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Superradiant Laser with Ultra-Narrow Linewidth Based on ^{40}Ca *

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An experiment setup for narrow linewidth superradiant laser based on the mechanism of an active optical clock is proposed with optical lattice trapped ^{40}Ca. We obtain the threshold pumping rate analytically and deduce the linewidth of the superradiant laser in a bad cavity regime. The proposed laser has an extremely narrow linewidth at millihertz level and a power level of order 10^{-9} W.

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With advantages in accuracy and stability compared to microwave atomic clocks, optical clocks have attracted increasing interest. Optical clocks have important applications not only for fundamental science, but also for applied technologies.[1−4] However, the linewidth of the probe laser limits the linewidth of spectrum for the clock transition.[3] Since the thermal noise of the cavity in conventional Pound–Drever–Hall locking technique is inevitable, the laser linewidth is limited in the subhertz level.[5,6] In order to fully take advantage of the ultranarrow clock transitions, the laser linewidth should be as narrow as possible. If laser linewidth is improved to millihertz level, the stability of atomic clocks can be improved by several orders of magnitude. Therefore, how to obtain the lasers with narrower linewidth becomes a challenge in precision measurement.

In 1958, Schawlow and Townes derived the formula of laser linewidth, which can be expressed as $\Delta\nu_{\rm ST} = \kappa/(2I_0)$, where κ is the cavity loss rate and I_0 is the intensity of laser in the cavity in units of the photon number.[7] After that, the laser linewidth was discussed in general cases, including good cavity case and bad cavity case.[8] Chen *et al.*[9−12] proposed the mechanism of the active optical clock to realize millihertz linewidth laser with reduced cavity thermal noise due to cavity pulling effect in bad cavity configuration. Instead of using conventional good cavity lasers, he made the experiment setup work in the bad cavity regime, in which case the cavity loss rate is much larger than the atomic damping rate. Due to the fact that the linewidth of the cavity is much wider than that of the laser gain profile, the active optical clock is insensitive to the cavity thermal noise. The laser can be directly used as an optical reference. The theory of superradiant laser was established by Fritz Haake *et al.*[13,14] They obtained the result that the intensity I of radiation is proportional to N^2 and the linewidth scales as $1/N^2$. The similar characters were discussed in a four-level superradiant laser.[15] Recently, following the concept of the active optical clock, steady-state superradiance has been analyzed with alkaline-earth-metal atoms to reach millihertz linewidth laser sources.[16−19]

Due to its ultra-narrow natural linewidth between $^3P_1$ and $^1S_0$, ^{40}Ca is a very promising choice for optical frequency standards.[20−23] Recently, Bose–Einstein condensation was realized in a gas of ^{40}Ca.[24] Based on the foundation discussed above, in this Letter, we study the superradiant laser based on the mechanism of the active optical clock with ^{40}Ca in detail, and discuss the power of superradiant laser, the threshold pumping rate, and the laser linewidth.

The proposed experimental setup is shown in Fig. 1.(a). The number of the ultracold ^{40}Ca atoms confined in an optical lattice is N. The wavelength of the optical lattice is 800.8 nm, which is the magic wavelength.[25] The cavity is a high Q optical resonator with resonance frequency ω_c. The cavity decay rate is κ, which is much larger than the atomic relaxation rate.

Energy level structure of ^{40}Ca is shown in Fig. 1(b). The upper level or excited state is $^3P_1$ described as $|e\rangle$, whereas the lower level or ground state is $^1S_0$ described as $|g\rangle$. The atoms at $^1S_0$ are excited into $^3P_2$ via $^1P_1$ by 423 nm and 1199 nm lasers. With 616.2 nm pumping laser and 610.3 nm re-pumping laser, the atoms can be pumped into $^3P_1$ via $^3S_1$. In fact, we can simplify the pumping procedure as an effective three level laser system. Atoms in $^1S_0$ are pumped into $^3S_1$, then decay to $^3P_1$ via spontaneous radiation. This procedure makes population inversion built between clock transition $^3P_1$–$^1S_0$ that is the precondition of laser. The effective pumping rate is w.

The corresponding Hamiltonian of this system is

$$H = \frac{\hbar\omega_a}{2}\sum_j \hat{\sigma}_z^j + \hbar\omega_c\hat{a}^\dagger\hat{a} + \hbar g\sum_j(\hat{a}^\dagger\hat{\sigma}_-^j + \hat{\sigma}_+^j\hat{a}), \quad (1)$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the field mode in the cavity respectively. The superscript j means the jth atom; ω_a is the atom transition frequency between the excited state $|e\rangle$ and the ground state $|g\rangle$. The Pauli matric $\hat{\sigma}_z^j = |e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|$ represents the population dif-

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ference between the excited state $|e\rangle$ and the ground state $|g\rangle$. The operator $\hat{\sigma}_-^j = |g_j\rangle\langle e_j|$ takes the jth atom in the upper state into the lower state while $\hat{\sigma}_+^j = |e_j\rangle\langle g_j|$ takes the jth atom in the lower state into the upper state. The coupling constant g is given by $g = \mu\sqrt{\frac{\omega_a}{2\hbar\epsilon_0 V}}$, where μ is the magnitude of the atomic dipole moment, ϵ_0 is the vacuum permittivity, and V is the effective mode volume of the laser cavity. For simplicity, the coupling constant g is assumed to be identical for all atoms. In our system, we use reasonable parameters for practical experimentation. The effective mode volume $V = (5\,\text{cm})\pi(1\,\text{mm})^2$. The atom-field coupling constant is $g = 482\,\text{s}^{-1}$ and $\gamma = 2\pi \times 370\,\text{Hz}$ for spontaneous decay rate.[24] For a finesse of $F = 2.5 \times 10^5$, the cavity decay rate is $\kappa = 3 \times 10^7\,\text{s}^{-1}$. Therefore, the single-atom cooperativity parameter is $C = g^2/\kappa\gamma = 3.3 \times 10^{-6}$ and N atoms confined in the optical lattice decay to the ground state $|g\rangle$ from the excited state $|e\rangle$ with rate $C\gamma$.

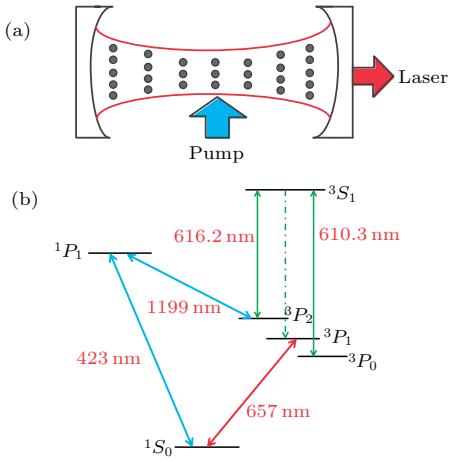

Fig. 1. (a) Experimental setup. Ultracold ^{40}Ca atoms are confined in an optical lattice. The cavity is a high Q cavity and the cavity decay rate is much larger than the atomic relaxation rate. (b) Energy level structure of ^{40}Ca. The atoms at $^1S_0$ are excited into $^3P_2$ via $^1P_1$ by 423 nm and 1199 nm lasers. With 616.2 nm pumping laser and 610.3 nm re-pumping laser, the atoms can be pumped into $^3P_1$ via $^3S_1$.

From Eq. (1), we calculate the laser power with the master equation, which was discussed in Ref. [16] in detail. The central equation can be obtained,

$$\langle\hat{\sigma}_+^i\hat{\sigma}_-^j\rangle_c\Big(\frac{w-\gamma}{w+\gamma}N\gamma C - \gamma - w - 2T_2^{-1} - 2\frac{N^2\gamma^2C^2}{w+\gamma}\langle\hat{\sigma}_+^i\hat{\sigma}_-^j\rangle_c\Big) = 0, \tag{2}$$

where T_2 is the effective relaxation rate for inhomogeneous processes. From Eq. (2) we obtain the threshold at which the pump overcomes the atomic losses, so that the steady-state superradiance is established. The boundary condition values at which $\langle\hat{\sigma}_+^i\hat{\sigma}_-^j\rangle_c$ will change sign. When the sign is positive, the regime is what we need. The solutions to Eq. (2) are denoted by $w_{\min}$ and $w_{\max}$. If the effective pumping rate is smaller than $w_{\min}$, we can not obtain the steady-state superradiance. However, the pumping rate can not be too large since the pump noise destroys the superradiance. Pump noise induced by pumping laser destroys the coherence faster than it establishes it. The maximum pumping rate is $w_{\max}$. Here $w_{\min}$ and $w_{\max}$ are calculated analytically,

$$w_{\min} = \frac{NC\gamma}{2} - \gamma - \frac{1}{T_2} - \sqrt{\frac{1}{T_2^2} + \frac{N^2C^2\gamma^2}{4} - \frac{NC\gamma}{T_2} - 2NC\gamma^2}, \tag{3}$$

$$w_{\max} = \frac{NC\gamma}{2} - \gamma - \frac{1}{T_2} + \sqrt{\frac{1}{T_2^2} + \frac{N^2C^2\gamma^2}{4} - \frac{NC\gamma}{T_2} - 2NC\gamma^2}. \tag{4}$$

In the following, we only discuss the regime at which the pumping rate is between $w_{\min}$ and $w_{\max}$. We take the effective relaxation rate for inhomogeneous processes $\frac{1}{T_2}$ as $1\,\text{s}^{-1}$.[26] There should be enough atoms to establish the superradiance. Figure 2(a) shows the function of the minimum pumping rate $w_{\min}$ with respect to the atom number, while Fig. 2(b) shows the function of the maximum pumping rate $w_{\max}$ with respect to the atom number. The minimum pumping rate $w_{\min}$ decreases and the maximum pumping rate $w_{\max}$ increases with the atom number increasing. This means that the superradiance regime becomes larger with the atom number increasing.

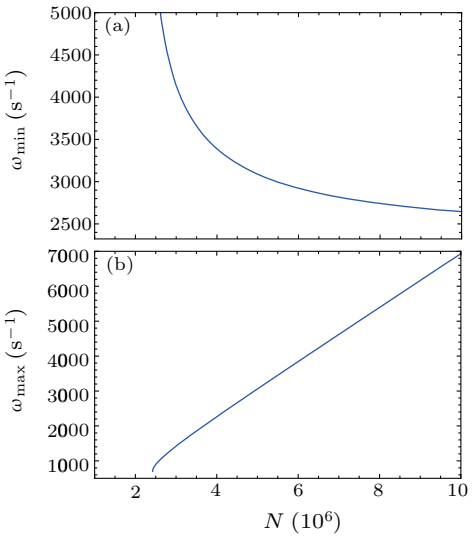

Fig. 2. (a) The minimum pumping rate $w_{\min}$ and (b) the maximum pumping rate $w_{\max}$ versus the atom number N.

At a special pumping rate $w_{opt} = NC\gamma/2$, which is called the optimal pumping rate, the maximum power of the laser[16] can be achieved at $P_{\max} = \hbar\omega_a N^2 C\gamma/8$. Since we need laser with power as large as possible, in the following we only consider the maximum power. Figure 3(a) shows the maximum laser power with respect to the atom number. When the atom number N is equal to 5×10^6, the laser power can reach

5×10^{-9} W, which is sufficient for applications in precision measurement.

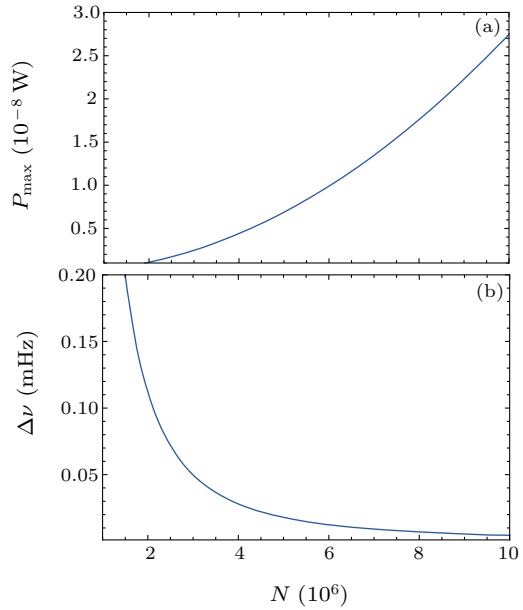

Fig. 3. (a) The maximum laser power $P_{\max}$ and (b) the laser linewidth versus the atom number N.

To obtain the linewidth of the superradiant laser, we use the standard Heisenberg–Langevin approach, which was discussed in Refs. [8,10,27] in detail. The laser linewidth for the bad cavity can be expressed as[28]

$$\Delta\nu = \frac{\hbar\omega_c\kappa^2}{4\pi P} N_{\rm sp}\left(1+\left[\frac{2(\omega_c-\omega_a)}{\gamma+\kappa}\right]^2\right)\left(\frac{\gamma}{\gamma+\kappa}\right)^2, \tag{5}$$

where P is the power of light which emits from the cavity, and $N_{\rm sp} = N_e/(N_e - N_g)$ is the spontaneous-emission factor. Using the near resonance approximation, the term $[\frac{2(\omega_c-\omega_a)}{\gamma+\kappa}]$ can be regarded as zero. The power P in Eq. (5) is replaced by $P_{\max} = \hbar\omega_a N^2 C\gamma/8$. We consider the denominator in parentheses. Under the condition of the bad cavity, κ is much larger than γ, therefore we can drop γ for simplicity. Lastly, we obtain the final formula of the superradiant laser linewidth in the bad cavity region,

$$\Delta\nu = \frac{2N_{\rm sp}\gamma}{\pi N^2 C}. \tag{6}$$

Based on the fact that the linewidth is inversely proportional to N^2, laser with linewidth much narrower than mHz can be achieved. For simplicity, we set $N_{\rm sp} = 1$. Figure 3(b) shows the changing action of laser linewidth with respect to the atom number N. We can see that the laser linewidth decreases with N increasing. The laser quantum-limited linewidth can even reach μHz level. When the atom number N is equal to 5×10^6, the laser linewidth is about 0.02 mHz. Some factors such as light shift and cavity pulling shift will affect the laser linewidth. Under the conditions we choose, the light shift induced fluctuations of the atomic transition frequency are negligible at the mHz level in calculation. The cavity pulling shift is $\Delta\nu_{\rm cp} = \frac{\gamma}{\kappa}(\nu_{\rm c} - \nu_0)$,[29] where $\nu_{\rm c}$ is the cavity frequency and ν_0 is the true resonance frequency. Assuming that the cavity mode drift is 1 Hz/s, the shift of the atomic transition frequency is 0.08 mHz/s.

In summary, we have studied the ultra-narrow linewidth laser based on ^{40}Ca in detail, including laser power and linewidth. A power level of order 10^{-9} W can be reached. We deduce the threshold pumping rate analytically and the formula of superradiant laser linewidth in a bad cavity region. This kind of superradiant laser has extremely narrow linewidth at mHz level. The theoretical research is significant for the following experiments we will focus on.

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