CHIN. PHYS. LETT. Vol. 31, No. 3 (2014) 037401

Josephson Effect in Graphene: Comparison of Real and Pseudo Vector Potential Barriers *

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(Received 15 August 2013)

The Josephson currents through real vector potential (RVP) and pseudo vector potential (PVP) barriers in graphene are investigated. In graphene, the pseudo vector potential may be caused by a local strain. The comparison of supercurrents induced by the two type-barriers is focused. As a result, we find that not only will the RVP induce a transition Josephson current from the $0 \to \pi$ state but also causes the difference in the phases of the order parameters of the two superconducting graphene layers to shift from $\varphi \to 2\varphi$. The critical current is PVP-independent around the neutrality point while it strongly depends on the RVP. The vector potential dependence of critical current is found to be perfectly linear for both PVP and RVP barriers.

PACS: 74.50.+r, 81.05.ue, 07.10.Pz DOI: 10.1088/0256-307X/31/3/037401

Since graphene was first prepared,[1] emerging of pseudo 2D relativistic fermion in graphene is one of the intriguing phenomena in solid state systems. This leads to several interesting phenomena like high energy systems and despite low energy.[2,3] Electrons in graphene are pseudo Dirac fermions with two valleys degenerated, k and k' valleys. Valley currents can be controlled by applied local strain and magnetic vector potential into graphene.[4–7] Local strain in graphene can create a vector-like potential called the pseudo vector potential (PVP) to act on electrons, depending on the valleys.[8–10] The real vector potential (RVP) may be accessible by using magnetic strips.[5,11,12] Graphene can be a superconducting material by means of a proximity effect.[13–17] Both fundamental science and applied physics of superconducting graphene have drawn much attention.[18–31] The Josephson effect has been generally known as the phenomenon of supercurrent led by superconductivity when the charge current moves without energy loss.[32] The first theoretical study on the Josephson effect in graphene-based superconductor/barrier/superconductor junction (SG/barrier/SG junction) has predicted that the critical current oscillates as a function of gate voltage.[28] The Josephson current switch has been investigated next in a magnetic barrier.[29] In this Letter, we study a new type of barrier of Josephson junction: (model I) a real vector potential barrier (RVP-barrier) and (model II) a pseudo vector potential barrier (PVP-barrier).

In our models, the SG/RVP-barrier/SG junction and SG/PVP-barrier/SG junction are shown in Figs. 1(a) and 1(b), respectively. The current is flowing in the y-direction. The motion of the quasiparticles with excited energy E may be described by the Dirac Bogoliubov-de Gennes (DBdG) equation[27,33]

$$\begin{pmatrix} \hat{H}_{0\lambda}(\boldsymbol{A}_{\rm R}+\boldsymbol{A}_{\rm S}) & \Delta(x,y) \\ \Delta^*(x,y) & -\hat{H}_{0\lambda}(-\boldsymbol{A}_{\rm R}+\boldsymbol{A}_{\rm S}) \end{pmatrix} \psi_\lambda(x,y) = E\psi_\lambda(x,y), \quad (1)$$

where the single electron Hamiltonian is given by

$$\hat{H}_{0\lambda}(\boldsymbol{A}_{\rm R}+\boldsymbol{A}_{\rm S}) = v_{\rm F}\boldsymbol{\sigma}\cdot\left(\hat{\boldsymbol{p}}_\lambda + \frac{e}{c}\boldsymbol{A}_{\rm R} - \lambda\frac{e}{c}\boldsymbol{A}_{\rm S}\right) + U(x,y), \quad (2)$$

with $\boldsymbol{A}_{\rm R}$ and $\boldsymbol{A}_{\rm S}$ being the RVP and the PVP, respectively. The Fermi velocity of graphene is of $v_{\rm F} \sim 10^6\,{\rm m/s}$. The notation $\lambda = +\ (-)$ denotes the k (k') valley. The momentum-like operator is defined as $\hat{\boldsymbol{p}}_\lambda = -i\hbar(\partial_x\hat{i} + \lambda\partial_y\hat{j})$ and $\boldsymbol{\sigma}$ is the Pauli matrices. The superconducting order parameter is $\Delta(x,y) = \Delta e^{i\varphi_1}\Theta(-y) + \Delta e^{i\varphi_2}\Theta(y-L)$, where $\Theta(y)$ is the Heaviside step function and L is the thickness of the barrier. The superconducting phase difference would be defined as $\varphi = \varphi_2 - \varphi_1$. The potential $V_{\rm G}$ and the Fermi level μ in the system are tuned via the top and back gates, respectively. The net potential is therefore defined as $U(x,y) = -\mu\Theta(-y) - (\mu + V_{\rm G})\Theta(y)\Theta(L-y) - \mu\Theta(y-L)$.

In the case of an RVP barrier, the RVP in the barrier may be realized by placing two magnetic strips[5,11,12] at the positions shown in Fig. 1(a). These would generate an RVP in the form

$$\boldsymbol{A}_{\rm R}(x,y,z) = B_0\sqrt{\hbar/eB_0}\hat{i}\Theta(y)\Theta(L-y), \quad (3)$$

where the effective normal magnetic field $B_Z = B_0\sqrt{\hbar/eB_0}(\delta(y) - \delta(y-L))$. When we consider the case of the PVP barrier, the PVP (or pseudo gauge

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potential) $\boldsymbol{A}_{\rm S} = A_{Sx}\hat{i} + A_{Sy}\hat{j}$ is generated by a local strain.[9] Here, hoping energies without strain is $t_0 \sim 2.7\,{\rm eV}$.[34] A small strain is applied only in the armchair direction (y-direction), giving rise to $A_{Sx} \neq 0$, $A_{Sy} = 0$. Using model hoping energies in Ref. [35] we obtain[12]

$$\begin{aligned}\boldsymbol{A}_{\rm S} &= \frac{c}{ev_{\rm F}}(3.37t_0 \times {\rm strain})\hat{i}\Theta(y)\Theta(L-y)\\ &= A_{Sx}\hat{i}\Theta(y)\Theta(L-y).\end{aligned} \tag{4}$$

To consider the time reversal operator acting on the Hamiltonian

$$H(\boldsymbol{A}_{\rm R}+\boldsymbol{A}_{\rm S}) = \begin{pmatrix} H_{0+}(\boldsymbol{A}_{\rm R}+\boldsymbol{A}_{\rm S}) & 0 \\ 0 & H_{0-}(\boldsymbol{A}_{\rm R}+\boldsymbol{A}_{\rm S}) \end{pmatrix}, \tag{5}$$

when electrons in a graphene superconductor undergo an RVP and a PVP, we will have[33]

$$\Im H(\boldsymbol{A}_{\rm R}+\boldsymbol{A}_{\rm S})\Im^{-1} = H(-\boldsymbol{A}_{\rm R}+\boldsymbol{A}_{\rm S}), \tag{6}$$

where $\Im = \begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix} C$ is the time reversal operator which interchanges the valleys. Here C is the complex conjugate operator. It can be seen that the time reversal symmetry would be broken only by applying the RVP.[33]

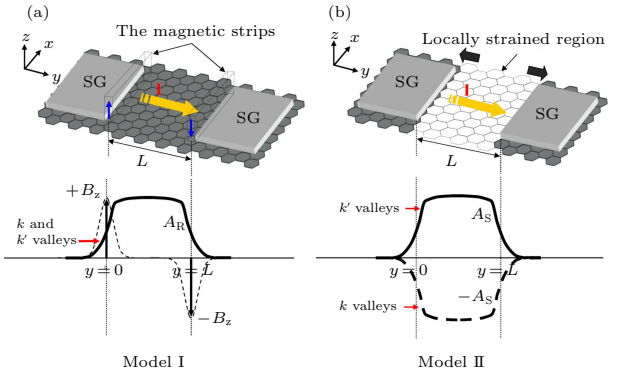

Fig. 1. The models of (a) an SG/RVP/SG junction and (b) an SG/PVP/SG junction.

The wave function $\psi(x,y)$, i.e. the solution to Eq. (1), would have the form $\psi(y)e^{ik_{||}x}$. The momentum in the x-direction $k_{||} = |\mu + E|\cos\theta/\hbar v_{\rm F}$ is a constant of motion, where θ is an angle of incidence in the SG regions. The wave vectors of an electron-like quasiparticle (ELQ) and a hole-like quasiparticle (HLQ) in the $\pm y$-direction are respectively given by $k_{ye} = \pm\sqrt{U^2 - (\hbar v_{\rm F}k_{||} + \delta_{\rm R} - \lambda\delta_{\rm S})^2}/\hbar v_{\rm F}$, and $k_{yh} = \pm\sqrt{U^2 - (\hbar v_{\rm F}k_{||} - \delta_{\rm R} - \lambda\delta_{\rm S})^2}/\hbar v_{\rm F}$, where $\delta_{\rm R(S)} = eA_{\rm R(S)x}v_{\rm F}/c$ with $c \sim 3\times10^8\,{\rm m/s}$ being the speed of light. The wave functions defined in the left-SG region, the right-SG region and the barrier are respectively given by

$$\begin{aligned}\psi_{\rm L}(x,y) &= (a\psi_{+{\rm h,S}} + b\psi_{-{\rm e,S}})e^{ik_{||}x+\kappa y},\\ \psi_{\rm R}(x,y) &= (c\psi_{-{\rm h,S}} + d\psi_{+{\rm e,S}})e^{ik_{||}x-\kappa y},\\ \psi_{\rm B}(x,y) &= (g_1\psi_{+{\rm e,B}} + g_2\psi_{-{\rm e,B}}\\ &\quad + g_3\psi_{+{\rm h,B}} + g_4\psi_{-{\rm h,B}})e^{ik_{||}x},\end{aligned} \tag{7}$$

where $\kappa = \sin\theta\sqrt{\Delta^2 - E^2}/\hbar v_{\rm F}$. The wave functions $\psi_{\pm{\rm e(h),S(B)}}$ are the solutions of $E(H)$ LQ in $\pm y$ directions for the SG (barrier) region. The constants a, b, g_{1-4}, c, and d are the coefficients. The Andreev energies are given by matching the wave functions with the usual boundary condition. The Andreev energies for models I and II are thus respectively obtained as

$$\begin{aligned}E_{\rm I} &= E \quad (A_{\rm R} \neq 0,\ A_{\rm S} = 0),\\ E_{\rm II} &= E \quad (A_{\rm R} = 0,\ A_{\rm S} \neq 0).\end{aligned} \tag{8}$$

The Josephson currents in the models I (II) can then be obtained using the usual formula[28,36]

$$I_{\rm I(II)}(\varphi,\theta) = -\sum_{\lambda=\pm1}\frac{e}{\hbar}\frac{\partial E_{\rm I(II),\lambda}}{\partial\varphi}\tanh\left(\frac{E_{\rm I(II),\lambda}}{2k_{\rm B}T}\right). \tag{9}$$

The total Josephson current in the system is calculated by integration over all incident angles projecting onto the y-direction i.e.,

$$I(\varphi) = \int_0^{\pi} d\theta\sin(\theta)I_{\rm I(II)}(\varphi,\theta). \tag{10}$$

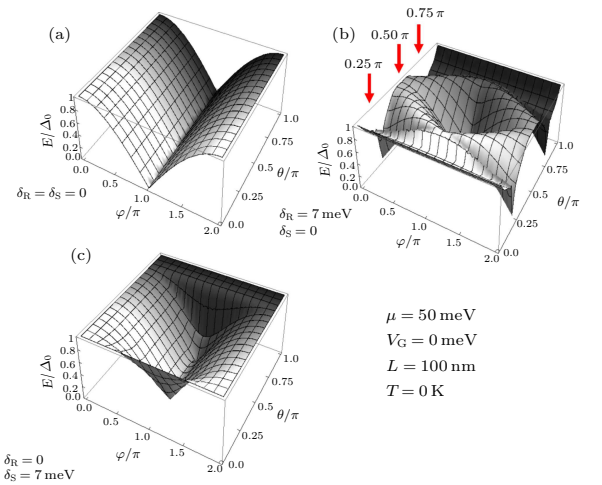

Fig. 2. Comparisons of Andreev energy levels affected by real and pseudo vector potentials in (a) SG/normal barrier/SG, (b) SG/RVP/SG, and (c) SG/PVP/SG junctions for k-valley.

In the numerical calculation, we take the magnitude of magnetic strips of $B_0 = 0.1\,{\rm T}$[5,11] giving rise to $\delta_{\rm R} = 7\,{\rm meV}$.[37] A magnitude of local strain is of $\sim$1%.[38] The Josephson current is normalized by $I_0 = 2e\Delta(T=0)/\hbar$. The energy levels affected by the presence of RVP or PVP are studied in Fig. 2. In

Fig. 2(a), the Andreev energy is found to solely depend on φ as a sinusoidal function $E = \Delta\cos(\varphi/2)$. As denoted by the arrows in Fig. 2(b), the Andreev energy for some angles of incidence can be approximated to $E(\theta = 0.25\pi) \cong +\Delta\cos(\varphi/2)$, $E(\theta = 0.50\pi) \cong -\Delta\cos(\varphi/2)$ and $E(\theta = 0.75\pi) \cong +\Delta\cos(\varphi/2)$.

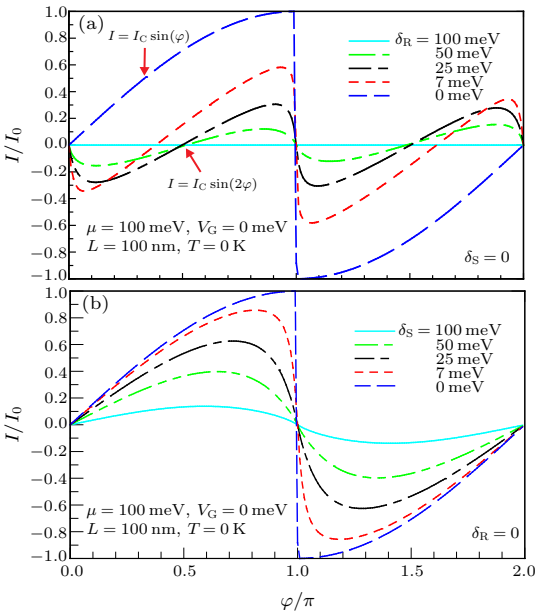

Fig. 3. Comparisons of current phase relations affected by real and pseudo vector potentials in (a) SG/RVP/SG and (b) SG/PVP/SG junctions.

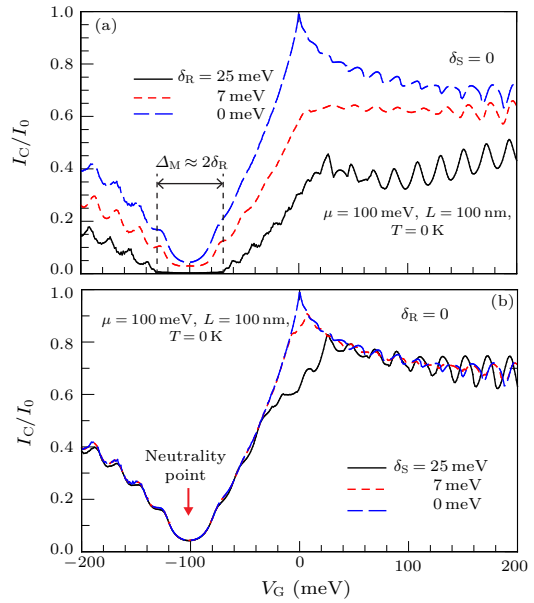

Fig. 4. Comparisons of critical currents under varying V_G affected by real and pseudo vector potentials in (a) SG/RVP/SG and (b) SG/PVP/SG junctions.

Applying PVP, the energy level exhibits very weak dependence on θ (see Fig. 2(c)). The RVP would induce a transition from $0 \to \pi$ state and would cause the difference in the phases of the order parameters of the two superconducting graphene layers to shift from $\varphi \to 2\varphi$ (see Fig. 3), in contrast to that in the RVP-barrier. This remarkable behavior, which shows a very different current-phase relation between RVP and PVP, could be used to test the theory of time reversal symmetry.[33] In Fig. 4, the critical current for the case of an RVP-barrier strongly depends on δ_R, suppressed completely for large δ_R (see Fig. 4(a)). The suppression of critical current takes place when $|\mu - V_G| \le \delta_R$ with the magnetic gap region of $\Delta_M = 2\delta_R$. Plot of I_C as a function of δ_S shows a very different behavior from that in the RVP-barrier (see Fig. 4(b)). The critical current is found to be strain-independent. The critical currents in Figs. 5(a)and 5(b) are found to perfectly linearly depend on both RVP and PVP around the zero vector potentials. The slopes are adjusted by varying the potential of the back gate μ.

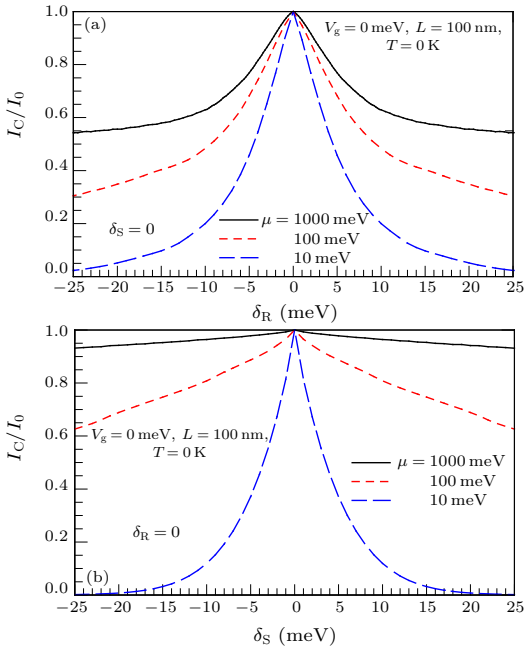

Fig. 5. Comparisons of critical currents under varying RVP or PVP in (a) SG/RVP/SG and (b) SG/PVP/SG junctions.

In summary, we have investigated the Josephson effect in SG/RVP (or PVP) barrier/SG junction. We find that only the RVP may induce a transition Josephson current from $0 \to \pi$ state and may cause the difference in the phases of the order parameters of the two superconducting graphene layers to shift from $\varphi \to 2\varphi$. The critical current is found to be PVP-independent around the neutrality point.

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