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Smart Rewiring: Improving Network Robustness Faster *

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Previous work puts forward a random edge rewiring method which is capable of improving the network robustness noticeably, while it lacks further discussions about how to improve the robustness faster. In this study, the detailed analysis of the structures of improved networks show that regenerating the edges between high-degree nodes can enhance the robustness against a targeted attack. Therefore, we propose a novel rewiring strategy based on regenerating more edges between high-degree nodes, called smart rewiring, which could speed up the increase of the robustness index effectively. The smart rewiring method also explains why positive degree-degree correlation could enhance network robustness.

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Recently, the focus of modern network science shifts to find the basic universal principles that govern how structural properties of complex networks influence its functionalities, such as virus spread,[1] rumor spread,[2] random walker,[3] synchronization[4,5] and controllability.[6,7] With the widespread use of infrastructure networks in daily life, i.e., electric power grids,[8] transportation systems,[9] communication networks[10] and the internet,[11] maintaining their efficiency is a matter of the utmost importance. However, the networked systems are always confronted with the potential risks of node attacks or failure, due to the fact that the destruction of a few individuals affects the normal functions of networks and systems. In particular, cascading failure and intentional attacks can easily damage its functions, such as network controllability[12–14] and connectivity.[15] Researchers have realized that simple modifications of the network topology, such as the connection pattern of nodes through links, have been shown to be an effective way to increase the robustness of connectivity under node or link attacks.[15–20] However, these methods improve network robustness by random edge rewiring, which could cause a great deal of extra cost to increase the robustness. Hence, we need to find a smart rewiring method to speed up the increased rate of robustness rather than random edge rewiring.

In real systems, when a fraction of power stations in the electricity network is malfunctioning due to either node cascade failures or malicious attacks, the whole network may be paralyzed due to the fact that it is broken into several isolated parts. Therefore, the robustness of networks means that it has the ability to mitigate the damage of connectivity so that it still has considerable connected components in the network after several malicious attacks. To avoid such a dramatic break-down and to reduce the fragility of complex networks, a procedure proposed by Schneider *et al.*[15] successfully optimizes the robustness of a network against malicious attacks by exchanging only a small number of links. We also use the robustness index R to assess the evolution of connectivity with the node removal, which is defined as

$$R = \frac{1}{N}\sum_{q=1/N}^{1} s(q), \tag{1}$$

where $s(q)$ is the fraction of the largest strong connected component in the remaining network after removing $q \times N$ nodes by malicious attack. We monitor the size of the largest strong connected component to estimate how the network function gets destroyed after some 'important' nodes are removed step by step. Although there are many methods to measure the 'importance' of nodes, we choose the most used maximal degree as the measure of node importance. Malicious attack means that the most important node is deleted at each attack step. Therefore, the largest strong connected component in the remaining network can evaluate the network robustness. If R is larger, it means that the network is more robust. The range of possible R values is between 0 and 0.5. When $R = 0$, it means that all the nodes in the network are completely isolated after removing the first important nodes, such as in the star network. If $R = 0.5$, it must meet two conditions: (i) the original network approaches to fully connected; (ii) the network only increases one isolated node at each attack step (only removing one node). Thus $R = 1/N\sum_{q=1/N}^{1} q = 1/N(1/N + \ldots + N/N) = (N+1)/2N \approx 0.5$.

To optimize the network robustness R, we use a simple hill-climbing search to modify a network topology. Note that the restriction to keep the degree of each node fixed is always important and is of real sig-

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nificance in the modification of artificial systems, for example, although adding links without any restrictions until the network is fully connected would enhance the robustness, this option must be useless for practical purposes due to the fact that the installation of power lines between each pair of power plants would skyrocket costs and sometimes be impractical. In this work, we follow the restriction to fix the degree of each node when swapping the links. Hence, in the original network, we swap the connections of two randomly chosen links, that is, the links e_{ij} and e_{kl} become e_{ik} and e_{jl} respectively, only if the robustness is increased, i.e., $R_{\rm new} > R_{\rm old}$. During the process of edge swapping, we perform a simple greedy choice: at each step we compare the robustness before and after the swap, and consider it as a successful step if the robustness has improved. If unsuccessful, the swap is reverted and another swapping step is tested. We repeat this procedure until no further substantial improvement is achieved for a large number of consecutive swapping trials. Note that any change of the network usually leads to an adjustment in the attack sequence. Previous work implied that this simple random rewiring method is capable of improving network robustness noticeably.[15,20]

To deeply understand the structures of improved networks, Fig. 1(a) indicates the fraction of remaining edges after removing $q \times N$ important nodes. We find that more edges remain in the improved network than those in the initial networks after each attack, which can help the improved network to maintain the remaining network's connectivity as much as possible. In fact, the number of nodes' degree cannot change after edge swapping due to the restriction of the rewiring edge. However, the phenomenon of why the improved network takes advantage of the remaining edges after removing the 5% high-degree nodes, can be explained that the connections among high-degree nodes in the improved networks are more strengthened than those of the initial networks after edge modifications. That is, in the improved networks, the high-degree nodes have a preference to attach to high-degree nodes. To test our hypothesis of improved networks' structure, the change of nodes' clustering coefficient and distribution of regenerated edges for the initial and improved networks are presented in Fig. 1.

Since the clustering coefficient is calculated as the number of closed triplets divided by the number of connected triplets (both open and closed), more edges among nodes, forming more close d triplets with high probability, may result in the increase of the clustering coefficient. Thus if the nodes' clustering coefficient of high-degree nodes in the improved network is higher than that of the corresponding nodes in the initial network, it indicates that there are more edges recreated among high-degree nodes. Figure 1(b) displays the nodes' clustering coefficient from the node with maximum degree to minimum degree, which shows the nodes' clustering coefficient in the improved networks: (i) clustering coefficient of high-degree nodes has a remarkable growth; (ii) clustering coefficient of low-degree nodes decreases noticeably. The results prove that the connections among high-degree nodes in the improved networks are more strengthened than those of the initial networks after edge modifications. The noticeable decrease of the clustering coefficient occurs in $k = 2$ or 3. Obviously, the edges, forming triangles in the original networks, are broken during the edge swapping. Note that when $k = 1$, the node's clustering coefficient is 0 for both initial and improved networks.

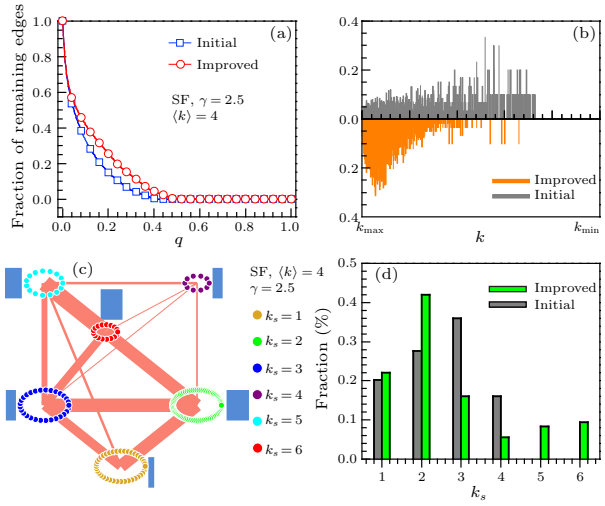


Fig. 1. (a) Plot of the fraction of remaining edges as a function of the fraction q of removed nodes for SF models of $\langle k \rangle = 4$ and $\gamma = 2.5$. (b) Stack bar of the each node's clustering coefficient from maximum degree to minimum degree for initial networks (grey bar) and improved networks (orange bar). The tested networks are the SF model with $\langle k \rangle = 4$ and $\gamma = 2.5$. The blank space of the Stack bar means that the clustering coefficient is 0 due to the fact that these nodes' degree is 1. (c) Sketch of the k-shell decomposition for an SF network with $\langle k \rangle = 4$ and $\gamma = 2.5$. Six colors distinguish six components of k-shells. The light red lines indicate the recreated edges connect with nodes belonging to two different k-shell components, while the light blue lines imply the regenerated edges connect with nodes from one internal component. Its width indicates the number of these regenerated edges. (d) Comparison of the faction of nodes belonging to different k-shells between the initial (grey bar) and improved network (green bar). The initial network only has 4 k-shells, while the improved network contains 6 k-shells.

On the other hand, the distribution of regenerated edges is presented in Fig. 1(c) through the k-shell components.[21–23] For example, Fig. 1(c) shows an improved SF network, in which $N = 200$, $\langle k \rangle = 4$ and $\gamma = 25$ are used in the greedy leaf removal procedure to decompose into 6 k-shell components. Here each oval component contains the set of vertices belonging to a given k-shell, while colors on the vertices distinguish different k-shells. The light red lines indicate the recreated edges connected with nodes belonging to two different k-shell components, while the light blue lines imply the regenerated edges connected with

nodes from one internal component. Its width indicates the number of these regenerated edges. Thus the width of these light lines demonstrates that most of these edges are regenerated between two different high k-shells, or one internal high k-shell. In Fig. 1(d), we compare the fraction of nodes belonging to different k-shell components in the initial and improved networks. The result clearly shows that the improved network contains 6 k-shell components, while the initial network only has $4k$ shells. This result could be explained that most of these regenerated edges connect with the high-degree nodes and finally compose the new higher k-shell component. In a word, the change of nodes' clustering coefficient and distribution of regenerated edges prove that there are more regenerated edges among the high-degree nodes after the optimization procedure.

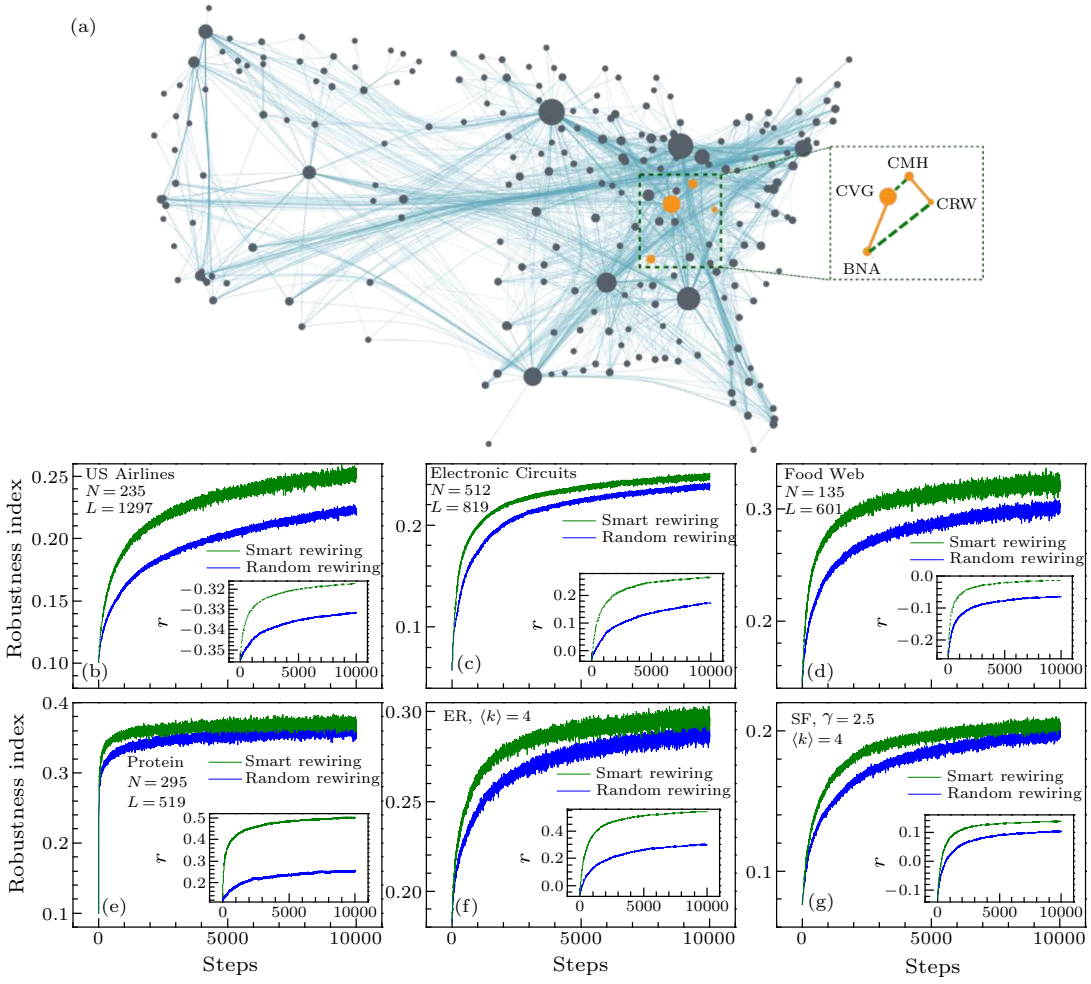


Fig. 2. (a) Example of the smart rewiring applied to a US air-transportation network ($N = 235$, $L = 1297$). Each node represents an airport and each edge indicates the airline between two airports. The scale of a node on the map relies on the number of its airlines. The inset displays the procedure of edge swapping according to the rule of smart rewiring. Connections CVG to CMH and BNA to CRW are added (dashed edges), while previous links from CVG to BNA and CMH to CRW are removed (solid edges). The Comparison of robustness index between smart rewiring and random rewiring for (b) the US airlines network, (c) the electronic circuits network, (d) the food web network, (e) the protein network, (f) ER networks with $\langle k\rangle$ and (g) SF networks with $\langle k\rangle = 4$ and $\gamma = 2.5$. The insets of (b)–(g) compare the degree-degree correlation between smart rewiring and random rewiring.

The edge swapping keeps the number of links and the degree of the nodes unchanged, and is capable of enhancing the robustness of network connectivity against malicious attacks. According to the above analysis, the improved networks tend to regenerate the edges among high-degree nodes. Therefore, we improve the random swapping method based on regenerating more edges between high-degree nodes. Here we propose a novel rewiring strategy, called smart rewiring, which could speed up the increase of R. The smart rewiring is defined as follows: (i) select randomly two edges $e_1 = (v_1, w_1)$ and $e_2 = (v_2, w_2)$, where two edges do not share a common node; (ii) computer the degree of four nodes, labeled as d_{v1}, d_{w1}, d_{v2} and d_{w2}. Then, rank the node's degree in descending order; (iii) remove the original edges e_1 and e_2; (iv) if the first two ranked nodes and the last two ranked nodes are not yet connected, then two new

edges between them will be created; (v) if the condition of step (iv) cannot be met, we can repeat step (i).

An illustration of this strategy is provided in the inset of Fig. 2(a). Here the four nodes represent four airports in US. The degrees of four nodes are ranked as $d_{\mathrm{CVG}} > d_{\mathrm{CMH}} > d_{\mathrm{BNA}} > d_{\mathrm{CRW}}$ in descending order. Due to the rule of smart rewiring, we need to remove the original edges (solid lines) and create two new edges (dashed lines) based on the ranked order of degree. Actually, the smart rewiring, in each swapping step, connects two nodes with the top two degrees among four selected nodes. Some simple examples of smart rewiring are presented in Fig. 2 for several real and model-generated networks. Especially for the US air-transportation network, although the fraction of the largest connected component of the original airlines network account for 100%, the robustness index R of the original network is 0.10. Though the network has better connectivity, the robustness against malicious attacks is weak. This is due to the fact that the degree sequence of the airlines network is of typical scale-free property. In Fig. 2(a), the scale of a node on the map relies on the number of its airlines. We can see that most of the small circles connect with a minority of large circles. Once the high-degree node is removed, the low-degree nodes could become isolated which is of maximum destructiveness to the connectivity.

From the results of Figs. 2(b)–2(g), we can see that the smart rewiring could increase the robustness index R faster than that of random rewiring, and the advantage of smart rewiring is noticeable. The smart rewiring could create more links between high-degree nodes which has been proven that it can enhance the robustness in the above analysis. However, the advantage of smart rewiring, for improving the robustness of the airlines network, could profit for another reason as well. In the airlines network, most of the edges connect with the high-degree node and low-degree node, and most of the nodes have low degrees. Thus after a swapping step of smart rewiring, a low-degree node does not connect with a high-degree node, while it tends to connect a node with a similar degree instead. Therefore, once the high-degree node is attacked, the former low-degree nodes could not be affected due to the fact that they may be connected with other nodes with a similar degree. Moreover in the improved network, the high-degree node tends to connect with other high-degree nodes. Thus the connectivity of the improved network, modified by smart rewiring, could not become worse noticeably than that of random rewiring after malicious attacks. For example, in the original network, once the airport CVG is attacked, the connectivity could become worse, due to the fact that the degrees of its neighbor nodes are low, which could become an isolated node with high probability. However, in the improving network, if CVG is attacked, the degrees of its neighbor nodes will be high or similar at least. It could not cause that most of its neighbor nodes with low degrees are isolated after attack. To sum up, the smart rewiring could reduce the difference of degree between nodes, that is, the improving network is more homogeneous and assortative.

In the insets of Figs. 2(b)–2(g), we also compare the degree-degree correlation[24,25] between the initial and smart rewiring methods. According to the above analysis, the smart rewiring reduces the difference of degree between nodes, and makes the improved network more assortative in essence. From the results of the insets of Figs. 2(b)–2(g), smart rewiring can increase the degree-degree correlation more quickly than that of random rewiring. Actually, the increase of degree-degree correlation creates more regenerated links between high-degree nodes, which explains why positive degree-degree correlation could enhance network robustness. On the other hand, the nodes' clustering coefficient for several real and model-generated networks modified by smart and random rewiring is presented in Fig. 3. We can see that the nodes' clustering coefficient of high-degree nodes optimized by smart rewiring is higher than that of a network modified by random rewiring. These results show that there are more edges recreated among high-degree nodes for the smart rewiring method.

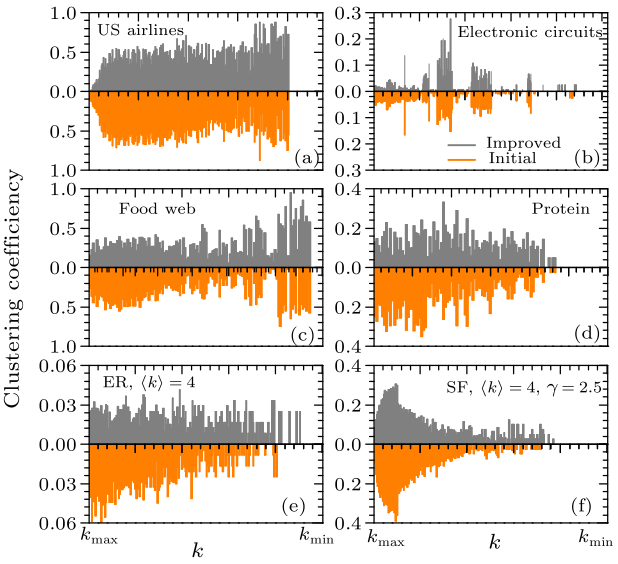


Fig. 3. Comparison of nodes' coefficient clustering between smart rewiring and random rewiring for (a) the US airlines network, (b) the electronic circuits network, (c) the food web network, (d) the protein network, (e) ER networks with $\langle k\rangle$ and (f) SF networks with $\langle k\rangle = 4$ and $\gamma = 2.5$.

Constructing and optimizing the robust topological structures are of great importance. In this work, we have introduced a unique measure for the robustness of network connectivity and used this measure to develop a method that significantly improves their

robustness against malicious attacks. The detailed analysis of the improved networks' structures shows that regenerating the edges between high-degree nodes can enhance the robustness against malicious attack. Therefore, we propose a novel rewiring strategy based on regenerating more edges between high-degree nodes, called smart rewiring, which could speed up the increase of the robustness index. Our approach has been found to be successfully useful as demonstrated on the real networked systems and synthetic networks. The smart rewiring method also explains why positive degree-degree correlation could enhance network robustness. As shown in our results, it is capable of enhancing the network robustness against node attacks. All the work can also be used to improve the robustness of network connectivity under edge removal. Although this method can be used for improving robustness against link failures, different attack strategies require different optimal network structures to mitigate the damage. From a practical point of view, the probability of node failure and link failure can hardly be known in reality. Therefore, future work will focus on a hybrid optimization method which can create a robust network to an attack combining node failures and link failures.

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