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Long-Lived Rogue Waves and Inelastic Interaction in Binary Mixtures of Bose–Einstein Condensates *

LIU Chong(刘冲)[1], YANG Zhan-Ying(杨战营)[1**], ZHAO Li-Chen(赵立臣)[2,3], YANG Wen-Li(杨文力)[4], YUE Rui-Hong(岳瑞宏)[5]

[1]*Department of Physics, Northwest University, Xi'an 710069*
[2]*Science and Technology Computation Physics Laboratory, Institute of Applied Physics and Computational Mathematics, Beijing 100088*
[3]*Department of Modern Physics, University of Science and Technology of China, Hefei 230026*
[4]*Institute of Modern Physics, Northwest University, Xi'an 710069*
[5]*Faculty of Science, Ningbo University, Ningbo 315211*

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We investigate bright, dark and two rogue waves in two-component Bose–Einstein condensates. It is found that one rogue wave of a new structure that contains two humps and two valleys around one center in the temporal-spatial distribution interacts with the conventional rogue wave. We present an effective way to obtain long-lived rogue waves through managing the nonlinear interaction with modulating trapping frequency and interchange atoms between thermal clouds and condensates. The results provide many possibilities to manipulate rogue waves experimentally in the condensate system.

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Rogue waves that appear occasionally in the ocean have attracted great attention.[1−14] These waves can reach amplitudes of two, three or more times higher than the average crest, appear from nowhere and disappear without a trace.[1−4] Theoretically, the behavior of rogue waves can be well described by the nonlinear Schrödinger equation (NLSE).[5,6] Experimentally, this novel phenomenon has been observed in nonlinear fiber optics[7,8] and water wave tanks.[9] Nowadays, studies on rogue waves have been extended to a variety of complex systems, such as two-mode nonlinear fibers[10−12] and multi-component Bose–Einstein condensates (BECs).[13] In this case, the coupled equations may describe rogue waves with higher accuracy than the uncoupled NLSE due to the fact that more than one wave amplitude needs to be considered. Interestingly, more abundant types of rogue waves[10−14] have been demonstrated in coupled systems.

The multi-component BEC is one of the coupled systems that provides an ideal platform to realize and observe rogue waves because of its remarkable features in experimental applications such as the interaction between mixtures of several components and Fesbash resonance management.[13,15] In this work, we investigate bright, dark and two rogue waves in the two-component BEC, which can be described by the coupled Gross–Pitaevskii (CGP) equations. We find an effective way to obtain long-lived rogue waves under a certain condition once they appear, i.e. rogue waves that can be sustained under the balance between the trapping frequency λ and gain or loss term $G_j(t)$ for the atoms transformed between condensates and thermal clouds. Moreover, we first find an inelastic collision between two rogue waves. It is interesting that two rogue waves coexist and interact with each other. It may be meaningful to realize rogue waves in future research about other coupled systems.

Considering a general two-component BEC, the behavior of the condensates can be described by the framework of the normalized form of the CGP equations as follows:[15]

$$i\frac{\partial \Psi_1}{\partial t}+\frac{\partial^2 \Psi_1}{\partial x^2}+V_1\Psi_1+[a_1|\Psi_1|^2+a|\Psi_2|^2]\Psi_1=0, \tag{1}$$

$$i\frac{\partial \Psi_2}{\partial t}+\frac{\partial^2 \Psi_2}{\partial x^2}+V_2\Psi_2+[a|\Psi_1|^2+a_2|\Psi_2|^2]\Psi_2=0, \tag{2}$$

where $V_j=\lambda(t)^2x^2+iG_j(t)$; and $\lambda(t)$ is the trapping frequency characterizing the strength of the parabolic potential. $G_j(t)$ represent the gain [or loss] term for the atom transformation between condensates and thermal clouds; a and a_1, a_2 denote the interaction nonlinear coefficients of the interspecies and intraspecies, respectively, which can be modulated effectively by Fesbash resonance management. When $\lambda(t)=G_j(t)=0$ and $a=a_1=a_2=$ const, the model reduces to the Manakov system and the vector rogue wave has been studied.[10−12] In this work, we present two kinds of rogue waves in a two-component BEC under the conditions $a_1=a=2\sigma e^{\int 2\lambda(t)+2G_1(t)dt}$ and $a=a_2=2\sigma e^{\int 2\lambda(t)+2G_2(t)dt}$, where σ is an arbitrary positive constant.

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**Corresponding author. Email: zyyang@nwu.edu.cn; yueruihong@nbu.edu.cn
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Case 1: bright and dark rogue waves solution and long-lived rogue waves in BECs.

We analytically present the bright and dark rogue waves solutions of the CGP equation as follows:

$$\Psi_j = \phi_j(t)[s_j \exp[i\theta_j] - \frac{1}{\sqrt{\sigma}} \frac{i(\mu-\bar{\mu})\Phi_1 \bar{\Phi}_{j+1}}{|\Phi_1|^2+|\Phi_2|^2+|\Phi_3|^2}]e^{i\Theta(x,t)}, \quad (3)$$

where

$$\begin{aligned}
\theta_j &= 2\sigma(s_j^2+s_{3-j}^2)\tau + k_j\xi - k_j^2\tau,\\
\Phi_1 &= (\Phi_{01}+\Phi_{02})\exp[i/3(\theta_1+\theta_2)],\\
\Phi_2 &= \frac{-\sqrt{\sigma}s_1}{\upsilon - i\mu/3 - i(2k_1-k_2)/3}\Phi_{01}\\
&\quad + \frac{-\sqrt{\sigma}s_1 - \frac{-\sqrt{\sigma}s_1}{\upsilon - i\mu/3 - i(2k_1-k_2)/3}}{\upsilon - i\mu/3 - i(2k_1-k_2)/3}\Phi_{02}\\
&\quad \times \exp[i/3(\theta_2-2\theta_1)],\\
\Phi_3 &= \frac{-\sqrt{\sigma}s_2}{\upsilon - i\mu/3 - i(2k_2-k_1)/3}\Phi_{01}\\
&\quad + \frac{-\sqrt{\sigma}s_2 - \frac{-\sqrt{\sigma}s_2}{\upsilon - i\mu/3 - i(2k_2-k_1)/3}}{\upsilon - i\mu/3 - i(2k_2-k_1)/3}\Phi_{02}\\
&\quad \times \exp[i/3(\theta_1-2\theta_2)].\\
\phi_j(t) &= e^{\int[\lambda(t)-G_j(t)]dt},\\
\Theta(x,t) &= -\frac{x^2}{2}\lambda(t) + xe^{\int 2\lambda(t)dt} - \int e^{\int 4\lambda(t)dt}dt,
\end{aligned}$$

and

$$\begin{aligned}
\Phi_{01} &= (A\xi + 2Ai\upsilon\tau + 2/3A(\mu-k_1-k_2)\tau)\\
&\quad \times \exp[\upsilon\xi + i\upsilon^2\tau + 2/3(\mu-k_1-k_2)\upsilon\tau],\\
\Phi_{02} &= A\exp[\upsilon\xi + i\upsilon^2\tau + 2/3(\mu-k_1-k_2)\upsilon\tau],\\
\tau(t) &= \int e^{\int 4\lambda(t)dt}dt,\\
\xi(x,t) &= xe^{\int 2\lambda(t)dt} - 2\int e^{\int 4\lambda(t)dt}dt.
\end{aligned}$$

Here A is a real number related to the initial velocity; s_j are arbitrary real constants which denote the backgrounds of the matter rogue waves; and k_j represent the wave vectors of the plane waves in the two components. The parameter μ satisfies the equation as follows:

$$\begin{aligned}
&[3(k_1+k_2)^2 + 4(c_3-c_1)]\mu^4\\
&+ [4(k_1+k_2)^3 + (k_1+k_2)(8c_3-6c_1) - 4c_2]\mu^3\\
&+ [4(k_1+k_2)^2c_3 + 4c_3^2/3 - c_1^2 - 6(k_1+k_2)c_2]\mu^2\\
&+ [4(k_1+k_2)c_3^2/3 - 2c_1c_2]\mu + 4c_3^3/27 - c_2^2 = 0,
\end{aligned}$$

where

$$\begin{aligned}
c_1 &= 2(2k_1-k_2)(2k_2-k_1) + (k_1+k_2)^2 + 9g_1s_1^2 + 9g_2s_2^2,\\
c_2 &= 9g_1s_1^2(2k_2-k_1) + (k_1+k_2)(2k_1-k_2)(2k_2-k_1) + 9g_2s_2^2(2k_1-k_2),\\
c_3 &= (k_1+k_2)^2 - (2k_1-k_2)(2k_2-k_1) + 9g_1s_1^2 + 9g_2s_2^2.
\end{aligned}$$

The parameter ν is given as

$$\nu = H_1(\mu)/H_2(\mu),$$

where

$$\begin{aligned}
H_1(\mu) &= i[2\mu^3 + 3(k_1+k_2)\mu^2 + c_1\mu + c_2],\\
H_2(\mu) &= 6\mu^2 + 6(k_1+k_2)\mu + 2c_3.
\end{aligned}$$

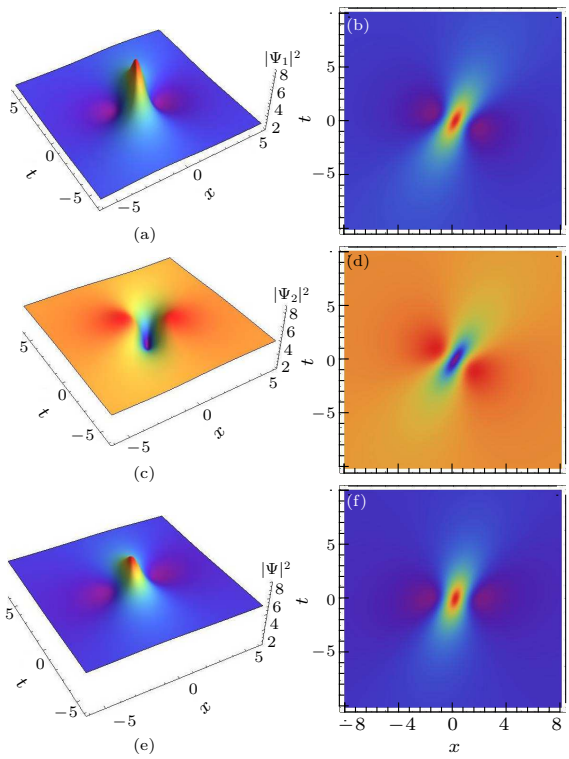

Fig. 1. (Color online) The plot of one rogue wave in a coupled system, (a) for the bright rogue wave in the Ψ_1 component, (c) for the dark rogue wave in the Ψ_2 component, and (e) for the whole density of two components. Here, (b), (d) and (f) are the density plots of (a), (c) and (f). The parameters are $G_j(t) = \lambda(t) = 0$, $\sigma = 1$, $s_1 = 1$, $s_2 = 2$, $A = 3$, $k_1 = -1$, $k_2 = 0.1$.

When $G_j(t) = \lambda(t) = 0$, Fig. 1 depicts rogue waves with different structures in each component. Figure 1(a) shows that the bright rogue wave of a hump and two valleys emerges near the central position in the Ψ_1 component. Obviously, these waves appear out of nowhere and disappear without a trace.[3] Note that a dark rogue wave appears in the Ψ_2 component with a deep valley and two small humps (see Fig. 1(c)), which

was predicted numerically in Ref. [13]. Moreover, the two humps and a valley of dark rogue waves in the Ψ_2 component correspond to the two valleys and the humps of the bright one in the Ψ_1 component. From Fig. 1(e), we can see that the whole density distribution is still in the classical shape.

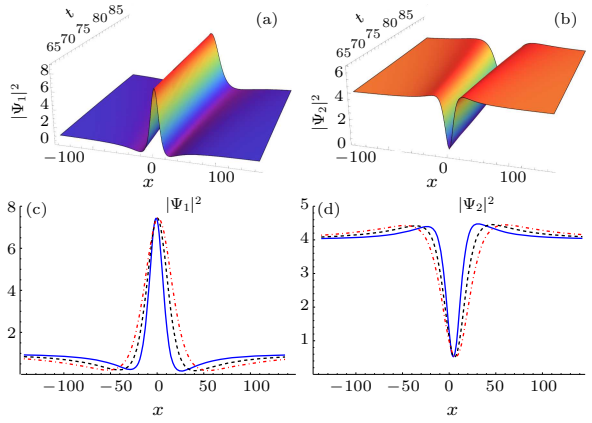

Fig. 2. (Color online) The evolutions of the bright rogue wave (a) and dark rogue wave (b) under Feshbach resonance management. The parameters are the same as those in Fig. 1 but $G_j(t) = \lambda(t) = -0.02$. Here, (c) and (d) give the evolutions of the bright and dark rogue waves at $t = 65, 75, 82$ with solid, dashed, and dot-dashed lines, respectively.

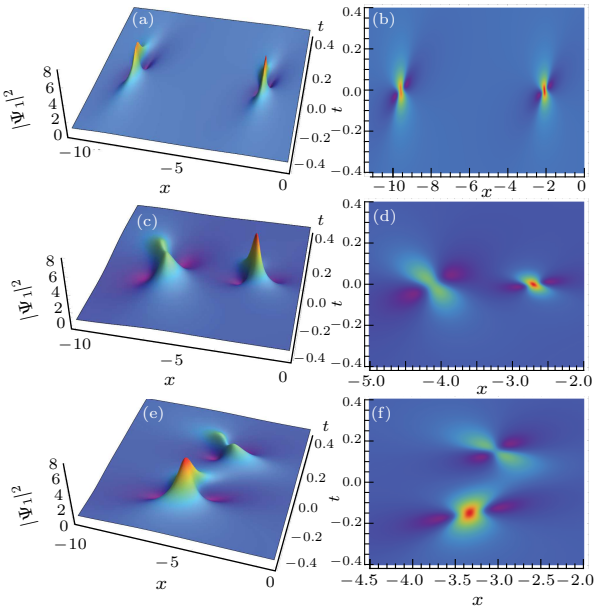

Fig. 3. (Color online) The evolutions of two rogue wave of different appearance in the coupled system with (a) $A_1 = A_2 = 1$, (c) $A_1 = A_2 = 0.52$, and (e) $A_1 = A_2 = 0.44$. Here, (b), (d) and (f) are the density plots of (a), (c) and (e). The other parameters are $G_j(t) = \lambda(t) = 0$, $\sigma = 9$, $s_1 = s_2 = 1$, $A_3 = 0.2$, $k_1 = 1.5$, $k_2 = -1.5$.

It is well known that Feshbach resonance management provides an effective way to change the sign of the scattering length.[13,15] Namely, one can modulate the nonlinearity in the CGP equation. Here we find one way to obtain long-lived rogue waves, which means that the shape of the rogue wave can be maintained under the condition $G_j(t) = \lambda = g$ and g is a nonzero constant, then the nonlinearity should be chosen exponentially as $a_1 = a = 2\sigma e^{4gt}$, $a = a_2 = 2\sigma e^{4gt}$. In Fig. 2 we show an example of such a condition for the gain term and parabolic potential coefficient $G_j(t) = \lambda(t) = -0.02$. It shows that bright and dark rogue waves can exist for a long time and its peak or valley remains invariable in Figs. 2(c) and 2(d). Compared with the case in Fig. 1, rogue waves appear near the central position, then they are sustained and have a long life. It is worth pointing out that long-lived rogue waves on the surface of the sea have been demonstrated by Yang *et al.*,[16] and this shows that long-lived rogue waves are indeed an analog of the line rogue waves.

Case 2: two rogue waves solution and inelastic interaction in BECs.

Recently, multirogue wave solutions have been obtained by different methods in Refs. [10,16,17]. In the following we report a novel phenomenon that is the inelastic interaction between two rogue waves. Under the condition $\sigma s_1^2 = \sigma s_2^2$, $|k_1 - k_2| = \sqrt{\sigma} s_1$, we obtain the solution of two rogue waves as follows:

$$\Psi_j = \phi_j(t)[1 + \frac{3\sqrt{3\sigma} s_1 W_j}{1 + \sigma s_1^2 |W_1|^2 + \sigma s_2^2 |W_2|^2}]\psi_j e^{i\Theta(x,t)}, \tag{4}$$

where

$$W_j = \frac{1}{K_j} + \frac{1}{K_j^2} + \frac{Q_j}{H},$$

$$K_j = -\frac{\sqrt{3\sigma} s_1}{2} + \frac{i(k_j - k_{3-j})}{2},$$

$$\begin{aligned} H = {} & \frac{1}{2} A_3 \xi^2 + \frac{2}{9}(\alpha - k_1 - k_2)^2 A_3 \tau^2 + (A_2 + A_3)\xi \\ & + \frac{2}{3}(\alpha - k_1 - k_2)(A_2 + A_3)\tau + i A_3 \tau \\ & + \frac{2}{3}(\alpha - k_1 - k_2) A_3 \tau \xi + A_1 + A_2 + A_3, \end{aligned}$$

$$Q_j = \frac{A_3}{K_j^3} - \frac{1}{K_j^2} M,$$

and

$$\begin{aligned} M = {} & \frac{1}{2} A_3 \xi^2 + \frac{2}{9}(\alpha - k_1 - k_2)^2 A_3 \tau^2 + A_2 \xi \\ & + \frac{2}{3}(\alpha - k_1 - k_2) A_2 \tau + i A_3 \tau \\ & + \frac{2}{3}(\alpha - k_1 - k_2) A_3 \tau \xi + A_1, \end{aligned}$$

$$\alpha = -(k_1 + k_2)/2 + i 3\sqrt{3\sigma} s_1/2.$$

Here A_1, A_2, A_3 are arbitrary real parameters. For $G_j(t) = \lambda(t) = 0$, if $A_3 \neq 0$, two rogue waves coexist and interact with each other, and depending on the values of A_1 and A_2, they have a different appearance, as shown in Fig. 3. When $A_3 = 0.2$, $A_1 = A_2 = 1$, Fig. 3(a) shows two rogue waves, R_1 and R_2, of classical shapes together with each other. Similarly, the two

rogue waves also disappear without a trace after the interaction with each other. Interestingly, by decreasing the values A_1 and A_2, two rogue waves, R_1 and R_2, attract and interact with each other. In particular, Figs. 3(c) and 3(e) show rogue wave R_1 with a classical shape together with another rogue wave R_2 with a new structure of two humps and two valleys. This indicates that the interaction between the two rogue waves is an inelastic collision. This comes from the effects of interspecies' interaction because this novel phenomenon cannot be observed in an uncoupled system. The results for Ψ_2 are similar and we do not show them. In $A_3 = 0$, A_1, $A_2 \neq 0$, this solution becomes the case where only one rogue wave exists in each component.

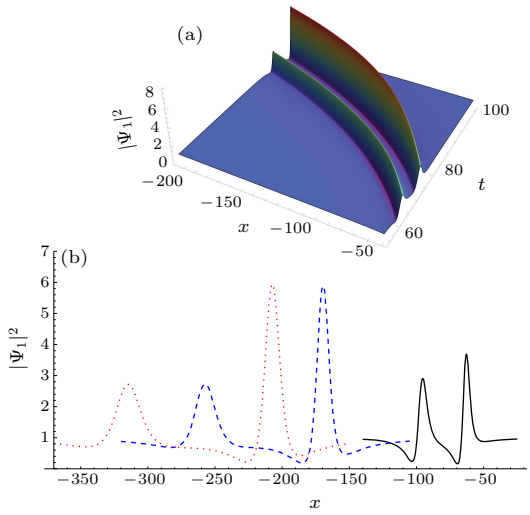

Fig. 4. (Color online) (a) The evolution of two rogue waves under the condition $G_j(t) = \lambda(t) = -0.02$, $A_1 = A_2 = 0.52$. The other parameters are the same as those in Fig. 3. (b) The profile of (a) at t=79, 103.5, 108.5 with solid, dashed, and dotted lines, respectively.

Considering the Feshbach resonance management used for the bright and dark rogue waves, we plot the evolution of rogue waves in the Ψ_1 component as shown in Fig. 4. One can obviously see that two rogue waves, R_1 and R_2, are sustained as t increases. This indicates that not only can one rogue wave (bright or dark) maintain its shape, but also two rogue waves can be sustained under certain conditions. Moreover, Fig. 4(b) shows the interaction between the self-accelerating rogue waves at different times with a strong variation in peak amplitudes. Note that the varying-velocity soliton solutions are characteristic of other nonlinear multi-component systems.[18,19] Here we can see that rogue waves R_1 and R_2 with small amplitudes interact with each other at $t = 79$. Then, the amplitude of R_1 increases rapidly while the amplitude of R_2 decreases gradually as t increases, which indicates some energy exchange between them. After that, two rogue waves achieve balance, i.e. maintain their shapes and their amplitudes remain unchanged.

In summary, we analytically present bright, dark and two rogue wave solutions in the two-component BEC. We find an effective way to trap rogue waves by means of modulating the trapping frequency and interchange atoms between thermal clouds and condensates. It is indicated that rogue waves maintain their shapes and cannot disappear once they appear. Moreover, the interaction between one rogue wave of a new structure, which contains two valleys and two humps around one center, and the classical rogue wave is reported.

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