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Dynamics Evolution Investigation of Mack Mode Instability in a Hypersonic Boundary Layer by Bicoherence Spectrum Analysis *

HAN Jian(韩健)[1,2], JIANG Nan(姜楠)[1,2,3]

[1]*Department of Mechanics, Tianjin University, Tianjin 300072*
[2]*Department of Mathematics, Tianjin University, Tianjin 300072*
[3]*Tianjin Key Laboratory of Modern Engineering Mechanics, Tianjin 300072*

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The instability of a hypersonic boundary layer on a cone is investigated by bicoherence spectrum analysis. The experiment is conducted at Mach number 6 in a hypersonic wind tunnel. The time series signals of instantaneous fluctuating surface-thermal-flux are measured by Pt-thin-film thermocouple temperature sensors mounted at 28 stations on the cone surface along streamwise direction to investigate the development of the unstable disturbances. The bicoherence spectrum analysis based on wavelet transform is employed to investigate the nonlinear interactions of the instability of Mack modes in hypersonic laminar boundary layer transition. The results show that wavelet bicoherence is a powerful tool in studying the unstable mode nonlinear interaction of hypersonic laminar-turbulent transition. The first mode instability gives rise to frequency shifts to higher unstable modes at the early stage of hypersonic laminar-turbulent transition. The modulations subsequently lead to the second mode instability occurrence. The second mode instability governs the last stage of instability and final breakdown to turbulence with multi-scale disturbances growth.

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Hypersonic flow instability and transition from laminar flow to turbulent flow are critical aspects in the design of hypersonic vehicles as they are characterized by obvious increases in surface heat flux and skin friction.[1] The higher surface heating and skin friction cause extra burdens for the thermal protection system and thus result in a reduction of payload,[2] as well as significant impacts on the performance control, fuel consumption, and structural design of hypersonic vehicles.[3] The accurate prediction of the laminar-turbulence transition position[4] is the key in designing hypersonic vehicles. The laminar-turbulence transition process is characterized by the development of instability modes in a boundary layer. Unlike the case in a subsonic flow, the instability in a hypersonic boundary layer includes more complicated factors such as the first mode (T-S waves), cross-flow vortices, Görtler vortices, and the second mode.[5] Among them, the second mode instability[6] is dominant and governs the occurrence and development of laminar-turbulence transition in two-dimensional or axisymmetric hypersonic boundary layer flow.

Wavelet transform[7] is a powerful mathematical technique with enormous applications in many fields of science and engineering. It convolutes time series signals with an analytic function named wavelet at a definite time and a definite scale by means of dilations and translations of the mother wavelet. It provides a two-dimensional unfolding of one-dimensional signals, resolving both time and temporal scale as independent variables. Thus this method comprises a decomposition of signals on both time and scale planes simultaneously. The wavelet transform technique overcomes the shortcomings of Fourier analysis by adding the time resolution and representing temporal characteristics of signals simultaneously.

A wavelet $\psi_{ab}(t)$ is the shift (by factor b) and dilatation (by factor a) of a mother wavelet $\psi(t)$:

$$\psi_{ab}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a}) \tag{1}$$

with $a, b \in R$ and $a > 0$.

The continuous wavelet transform is defined as the scalar product of signal $s(t)$ with respect to a family of wavelet $\psi_{ab}(t)$:

$$\psi_s(a,b) = \int_{-\infty}^{+\infty} s(t)\bar{\psi}_{ab}(t)dt, \tag{2}$$

where $\bar{\psi}_{ab}(t)$ indicates the complex conjugate of $\psi_{ab}(t)$. Wavelet analysis extends Fourier analysis by shifting time windows with a proper time interval (window width). This is appropriate for non-stationary signals with multi-scale local singular structure or instantaneous disturbance.

Since the modulation of Mack modes during the nonlinear evolution may be important in a hypersonic transition, the wavelet bispectrum[8–12] is a suitable technique which combines the dual time-frequency analysis of the wavelet transform with the ability to

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**Corresponding author. Email: hanjian@tju.edu.cn

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quantify quadratic phase coupling through the bispectrum. The wavelet bispectrum is defined as

$$B_s(a_1, a_2) = \int_{-\infty}^{+\infty} \psi_s(a, b)\psi_s(a_1, b)\psi_s(a_2, b)db, \quad (3)$$

where $1/a = 1/a_1 + 1/a_2$ is equal to $f = f_1 + f_2$, because of $c/a = f$, $c/a_1 = f_1$, $c/a_2 = f_2$; $\psi_f(a, b)$, $\psi_f(a_1, b)$, $\psi_f(a_2, b)$ are the wavelet coefficients corresponding to different frequencies of f, f_1, and f_2, respectively.

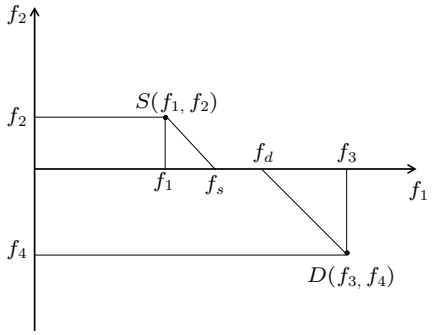


Fig. 1. Schematic of bicoherence showing regions of frequency sum and difference interactions.

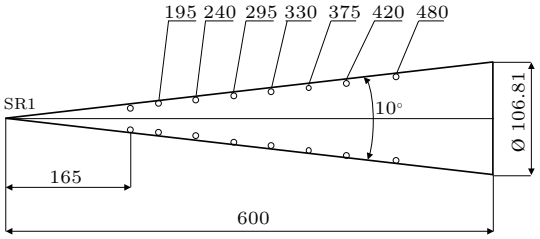


Fig. 2. Schematic diagram of the cone model.

The normalized bispectrum, or bicoherence spectrum, is defined as

$$b_s(a_1, a_2) = \frac{B_s(a_1, a_2)}{\sqrt{P_s(a)P_s(a_1)P_s(a_2)}}, \quad (4)$$

with

$$P_s(a) = \int_{-\infty}^{+\infty} |\psi_s(a, b)|^2 db. \quad (5)$$

The bicoherence spectrum is used to detect the nonlinear disturbance interactions. The bicoherence measures the coherence degree amongst a wave triad due to wave coupling. Figure 1 shows the quadratic interaction between waves at frequencies f_1 and f_2. If the waves present at f_1, f_2, and $f = f_1 + f_2$ are characterized as statistically independent random processes, the bicoherence will be close to zero. If the triad of waves f_1, f_2, and $f = f_1 + f_2$ are nonlinearly coupled, the statistical average will yield a nonzero value of the bicoherence. The bicoherence can be interpreted as a quantitative measure of the fraction of power at $f = f_1 + f_2$ due to a quadratic interaction between waves at f_1 and f_2 (see Fig. 1).

We performed the experiment in a blow-down type hypersonic wind tunnel. This tunnel is designed for Mach numbers (M_∞) of 5–15 with the unit Reynolds numbers Re_1 of 5×10^5–7×10^7) m^{-1}. There are a set of conic axis-symmetric nozzles with different shapes. An ohmic heater is used for maintaining air temperature in the settling chamber up to 700–1300 K. The open-jet test section is 1600 mm long with a nozzle end diameter of 480 mm. The stagnation pressure P_0 and stagnation temperature T_0 were measured and kept constant during the experiment. The stagnation pressure P_0 was 2.69 MPa, and the stagnation temperature T_0 was 702 K. The unit Reynolds number (Re_1) was 1.3×10^7 m^{-1}.

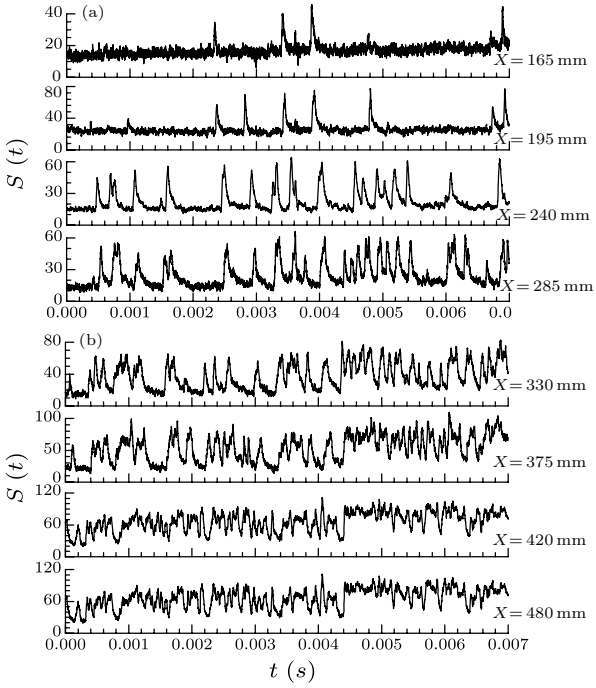


Fig. 3. Signals of instantaneous fluctuating surface-thermal-flux at different stations along one generatrix on the cone.

The model was a 5° half-angle sharp cone of 600 mm in length and 106.8 mm in diameter. Figure 2 shows the schematic of the model. The experiment model, consisting of a cone nose of 1 mm in bluntness radius, was equipped with 28 Pt-thin-film thermocouple sensors on the surface to measure the wall temperature. The thermocouple sensors were mounted along a generatrix. The first thermocouple was located at 165 mm from the cone apex. The output signals were digitized by a 12-bit analog-to-digital converter data acquisition card with ±5 V range and 1 MHz sampling rate. At each measurement station, 524288 samples were recorded.

Figure 3 shows the instantaneous fluctuating surface-thermal-flux signals measured by Pt-thin-film thermocouple sensors at 8 stations along one generatrix. For clarity purposes, we only plot a section

of 7 ms of the signals in these figures. We can see that the high-magnitude unstable disturbances occur at different times and at different stations. More large-amplitude and multi-scale disturbances are triggered downstream with the development of instability.

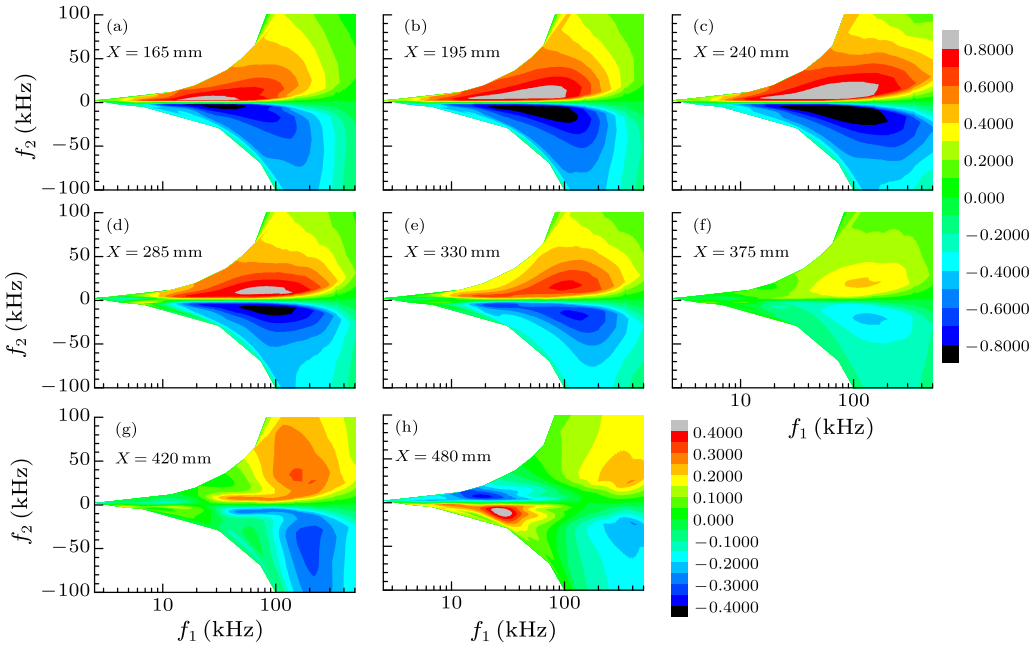


Fig. 4. Bicoherence spectra at different stations along one generatrix on the cone: (a) $X = 165$ mm, (b) $X = 195$ mm, (c) $X = 240$ mm, (d) $X = 285$ mm, (e) $X = 330$ mm, (f) $X = 375$ mm, (g) $X = 420$ mm, (h) $X = 480$ mm.

Table 1. The sum and difference bicoherence at 8 stations along one generatrix in units of kHz.

	Station 1 X=165 mm		Station 2 X=195 mm		Station 3 X=240 mm		Station 4 X=285 mm		Station 5 X=330 mm		Station 6 X=375 mm		Station 7 X=420 mm		Station 8 X=480 mm	
f_1	20.4	23.0	48.8	55.2	54.8	61.5	77.8	87.9	116.9	131.3	124.0	145.6	165.9	198.4	375.1	398.0
f_2	2.6	−2.6	6.4	−6.4	6.7	−6.7	10.1	−10.1	14.4	−14.4	21.6	−21.6	32.5	−32.5	22.9	−22.9
Sum	23.0		55.2		61.5		87.9		131.3		145.6		198.4		398.0	
Difference		20.4		48.8		54.8		77.8		116.9		124.0		165.9		375.1

Figure 4 shows the bicoherence at 8 stations along one generatrix on the cone surface. At $X = 165$ mm, there are two low-frequency module peaks of bicoherence (see Fig. 4(a)). One peak is at $f_1 = 20.4$ kHz with $f_2 = 2.6$ kHz, and the sum frequency $f_1 + f_2 = 23$ kHz, where f_2 is the sideband of f_1. The other peak is at $f_1 = 23$ kHz, $f_2 = 2.6$ kHz and the difference frequency $f_1 - f_2 = 20.4$ kHz. At $X = 195$ mm, the low-frequency peaks of bicoherence extend to slightly higher frequencies. One peak is at $f_1 = 48.8$ kHz, $f_2 = 6.4$ kHz, and the sum frequency $f_1 + f_2 = 55.2$ kHz, where f_2 is also the sideband of f_1. The other peak is at $f_1 = 55.2$ kHz, $f_2 = 6.4$ kHz, and the difference frequency $f_1 - f_2 = 48.8$ kHz. At the downstream station $X = 240$ mm, the energy spreads into the sidebands and the energy increases at the first mode instability. The low-frequency peaks of bicoherence extend to even higher frequencies with one peak at $f_1 = 54.8$ kHz, $f_2 = 6.7$ kHz and the sum frequency $f_1 + f_2 = 61.5$ kHz. The other peak is at $f_1 = 61.5$ kHz, $f_2 = 6.7$ kHz and the difference frequency $f_1 - f_2 = 54.8$ kHz. The bicoherence at $X = 285$ mm characterizes the enhanced level of nonlinear interactions where the sum frequency peak of bicoherence is at $f_1 = 77.8$ kHz, $f_2 = 10.1$ kHz and $f_1 + f_2 = 87.9$ kHz, while the difference frequency peak is at $f_1 = 87.9$ kHz, $f_2 = 10.1$ kHz and $f_1 - f_2 = 77.8$ kHz. At $X = 330$ mm, the sum frequency peak of bicoherence is at $f_1 = 116.9$ kHz, $f_2 = 14.4$ kHz and the sum frequency $f_1 + f_2 = 131.3$ kHz while the difference frequency $f_1 - f_2 = 116.9$ kHz with $f_1 = 131.3$ kHz, $f_2 = 14.4$ kHz. At $X = 375$ mm, the sum frequency peak of bicoherence is at $f_1 = 124.0$ kHz, $f_2 = 21.6$ kHz with the sum frequency $f_1 + f_2 = 145.6$ kHz while the difference frequency $f_1 - f_2 = 124.0$ kHz with $f_1 = 145.6$ kHz, $f_2 = 21.6$ kHz. At $X = 420$ mm, the sum frequency peak of bicoherence is at $f_1 = 165.9$ kHz, $f_2 = 32.5$ kHz with the sum $f_1 + f_2 = 198.4$ kHz while the difference frequency $f_1 - f_2 = 165.9$ kHz with $f_1 = 198.4$ kHz, $f_2 = 32.5$ kHz meaning the second mode instability is weakly modulated at this station. At the last station, $X = 480$ mm, there are two pairs of local frequency peaks of bicoherence. The first one is the second mode

instability with the sum frequency $f_1+f_2 = 398.0\,\text{kHz}$ at $f_1 = 375.1\,\text{kHz}$, $f_2 = 22.9\,\text{kHz}$, and the difference frequency $f_1 - f_2 = 375.1\,\text{kHz}$ at $f_1 = 398.0\,\text{kHz}$, $f_2 = 22.9\,\text{kHz}$. The most significant is that a new first mode instability interaction is observed at this station. The first mode instability has the sum frequency $f_1 + f_2 = 32.3\,\text{kHz}$ at $f_1 = 21.6\,\text{kHz}$, $f_2 = 10.7\,\text{kHz}$ and the difference frequency $f_1 - f_2 = 17.5\,\text{kHz}$ at $f_1 = 28.9\,\text{kHz}$, $f_2 = 11.4\,\text{kHz}$. This suggests that some low-frequency modulations of the first mode instability are also regenerated and the triggered multi-scale disturbances lead to the breakdown to turbulent flow. The sum and difference bicoherence evolutions at these 8 stations are summarized in Table 1.

In summary, the bicoherence spectrum analysis method based on wavelet transform is a powerful tool for studying the unstable mode nonlinear interactions in hypersonic laminar-turbulent transition. At hypersonic speeds, the nonlinear interactions of Mack mode disturbances are dominant in the hypersonic laminar-turbulence transition process. The nonlinear interactions include the first mode with its sideband and the high-frequency second mode. The first mode interaction with its sideband first leads to frequency shifts to higher unstable modes in the early stage of hypersonic transition. The modulations subsequently lead to the second mode instability occurrence. The second mode governs the last stage instability and finally breakdown to turbulence of hypersonic laminar-turbulent transition with multi-scale disturbances growth.

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