rm i}}, \tag{7}$$

$$n_{\rm i} \equiv 1, \tag{8}$$

$$\mu E - u_{{\rm i},\rho} + u_{{\rm i},\varphi}B\beta_{\rm i} = 0, \tag{9}$$

$$u_{{\rm i},\varphi} + u_{{\rm i},\rho}B\beta_{\rm i} = 0, \tag{10}$$

$$\frac{\partial n_{\rm d}}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho n_{\rm d}u_{{\rm d},\rho}) = D\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\frac{\partial n_{\rm d}}{\partial \rho}), \tag{11}$$

$$\frac{\partial u_{{\rm d},\rho}}{\partial t} = -E + \frac{au_{{\rm i},\rho}}{b + u_{{\rm i},\rho}^3} - a_0 u_{{\rm d},\rho} - \frac{\delta}{n_{\rm d}}\frac{\partial n_{\rm d}}{\partial \rho}, \tag{12}$$

$$\frac{\partial u_{{\rm d},\varphi}}{\partial t} = \frac{au_{{\rm i},\varphi}}{b + u_{{\rm i},\varphi}^3} - a_0 u_{{\rm d},\varphi}, \tag{13}$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E) = (1 - n_{\rm e} - n_{\rm d}), \tag{14}$$

where we have introduced the following dimensionless variables:

$$\frac{Zn_{\rm d}}{n_{\rm i0}} = \hat{n}_{\rm d},\ \frac{\tau_{\rm i}\sqrt{\varepsilon}u_{\rm d}}{\lambda_{\rm di}\omega_{\rm pd}} = \hat{u}_{\rm d},\ \frac{u_{\rm i}}{u_{\rm thi}} = \hat{u}_{\rm i},\ \frac{n_{\rm e}}{n_{\rm i0}} = \hat{n}_{\rm e},$$
$$\frac{\lambda_{\rm di}eE}{T_{\rm e}} = \hat{E},\ \frac{\tau_{\rm i}\rho}{\lambda_{\rm di}} = \hat{\rho},\ \frac{\omega_{\rm pd}t}{\sqrt{\varepsilon}} = \hat{t},\ \frac{B}{B_0} = \hat{B},$$
$$\tau_{\rm i} = \frac{T_{\rm i}}{T_{\rm e}},\ \varepsilon = \frac{Zn_{\rm d0}}{n_{\rm i0}},\ \lambda_{\rm di} = \sqrt{\frac{T_{\rm i}}{4\pi n_{\rm i0}e^2}},$$
$$\omega_{\rm pd} = \sqrt{\frac{4\pi n_{\rm d0}Z^2e^2}{m_{\rm d}}},\ u_{\rm thi} = \sqrt{\frac{T_{\rm i}}{m_{\rm i}}},$$

with

$$D_0 = \frac{D\tau_{\rm i}^2\sqrt{\varepsilon}}{\lambda_{\rm di}^2\omega_{\rm pd}},\ a_0 = \frac{v_{\rm dn}\sqrt{\varepsilon}}{\omega_{\rm pd}},\ \delta = \frac{\tau_{\rm i}\tau_{\rm d}}{Z},\ \mu = \frac{\omega_{pi}}{v_{\rm in}\tau_{\rm i}},$$
$$\omega_{pi} = \sqrt{\frac{4\pi n_{\rm i0}e^2}{m_{\rm i}}},\ \tau_{\rm d} = \frac{T_{\rm d}}{T_{\rm e}},\ \beta_{\rm i} = \frac{eB_0}{\nu_{\rm in}m_{\rm i}},\ a = \frac{m_{\rm d}\nu_{\rm di}\tau_{\rm i}}{Zm_{\rm i}\omega_{\rm pd}}.$$

In these equations, $\lambda_{\rm di}$ is the ion Debye length, $\omega_{\rm pd}$ is the dust plasma frequency, $u_{\rm thi}$ is the ion thermal velocity, and $\beta_{\rm i}$ is the Hall parameter of the ion. Thus we obtain a fluid model for dust-void formation in a static magnetic field.

Our model offers separate views of radial and azimuthal directions in a cylindrical coordinate with an axial symmetry. We consider the evolution of the dust system by numerically integrating the governing Eqs. (7)–(14). Assuming an open boundary for the simulation domain, these equations are evolved from a homogeneous field-free equilibrium with $E = 0$, $n_{\rm d} = 0.005$, and $u_{\rm d} = 0$. For the parameters relevant for the microgravity experiment, we obtain $a = 5$, $b = 1.6$, $a_0 = 2$, $\delta \approx 0.001$, $\mu = 1.5$, $D_0 \approx 10^{-2}$, and $\tau = 0.125$.

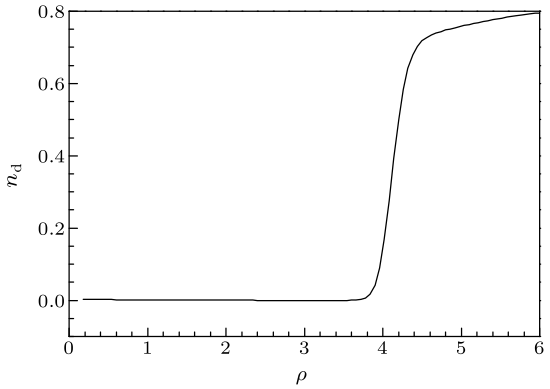

Fig. 1. The dimensionless dust density distribution in the saturated state of the instability in a static magnetic field. Note the sharp boundary of the void region at $\rho = 4$.

Firstly, we focus on the evolution of dust density. The numerical results show that the homogeneous dust density evolves rapidly after the growth of the instability to a sharp-boundary void in the magnetic field. Figure 1 shows the dust density distribution in a static magnetic field when the system attains a steady state, which is similar to the results in the unmagnetized one-dimensional Cartesian coordinates in Ref. [10]. This means that in the radial direction, the inward electrical force and the outward ion drag force on negatively charged dust particles make the initial uniform equilibrium dust density evolve to a void with a sharp boundary, without obvious effects of the static magnetic field.

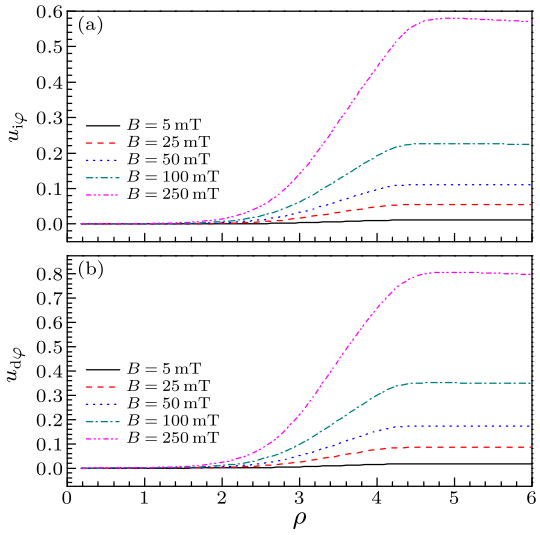

Fig. 2. The dimensionless azimuthal components of ion velocity (a) and dust velocity (b) distributions in different strengths of the magnetic field.

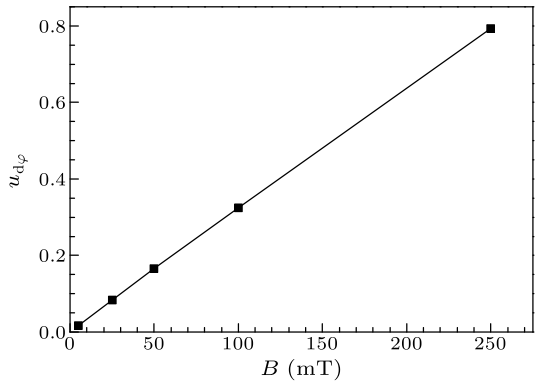

Fig. 3. The dimensionless azimuthal component of velocity of dusts in the dust cloud around the void in different strengths of the magnetic field.

Then we investigate the rotation of dusts. Figure 2 shows the distribution of azimuthal components of ion velocity and dust velocity respectively in different magnetic field strengths. In the azimuthal direction, the azimuthal component of ion drag force pushes the dusts rotating around the axis of the magnetic field, which means that the rotation of dusts is caused by the drag of azimuthal ion flow. Meanwhile, the azimuthal velocity of the ion is proportional to the strength of the magnetic field, as a consequence, the azimuthal velocity of dusts also increases with the field strength. Figure 3 shows the azimuthal velocity

of dusts in the dust cloud around the void increases linearly with the strength of the magnetic field from $B = 5\,\mathrm{mT}$ to $B = 250\,\mathrm{mT}$. Also, these two velocities will change to the opposite direction if the direction of the magnetic field flips. These results are consistent with the experimental observations.

To better illustrate the rotation of dusts, we calculate the angular velocity of dust rotation. In Fig. 4, there is a radial distribution in the angular velocity of dusts around the dust void at $\rho > 4$, which means a sheared rotation, as described in the experiments mentioned above.[5,13,20] This is due to the fact that the dusts near the void suffer from sheared azimuthal ion drag, which is caused by the azimuthal ion flow induced by the magnetic field. We also find that the rotation slows down when ν_{in} increases, which is also consistent with the experimental observation that the angular velocity of the rotating dust particles decreases with increasing the gas pressure.[13]

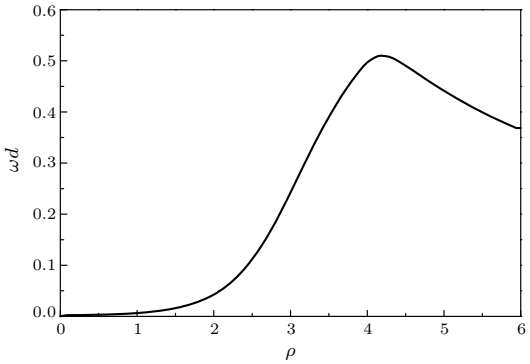

Fig. 4. The angular velocity of dust rotation, which is normalized by $2\pi\omega_{\mathrm{pd}}/\sqrt{\varepsilon}$. Here $B = 100\,\mathrm{mT}$ is applied according to the experiments.

In summary, we have presented a nonlinear time-dependent model that describes the evolution of dust voids and motion behavior of dusts in external magnetic fields. The model reveals that the void can still form in a static magnetic field, while the dusts around the void rotate due to the magnetic field. The ion drag force plays a crucial role in these processes. The radial component of the ion drag force balances the electric force on the dusts, leading to the formation of voids. While the azimuthal component of the ion drag force, increasing with the strength of the magnetic field, pushes the dusts rotating at the same direction with angular velocity gradients. Due to the fact that our model demonstrates an independence of the dust motion in radial and azimuthal directions, we beg to differ from the conclusion of the model in Ref. [22] stating that 'the ion drag force on charged dust grains is in the poloidal direction, and thus it cannot play an important role in the creation of this new type of dust void in a magnetized dusty plasma'. The ion drag force on charged dust grains has components in both the radial and azimuthal directions and thus it plays an important role in the creation of the dust void in a magnetized dusty plasma as well as the sheared rotation of the dusts.

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