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Three-Dimensional Sound Propagation and Scattering in Two-Dimensional Waveguides *

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A coupled-mode method for three-dimensional acoustic propagation and scattering in two-dimensional waveguides is presented. This method synthesizes the three-dimensional field solution by using Fourier transform techniques based on a sequence of two-dimensional problems, each of which is solved by a numerical model recently developed by Luo et al. [Chin. Phys. Lett. 29 (2012) 014302]. Numerical results indicate that the present model is remarkably accurate, and thus can serve as benchmark against other numerical models. In addition, this model can be applied to realistic problems, and can also be used to analyze horizontal refraction in some range-dependent waveguides in reality, such as the continental shelf environment, ridge-like bathymetry, and underwater trenches.

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Over the past several decades, a large number of numerical models have been developed for underwater acoustic propagation. Most of these models provide solutions for two-dimensional (2D) problems, and they provide satisfactory solutions for the majority of propagation problems where the environmental dependence on azimuth is insignificant. However, acoustic propagation and scattering in the ocean usually involves fully three-dimensional (3D) effects. The full solution of general 3D propagation and scattering is very intensive computationally. The finite difference method[1] and the finite element method[2] can be applied to such problems, while the computational size may be excessively large for long-range propagation. The $N\times$2D approach introduced first by Perkins and Baer[3] is highly efficient in solving 3D propagation problems. It assumes out-of-plane scattering to be insignificant, and is therefore applicable only to problems with weak transverse environmental variability. Various 3D approaches have been developed. Some of these 3D models, such as the adiabatic mode parabolic equation approach,[4] the coupled mode parabolic equation approach,[5] and FOR3D, a 3D parabolic equation model presented in Refs. [6,7], can provide approximate solutions for 3D problems.

The 3D problem involving a 2D waveguide is sometimes referred to as the two-and-half-dimensional (2.5D) problem.[8–10] In some special cases, analytical solutions are available for such 2.5D problems, for example, acoustic propagation in a perfect wedge,[11,12] or in a penetrable wedge.[13] In general, however, numerical methods must be employed. The 2.5D problem of acoustic propagation can be solved in a straightforward manner by using the Fourier transform method to reduce the problem to a sequence of 2D ones, the solutions of which can then be used to synthesize the 3D field. The methods in Refs. [8–10] and in the present study belong to this category.

Recently, a coupled-mode method with the use of the direct global matrix (DGM) approach was proposed by Luo *et al.*,[14,15] hereafter referred to as the DGMCM model. This method is capable of handling 2D problems with either a point source in cylindrical geometry or a line source in plane geometry. Due to the use of the DGM approach, this method is numerically stable. In addition, the introduction of the general source condition enables this method to handle the scenario where a line source is located right on top of a sloping bottom. The DGMCM model is capable of accurately and efficiently modeling propagation and scattering in very complicated waveguides, but it is restricted to 2D problems.

In this Letter, we extend the DGMCM model to a version capable of handing the 2.5D problem. Even with the restricted waveguide geometry, the extended model is nonetheless more physically realistic. We first give a summary of the 2D DGMCM theory. Second, we extend this 2D model to be able to solve the 2.5D problem with the use of the Fourier synthesis technique, and discuss the numerical implementation. Third, we apply this 2.5D model to solve an ideal wedge problem, and compare the result with the analytical solution to demonstrate the accuracy of this method.

The formulation of 2D line-source problems will be used to compute the 3D acoustic field. Thus, we give the DGMCM theory for the line-source problem in plane geometry. In Cartesian coordinates, with the location of a harmonic line source of angular frequency ω denoted by (x_s, z_s), the Helmholtz equation is[16]

$$\frac{\partial^2 p}{\partial x^2} + \rho(z)\frac{\partial}{\partial z}\left(\frac{1}{\rho(z)}\frac{\partial p}{\partial z}\right) + \frac{\omega^2}{c^2(z)}p = -\delta(x-x_s)\delta(z-z_s). \quad (1)$$

We divide the range-dependent waveguide into a number of range-independent segments, and represent the field solution in segment j as

$$p^j(x,z) = \sum_{n=1}^{N}[a_n^j E_{1n}^j(x) + b_n^j E_{2n}^j(x)]\,\Psi_n^j(z), \quad (2)$$

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where N is the total number of normal modes involved in mode coupling, a_n^j and b_n^j are modal amplitudes corresponding to the forward and backward propagating waves respectively, $\Psi_n^j(z)$ are the local vertical eigenfunctions, $E_{1n}^j(x)$ and $E_{2n}^j(x)$ are normalized exponential functions defined as follows:

$$E_{1n}^j(x) = e^{ik_{xn}^j(x-x^{j-1})}, \tag{3}$$

$$E_{2n}^j(x) = e^{-ik_{xn}^j(x-x^j)}, \tag{4}$$

with k_{xn}^j denoting the horizontal wavenumbers. The solutions for $\boldsymbol{a}^j$ and $\boldsymbol{b}^j$ in each segment are obtained by solving the following linear system of equations,

$$\boldsymbol{C}\boldsymbol{x} = \boldsymbol{v}, \tag{5}$$

where $\boldsymbol{x}$ is the column vector

$$\boldsymbol{x} = [\boldsymbol{b}^1\ \boldsymbol{a}^2\ \boldsymbol{b}^2\ \boldsymbol{a}^3\ \cdots\ \boldsymbol{b}^{j_s}\ \boldsymbol{a}^{j_s+1}\ \cdots\ \boldsymbol{b}^{J-1}\ \boldsymbol{a}^J]^{\mathrm{T}}, \tag{6}$$

$\boldsymbol{v}$ is the column vector

$$\boldsymbol{v} = [\boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \boldsymbol{0}\ \ \cdots\ \ \boldsymbol{0}\ \ \boldsymbol{s}\ \ \cdots\ \ \boldsymbol{0}\ \ \boldsymbol{0}]^{\mathrm{T}}, \tag{7}$$

with the superscript T denoting transpose, and $\boldsymbol{s}$ being the column vector with entries

$$s_n = -\frac{i}{\rho(z_{\mathrm{s}})}\frac{\Psi_n^{j_s}(z_{\mathrm{s}})}{k_{xn}^{j_s}}, \quad n = 1, 2, \ldots, N, \tag{8}$$

and $\boldsymbol{C}$ is the global coefficient matrix

$$\boldsymbol{C} = \begin{bmatrix}
\widetilde{\boldsymbol{C}}^1 & -\boldsymbol{I} & -\boldsymbol{E}_2^2 & & & & & & & \\
-\widehat{\boldsymbol{C}}^1 & -\boldsymbol{I} & \boldsymbol{E}_2^2 & & & & & & & \\
 & \widetilde{\boldsymbol{C}}^2\boldsymbol{E}_1^2 & \widetilde{\boldsymbol{C}}^2 & -\boldsymbol{I} & -\boldsymbol{E}_2^3 & & & & & \\
 & \widehat{\boldsymbol{C}}^2\boldsymbol{E}_1^2 & -\widehat{\boldsymbol{C}}^2 & -\boldsymbol{I} & \boldsymbol{E}_2^3 & & & & & \\
 & & \ddots & \ddots & \ddots & \ddots & & & & \\
 & & & & \widetilde{\boldsymbol{C}}^{j_s}\boldsymbol{E}_1^{j_s} & \widetilde{\boldsymbol{C}}^{j_s} & -\boldsymbol{I} & -\boldsymbol{E}_2^{j_s+1} & & \\
 & & & & \widehat{\boldsymbol{C}}^{j_s}\boldsymbol{E}_1^{j_s} & -\widehat{\boldsymbol{C}}^{j_s} & -\boldsymbol{I} & \boldsymbol{E}_2^{j_s+1} & & \\
 & & & & & \ddots & \ddots & \ddots & \ddots & \\
 & & & & & & \widetilde{\boldsymbol{C}}^{J-1}\boldsymbol{E}_1^{J-1} & \widetilde{\boldsymbol{C}}^{J-1} & -\boldsymbol{I} \\
 & & & & & & \widehat{\boldsymbol{C}}^{J-1}\boldsymbol{E}_1^{J-1} & -\widehat{\boldsymbol{C}}^{J-1} & -\boldsymbol{I}
\end{bmatrix}. \tag{9}$$

Coupling matrices $\widetilde{\boldsymbol{C}}^j$ and $\widehat{\boldsymbol{C}}^j$ are respectively defined as

$$\widetilde{C}_{\nu n}^j = \int_0^\infty \frac{1}{\rho^{j+1}(z)}\Psi_\nu^{j+1}(z)\,\Psi_n^j(z)dz, \tag{10}$$

$$\widehat{C}_{\nu n}^j = \frac{k_{xn}^j}{k_{x\nu}^{j+1}}\int_0^\infty \frac{1}{\rho^j(z)}\Psi_\nu^{j+1}(z)\,\Psi_n^j(z)dz. \tag{11}$$

In the following, we introduce the formulation for the 2.5D problem as depicted in Fig. 1. The acoustic properties in the waveguide are dependent on the coordinates x and z, but independent of the coordinate y. When a point source is located at $(x_{\mathrm{s}}, 0, z_{\mathrm{s}})$, the complex pressure satisfies the following 3D Helmholtz equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \rho(z)\frac{\partial}{\partial z}\Big(\frac{1}{\rho(z)}\frac{\partial p}{\partial z}\Big) + \frac{\omega^2}{c^2(z)}p = -\delta(x-x_{\mathrm{s}})\delta(y)\delta(z-z_{\mathrm{s}}). \tag{12}$$

The Fourier transform is defined as[10]

$$\tilde{p}(x, k_y, z) = \int_{-\infty}^{\infty} p(x, y, z)e^{-ik_y y}dy. \tag{13}$$

Since the waveguide is independent of the coordinate y, by taking the Fourier transform of Eq. (12), we obtain

$$\frac{\partial^2 \tilde{p}}{\partial x^2} + \rho(z)\frac{\partial}{\partial z}\Big(\frac{1}{\rho(z)}\frac{\partial \tilde{p}}{\partial z}\Big) + \Big(\frac{\omega^2}{c^2(z)} - k_y^2\Big)\tilde{p} = -\delta(x-x_{\mathrm{s}})\delta(z-z_{\mathrm{s}}). \tag{14}$$

Comparing with Eq. (1), we observe that the resulting problem is 2D for each specified value of k_y. Here we solve Eq. (14) using the DGMCM model,[14,15] with $k^2(z) - k_y^2$ playing the role that $k^2(z)$ usually plays, where the wavenumber $k(z) = \omega/c(z)$.

After the pressure $\tilde{p}(x, k_y, z)$ has been computed for a sequence of k_y values, the solution $p(x, y, z)$ can be obtained by the inverse Fourier transform,[10]

$$p(x, y, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{p}(x, k_y, z)e^{ik_y y}dk_y. \tag{15}$$

As clearly explained in Refs. [9,10], $\tilde{p}(x, k_y, z)$ in Eq. (15) has singularities at the eigenvalues of the 2D problem. We use contour integration to avoid the singularities, which is analogous to the method used in the SAFARI model.[17] Here k_y is taken along the contour in the complex plane described by the functional form,[9]

$$k_y = s - i\epsilon\tanh(\delta s), \quad -\infty < s < \infty, \tag{16}$$

where s is the real part of k_y, and ϵ and δ are the parameters governing the integration contour. The integration contour is above the real axis for $s < 0$

and below the real axis for $s > 0$. Equation (15) thus becomes

$$p(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{p}[x,k_y(s),z]e^{ik_y(s)y} \cdot [1 - i\epsilon\delta \mathrm{sech}^2(\delta s)]ds. \quad (17)$$

In implementation, the integration employs a trapezoidal rule, and symmetry may be exploited to avoid solving Eq. (14) for $s < 0$.

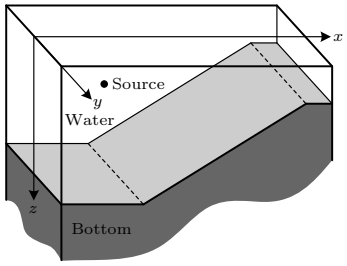


Fig. 1. Geometry of the 2.5D problem involving a point source in a waveguide in which the acoustic properties vary with the coordinates x and z, but not with the coordinate y.

Consider the wedge problem as illustrated in Fig. 2. The environment consists of a homogeneous water column with sound speed $c_w = 1500\,\mathrm{m/s}$ and density $\rho_w = 1.0\,\mathrm{g/cm^3}$, bounded by plane, pressure-release boundaries. The wedge angle is approximately 3.0°. We set the origin on the sea surface where the water depth is 252.41 m. The point source is located at range (x) 1.0 km, cross range (y) 0 km, and depth (z) 100 m, where the water depth is 200 m. The source frequency is 25 Hz.

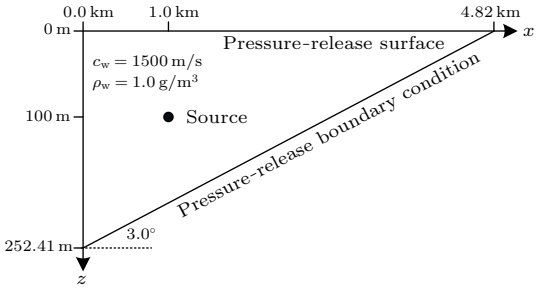


Fig. 2. Geometry of the ideal wedge problem involving pressure-release boundaries.

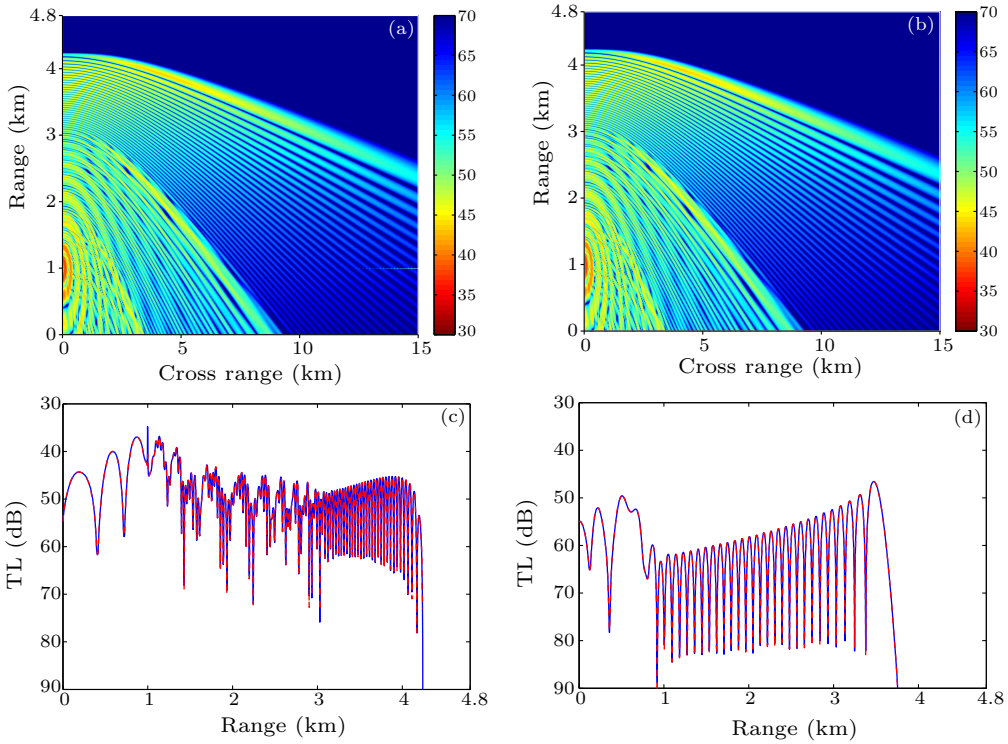


Fig. 3. Field solutions for the 3.0° wedge problem with pressure-release boundaries. The source is located at range 1.0 km, cross range 0 km, and depth 100 m. The source frequency is 25 Hz. (a) TL in the horizontal plane at depth 30 m by the present model; (b) TL in the horizontal plane at depth 30 m by the analytical solution; (c) TL versus range at $y = 0$ km; (d) TL versus range at $y = 7.5$ km. In (c) and (d), the solid line in blue corresponds to the analytical solution, and the dashed line in red corresponds to the solution of the present model.

In the implementation of the present model, 513 values of k_y are used to compute the integral (17) for $s \geq 0$, and values of the integrand for $s < 0$ are computed using symmetry. To obtain convergent 2D solutions for each k_y value, a total of 30 normal modes is involved in mode coupling and a total of 400 segments is used to represent the slope. For such an ideal wedge problem, the analytical solution presented in Refs. [11,12] is used as the reference solution. Because the wedge angle is a submultiple of π, the diffracted component of the sound field is identically zero.[11,12] Thus, the diffracted component need not be considered in this analytical solution. Figure 3 shows the results of transmission loss (TL) in the horizontal plane at depth of 30 m (Figs. 3(a) and 3(b)), and TL versus range along 0-km and 7.5-km cross range (Figs. 3(c) and 3(d)). Sound field in the horizontal plane is symmetric with respect to the x axis, so we only give the

results in the half plane in Figs. 3(a) and 3(b). As can be seen, the agreement between the solution by the present model and the reference solution is excellent, indicating that the present model is remarkably accurate.

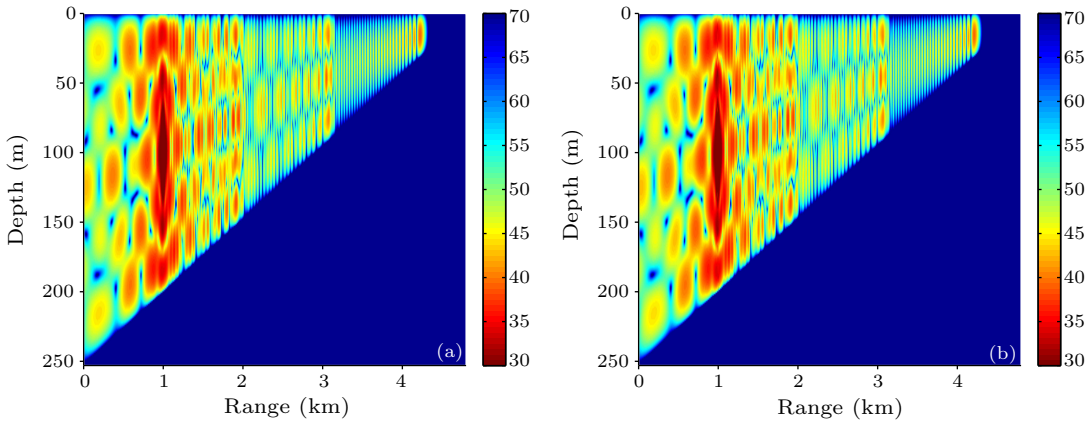


Fig. 4. Field solutions in the vertical plane of cross range $y = 0$ km for the 3.0° wedge problem with pressure-release boundaries. The source is located at range 1.0 km, cross range 0 km, and depth 100 m. The source frequency is 25 Hz. (a) TL by the present model; (b) TL by the analytical solution.

A comparison of the field results in the vertical plane $y = 0$ km by the present model and by the reference solution is provided in Fig. 4, which also shows excellent overall agreement. Moreover, the mode stripping effect, which is clearly explained in Ref. [18], is evident in Fig. 4 in the upslope region, that is, from 1 km to 4.82 km.

The ideal wedge problem is a particularly stringent test for numerical propagation and scattering models. For this kind of 2.5D wedge problem, the analytical solution presented in Refs. [11,12] is extremely useful as a primary benchmark. The excellent agreement between the analytical solution and the present model establishes the accuracy of the latter. Consequently, with the use of closed-form expressions for local modal solutions and their associated coupling matrices, the present model can serve as a secondary benchmark against other 2.5D and 3D numerical models. Note that the present model can be applied to general range-dependent waveguides, not necessarily wedge-shaped waveguides. Moreover, this model can be easily generalized to more realistic waveguides. For this purpose, a numerical normal mode model such as KRAKEN[19] should be adopted to provide local modal solutions and their associated coupling matrices.

In summary, a coupled-mode model for 3D sound propagation and scattering in a 2D range-dependent waveguide has been presented. The 3D field in the waveguide can be synthesized by a sequence of 2D problems, each of which is solved by DGMCM.[14,15] The numerical results indicate that the present method is remarkably accurate. Thus, this model can serve as benchmark against other 2.5D and 3D numerical models. In the future, the present model will be generalized to solve realistic 2.5D problems. Moreover, the present model will be used to analyze the phenomenon of horizontal refraction in some range-dependent waveguides in reality, for instance, the continental shelf environment, ridge-like bathymetry, and underwater trenches.

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