CHIN. PHYS. LETT. Vol. 30, No. 5 (2013) 050501

Local Property of Recurrence Network for Investigating Gas-Liquid Two-Phase Flow Characteristics *

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(Received 23 January 2013)

Understanding the dynamics of gas-liquid two-phase flows is a challenge in the fields of nonlinear dynamics. We first construct and analyze a recurrence network from Chen's chaotic system and find that the network local statistic is feasible to characterize chaotic dynamics associated with unstable periodic orbits. Then we construct recurrence networks from gas-liquid two-phase flow experimental signals and associate the network topological statistic with the flow pattern dynamics. The results indicate that the recurrence network could be a powerful tool for the dynamic characterization of experimental gas-liquid two-phase flows.

PACS: 05.45.Tp, 47.55.Ca DOI: 10.1088/0256-307X/30/5/050501

Gas-liquid two-phase flow widely exists in various practical applications such as spray processes, natural gas networks, and nuclear reactor cooling. In gas-liquid two-phase flow, the deformable interfaces of the two phases can acquire a variety of characteristic distributions which are called flow patterns. Understanding the dynamics of flow patterns is crucial to some important problems such as predicting pressure drop. Due to the interplay among many complex factors such as fluid turbulence and phase interfacial interaction, gas-liquid two-phase flow exhibits a highly unsteady and complicated flow structure. Hence, uncovering the dynamic behavior of various flow patterns arising in gas-liquid two-phase flow is a challenging and important problem.

Existing methods aiming at exploring gas-liquid two-phase flow include those based on the hydrodynamic model,[1] Shannon entropy,[2] wavelet transform,[3−4] probability density function analysis and Fast Fourier Transform.[5,6] Despite the existing results, the dynamical mechanisms generating various patterns in gas-liquid two-phase flows are still elusive up to now. In this regard, developing an effective approach to characterize dynamic flow behavior underlying gas-liquid flow patterns becomes quite necessary and important. Complex network theory has undergone a remarkable development over the last decade and has contributed significantly to our understanding of complex systems.[7−18] The great success of network theory in various fields of research has recently motivated attempts to generalize this concept for a direct application to time series.[19−31] Complex network analysis of time series has been successfully applied in many research fields, such as climate system,[32] financial system,[33] brain network,[34] human gait,[35] human ventricular fibrillation,[36] gene system,[37] and multiphase flow system.[38−40] In this Letter, we systematically carry out gas-liquid two-phase flow experiments for measuring the signals corresponding to different flow patterns. Based on the experimental signals, we use the analytical framework of the recurrence network to characterize the chaotic dynamic behavior in gas-liquid flow patterns.

The vertical gas-liquid two-phase flow experiment in a 125-mm diameter pipe has been carried out in the multiphase flow loop laboratory of Tianjin University. The experimental mediums are air and tap water. Experiments are implemented by initially injecting both gas and water flows into the pipe. The water flow rate is fixed and the gas flow rate is gradually increased. Each time the ratio between the gas and water flow rates reaches a certain preset value, a conductance fluctuating signal is collected from our designed vertical multi-electrode array conductance (VMEA) sensor[41] and the flow pattern is visualized by the high-speed video camera recorder (VCR). The sampling frequency is 400 Hz and the sampling time between two measuring points is 60 s. We have observed five different vertical gas-water flow patterns in the experiment, i.e., gas-liquid bubble flow, bubble-slug transitional flow, slug flow, slug-churn transitional flow and churn flow. The conductance fluctuating signals of VMEA sensor corresponding to the five gas-water flow patterns are shown in Fig. 1, where U_{sg} and U_{sw} represent gas superficial velocity and water superficial velocity, respectively. Based on the conductance fluctuating signals, we will conduct recurrence networks analysis to characterize the dynamic flow be-

*Supported by the National Natural Science Foundation of China under Grant Nos 41174109, 61104148 and 50974095, the National Science and Technology Major Projects under Grant No 2011ZX05020-006, and the Specialized Research Fund for the Doctoral Program of Higher Education of China under Grant No 20110032120088.

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havior in gas-liquid two-phase flow.

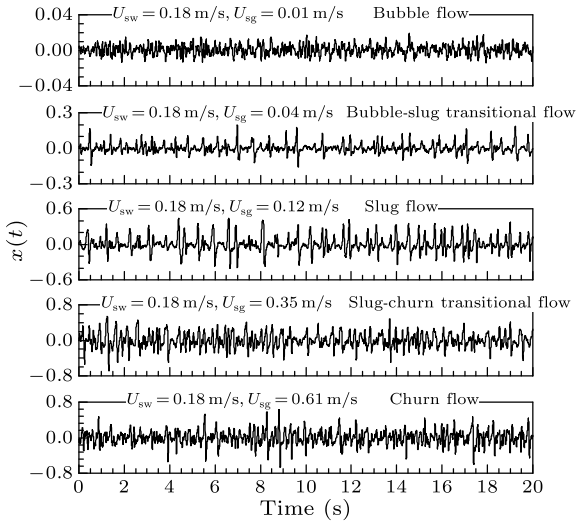


Fig. 1. The conductance fluctuating signals in five gas-water flow patterns.

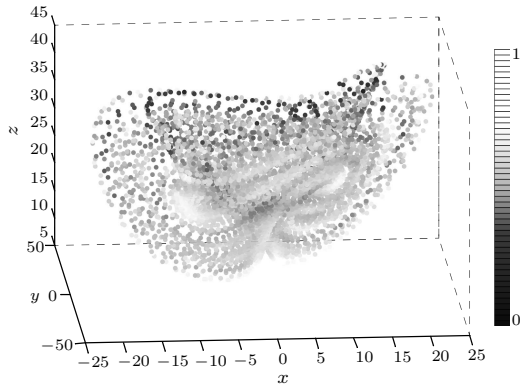


Fig. 2. Color-coded representation of the local clustering coefficients $C^{\mathrm{L}}(v)$ for the recurrence network generated from Chen's chaotic system.

Now we introduce how to transform a time series into a complex network in the framework of a recurrence network. When dealing with a time series $x(t)$ $(t = 1,\ldots,M)$, one may use a suitable m-dimensional time delay embedding of $x(t)$ with delay τ,[42]

$$\boldsymbol{X}(t) = (x(t), x(t+\tau), \cdots, x(t+(m-1)\tau)), \quad (1)$$

to obtain a phase space with N phase space vector points $\boldsymbol{X}(t)$, $t = 1,2,\ldots,N$, where

$$\mathrm{N} = M - (m-1)\tau. \quad (2)$$

According to Refs. [28,29], in the formalism of recurrence network, a state in the phase space vector $\boldsymbol{X}(t_i)$ is said to be recurrent if there is $t_j \neq t_i$, such that

$$d(\boldsymbol{X}(t_i), \boldsymbol{X}(t_j)) < \varepsilon \quad (3)$$

for some distance measure d in phase space, where

$$d(\boldsymbol{X}(t_i), \boldsymbol{X}(t_j)) = ||\boldsymbol{X}(t_i) - \boldsymbol{X}(t_j)||. \quad (4)$$

The structure of recurrences in phase space can be simply encoded in terms of the recurrence matrix

$$R_{ij} = \Theta(\varepsilon - d(\boldsymbol{X}(t_i), \boldsymbol{X}(t_j))), \quad (5)$$

where Θ is the Heaviside function. Following the above considerations, this matrix can be re-interpreted as the adjacency matrix of an unweighted complex network associated with the given time series which is called (ε) recurrence network. To be more specific, the adjacency matrix of the recurrence network is given by

$$A_{ij} = R_{ij} - \delta_{ij}, \quad (6)$$

where δ_{ij} is the Kronecker delta introduced to avoid artificial self-loops in the network.[28,29] That is, individual values of the respective observable in phase space can be directly considered as nodes of the recurrence network, while the existence of an edge serves as an indicator of a recurrence, i.e., pairs of states whose values do not differ by more than a small value ε in the phase space. In order to investigate the dynamics of a time series in terms of recurrence network, Donner *et al.*[28] defined the local clustering coefficient in terms of conditional probabilities, denoted as $C^{\mathrm{L}}(v)$,

$$\begin{aligned} C^{\mathrm{L}}(v) &= P(A_{ij} = 1 | A_{vi} = 1, A_{vj} = 1) \\ &= \frac{P(A_{ij} = 1, A_{vi} = 1, A_{vj} = 1)}{P(A_{vi} = 1, A_{vj} = 1)}, \end{aligned} \quad (7)$$

where A_{ij} is the network adjacent matrix. With Bayes' theorem, we can obtain

$$\begin{aligned} &P(A_{vi} = 1, A_{vj} = 1) \\ &= \frac{1}{(N-1)(N-2)} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} A_{vi} A_{vj}, \end{aligned} \quad (8)$$

where N is the number of nodes in the network, and a similar expression for $P(A_{ij} = 1, A_{vi} = 1, A_{vj} = 1)$. For more details one can see Ref. [28]. We use the above method to generate a recurrence network from Chen's chaotic system,[43]

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= (c - a)x - xz + cy, \\ \dot{z} &= xy - bz, \end{aligned} \quad (9)$$

where $a = 35$, $b = 3$, $c = 28$. We calculate the local clustering coefficient for the recurrence network generated from Chen's chaotic system and present the result in Fig. 2. As can be seen, there are remarkably high values of local clustering coefficient in the phase space and the presence of distinct structures in the spatial profile of the local clustering coefficient is related to the emergence of unstable periodic orbits.[28,44] Unstable periodic orbits (UPOs) embedded in the chaotic attractor are fundamental to the understanding of the

chaotic dynamics. It has been known that when the infinite set of periodic orbits embedded in a chaotic set are all unstable, their stabilities, as determined by the corresponding largest eigenvalues, are typically quite heterogeneous. Correspondingly, we can see heterogeneous distribution of the local clustering coefficient in the chaotic attractor, as shown in Fig. 2. In fact, a small set of UPOs can be significantly less unstable than others. In the phase space, a UPO appears as a closed loop. A chaotic trajectory tends to spend substantially more time near weakly unstable orbits. As a result, there can be many recurrences near such a UPO, giving rise to a cluster in the corresponding network, which can be captured by a high value of local clustering coefficient in the sense that the neighborhoods of a node in this region usually densely connect with each other. Therefore, the local clustering coefficient of a recurrence network can be used to characterize chaotic dynamics associated with unstable periodic orbits.

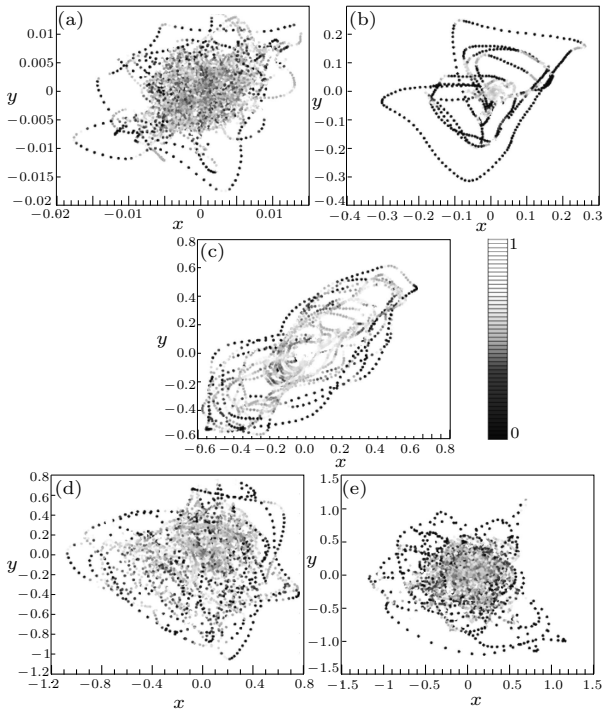


Fig. 3. Color-coded representation of the local clustering coefficients in the phase space for the recurrence network generated from five different gas-liquid flow patterns: (a) bubble flow, (b) bubble-slug transitional flow, (c) slug flow, (d) slug-churn transitional flow, and (e) Churn flow.

We construct two-phase flow recurrence networks corresponding to different flow patterns from experimental signals. Representing the conductance fluctuation signal through a corresponding recurrence network, we can then study the dynamical behaviors of gas-liquid flow patterns from complex network analysis, which is quantified via the local clustering coefficient. Figure 3 shows the structures in the spatial profile of the local clustering coefficient for the recurrence networks generated from the signals of five different flow patterns. From Fig. 3, we can extract the mean value of local clustering coefficients for the network, denoted as $\langle C^{\mathrm{L}}(v)\rangle$. A signal measured from a flow condition corresponds to a recurrence network. In order to characterize the dynamic behavior in the transition of different flow patterns, we construct numbers of recurrence networks from various experimental signals measured from different flow conditions. We then calculate the mean value of local clustering coefficients for each generated network. Figure 4 displays the distributions of $\langle C^{\mathrm{L}}(v)\rangle$ in semi-log scale for numbers of generated networks with the increase of gas superficial velocity, in which the U_{sg} and U_{sw} represent gas superficial velocity and water superficial velocity, respectively.

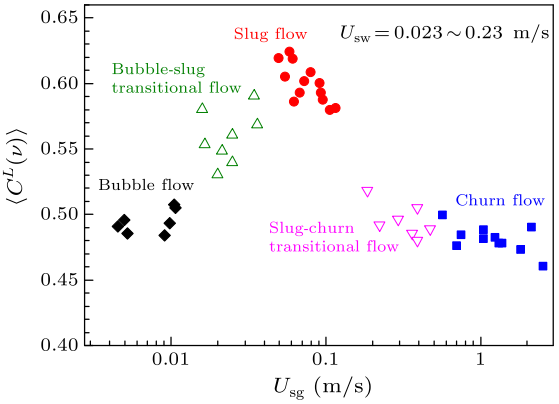


Fig. 4. The distributions of $\langle C^{\mathrm{L}}(v)\rangle$ for recurrence networks generated from different flow patterns.

Gas-liquid bubble flow occurs at low gas flow rates where the gas phase is approximately uniformly distributed in the form of discrete bubbles in a continuum of liquid phase. Owing to the stochastic motions of large numbers of small air bubbles, the dynamical behavior of bubble flow is very complex and no obvious periodic orbits exist in the phase space from bubble flow, corresponding to the weak heterogeneous distributions of local clustering coefficients, as shown in Fig. 3(a), and small value of $\langle C^{\mathrm{L}}(v)\rangle$ is shown in Fig. 4. In the transition from bubble flow to slug flow, small air bubbles begin to coalesce to form a large one. Correspondingly, a small number of unstable periodic orbits with small local clustering coefficients gradually appear, as shown in Fig. 3(b). With an increase in gas flow rate, slug flow appears. Because of the quasi-periodic oscillation behavior associated with unstable periodic orbits between gas slug and liquid slug, the dynamical behavior of slug flow becomes relatively simple compared with that of bubble flow, which explains the strong heterogeneous distributions of local clustering coefficients, as shown in Fig. 3(c). The increase of the $\langle C^{\mathrm{L}}(v)\rangle$ as the flow pattern evolves from

bubble flow to slug flow, as shown in Fig. 4, agrees well with the results presented in the analysis of Chen's chaotic system. When the gas superficial velocity is high, churn flow gradually occurs. As a highly disordered flow, churn flow is mainly composed of a discrete gas phase and a continuous liquid phase of highly turbulent kinetic energy. Due to the influence of the turbulence effect, the slugs are collapsed and the oscillation behavior becomes rare and the dynamical behavior of churn flow becomes more complex than that of slug flow. Consequently, the value of $\langle C^{\mathrm{L}}(v)\rangle$ decreases and the heterogeneity of the distributions of local clustering coefficients becomes weaker as the flow pattern evolves from slug flow to churn flow, as shown in Figs. 3(d), 3(e) and 4.

In summary, we have constructed a recurrence network from experimental signals to uncover the dynamic flow behavior underlying different gas-liquid flow patterns. Recurrence network analysis of Chen's chaotic system indicates that the local clustering coefficient can be used to characterize chaotic dynamics associated with unstable periodic orbits. Recurrence network analysis of experimental signals suggests that the local clustering coefficient can faithfully represent the distinct dynamical states of the gas-liquid two-phase flow and further can be used to uncover the dynamic flow behavior associated with chaotic UPOs. Our research opens a wide field for novel applications of recurrence network theory in the experimental signal analysis.

References

[1] Fernandes R C, Semiat R and Dukler A E 1983 *AIChE J.* **29** 981
[2] Zhang Z Y and Shi L 1999 *J. Appl. Phys.* **85** 7544
[3] Park S H, Kang Y and Kim S D 2001 *Chem. Eng. Sci.* **56** 6259
[4] Kulkarni A A, Joshi J B, Kumarb V R and Kulkarni B D 2001 *Chem. Eng. Sci.* **56** 5305
[5] Mandal T K, Bhuyan M K, Das G and Das P K 2008 *Chem. Eng. Res. Des.* **86** 269
[6] Mandal T K, Das G and Das P K 2010 *Int. J. Multiphase Flow* **36** 661
[7] Watts D J and Strogatz S H 1998 *Nature* **393** 440
[8] Albert R and Barabási A L 2002 *Rev. Mod. Phys.* **74** 47
[9] Huang L, Park K, Lai Y C, Yang L and Yang K Q 2006 *Phys. Rev. Lett.* **97** 164101
[10] Zhao H and Gao Z Y 2006 *Chin. Phys. Lett.* **23** 2311
[11] Jiang Z Q, Zhou W X, Xu B and Yuan W K 2007 *AIChE J.* **53** 423
[12] Zhou J, Lu J A and Lu J H 2008 *Automatica* **44** 996
[13] Guo L and Cai X 2009 *Chin. Phys. Lett.* **26** 088901
[14] Yu W, Cao J, Chen G, Lü J, Han J and Wei W 2009 *IEEE Trans. Syst. Man Cybern. Part* B **39** 230
[15] Lu X B, Wang X F and Fang J Q 2010 *Physica* D **239** 341
[16] Shang M S, Chen D B and Zhou T 2010 *Chin. Phys. Lett.* **27** 058901
[17] Rong Z H, Wu Z X and Wang W X 2010 *Phys. Rev.* E **82** 047101
[18] Ye Z Q, Zhang K, Hu S N and Yu J 2012 *Chin. Phys. Lett.* **29** 098901
[19] Zhang J and Small M 2006 *Phys. Rev. Lett.* **96** 238701
[20] Zhang J, Sun J, Luo X, Zhang K, Nakamura T and Small M 2008 *Physica* D **237** 2856
[21] Yang Y and Yang H J 2008 *Physica* A **387** 1381
[22] Lacasa L and Toral R 2010 *Phys. Rev.* E **82** 036120
[23] Liu C, Zhou W X and Yuan W K 2010 *Physica* A **389** 2675
[24] Li P and Wang B H 2007 *Physica* A **378** 519
[25] Xu X, Zhang J and Small M 2008 *Proc. Natl. Acad. Sci. USA* **105** 19601
[26] Marwan N, Donges J F, Zou Y, Donner R V and Kurths J 2009 *Phys. Lett.* A **373** 4246
[27] Gao Z K and Jin N D 2009 *Chaos* **19** 033137
[28] Donner R V, Zou Y, Donges J F, Marwan N and Kurths J 2010 *New J. Phys.* **12** 033025
[29] Donner R V, Heitzig J, Donges J F, Zou Y, Marwan N and Kurths J 2011 *Eur. Phys. J.* B **84** 653
[30] Donges J F, Heitzig J, Donner R V and Kurths J 2012 *Phys. Rev. E* **85** 046105
[31] Shimada Y, Ikeguchi T and Shigehara T 2012 *Phys. Rev. Lett.* **109** 158701
[32] Donges J F, Zou Y, Marwan N and Kurths J 2009 *Europhys. Lett.* **87** 48007
[33] Song D M, Jiang Z Q and Zhou W X 2009 *Physica* A **388** 2450
[34] Sun W G, Cao J T and Wang R B 2011 *Chin. Phys. Lett.* **28** 068701
[35] Zhang J, Zhang K, Feng J F and Small M 2010 *PLoS Comput. Biol.* **6** e1001033
[36] Li X and Dong Z 2011 *Phys. Rev.* E **84** 062901
[37] Hempel S, Koseska A, Kurths J and Nikoloski Z 2011 *Phys. Rev. Lett.* **107** 054101
[38] Gao Z K, Jin N D, Wang W X and Lai Y C 2010 *Phys. Rev.* E **82** 016210
[39] Gao Z K and Jin N D 2011 *Chem. Eng. Sci.* **66** 2660
[40] Gao Z K and Jin N D 2012 *Physica* A **391** 3005
[41] Jin N D, Xin Z, Wang J, WangZ Y Jia X H and Chen W P 2008 *Meas. Sci. Technol.* **19** 045403
[42] Takens F 1981 *Dynamical Syst. Turbulence Lecture Notes Math.* **898** 366
[43] Chen G and Ueta T 1999 *Int. J. Bifurcation Chaos Appl. Sci. Eng.* **9** 1465
[44] Hunt B and Ott E 1996 *Phys. Rev. Lett.* **76** 2254